

Hadamard Matrix Arrangement and Hadamard Conjecture for Designing of Thinned Linear Antenna Array

Saurabh Bhutani¹, Prabhakar Dubey², Navneet Kr. Pandey³

Student, M.Tech (Electronics Engineering), R.R.I.M.T, Lucknow, Uttar Pradesh, INDIA

Professor, ECE Department, R.R.I.M.T, Lucknow, Uttar Pradesh, INDIA

Student, M.Tech (Electronics Engineering), R.R.I.M.T, Lucknow, Uttar Pradesh, INDIA

Abstract: Reduced Side lobe-level i.e. thereby increasing the directivity can be obtained in the case of thinned linear array. This paper tells about the arrangement of thinned array using Hadamard matrix and Hadamard Conjecture. Using Hadamard Matrix arrangement, the various states of antenna elements are arranged and the inter-element spacing of array in both x-direction and y-direction are varied to optimize SLL (side lobe level) with a smallest value. The result for 4x4 planar arrays is compared with fully filled antenna array. Thus a low side lobe level can be achieved using Hadamard matrix arrangement.

Keywords: Directivity; Hadamard conjecture; Hadamard matrix; thinned linear array; side lobe level; inter-element spacing

IV. Introduction

The Directivity or the gain for a lossless antenna is one of the parameter for the selection of antenna for various types of applications among various other important parameters. While increasing the gain of antenna some constraints are fixed i.e. the length of an antenna cannot be increased beyond a certain limit in order to increase the gain of an antenna, thus to increase the overall gain of the antenna system we try to form antenna array system where we can easily control various antenna parameters like efficiency, radiation pattern, beam efficiency, radiation intensity etc. The radiation pattern of an antenna consists of Major beam or major lobe, null and side lobes. The side lobes consume a substantial amount of power within them which is of no use to the antenna and thus it can cause interference in signal coming or going out from the antenna in undesired direction therefore, these side lobe levels must be lowest in antenna array systems.[1]-[2]. There are various antenna array synthesis methods to decrease SLL keeping the directivity or gain of system constant.[1]-[3]. If in an antenna array system, each and every element is excited (or in working condition as energy is provided) then that type of antenna array is called fully filled or fully populated array. In the case of thinned linear antenna array in order to get a narrow beam of radiation with low SLL, some antenna elements are *ON* while others are *OFF* without decreasing the performance of array. When the antenna element is in *OFF* state then that element is either terminated by a matched load or it is open circuited. Among the various methods reducing SLL of an antenna array system thinning using optimization techniques is most popularly used. The various popular optimization techniques include particle swarm optimization [11]-[06], ant colony optimization and based on different sets of Hadamard difference set [14]-[08]. Thin planar antenna arrays are designed using Hadamard matrix. The two states i.e. on and off states of the antenna elements correspond to '1' and '-1' values of the Hadamard matrix. In the case of 2-D the inter element spacing in the array is varied in both the directions. The results are found in this case are compared with fully filled array.

II. Hadamard Matrix

A $n \times n$ Hadamard matrix H is of the order n with values from (± 1) such that $HH^T = n \times I_n$ (1)

Here in this case:

I_n is the $n \times n$ unit matrix or identity matrix.

H^T is the transpose matrix of H .

Hadamard matrix can be defined by [4]

$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (2)$$

Hadamard matrix of order 4 is defined as [9]

$$H_4 = \begin{bmatrix} H & H \\ H & -H \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (3)$$

Hadamard matrix is a square matrix where the entries are +1 and -1 and whose rows are mutually orthogonal in nature. Thus this means that each pair of rows in Hadamard matrix represents two perpendicular vectors i.e. each pair of rows has matching entries in exactly half of their columns but at the same time it has mismatched entries in the other columns. In brief we can say that the corresponding properties hold for columns as well as rows.

- i) Any two columns or rows are orthogonal.
- ii) Every pair of columns or every pair of rows differs in exactly n/2 places.

III. Planar Linear Antenna Array

In a Linear planar rectangular 2-D antenna array with elements $M \times N$ are grouped together in a configuration having M rows and having an inter-element distance of d_x between the rows and having N columns having with an inter- element distance of d_y between the columns, shown in Fig. 01.

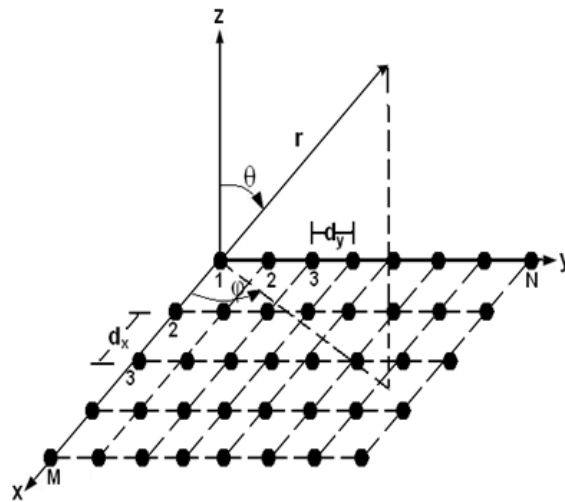


Figure 01. Planar antenna array

The planar antenna array is regarded as antenna array along any plane i.e. either a linear antenna array along the x-axis having an inter element distance value of d_x or a linear antenna array along the y-axis with an inter-element distance value of d_y . Let us consider the position of an element (m,n) in the planar antenna array r_{mn} where m is any value in X-direction ($m=1,2,3,4,5,\dots,M$) and n is any value in Y-direction ($n=1,2,3,4,5,\dots,N$), hence the overall equation can be given as

$$r_{mn} = (m - 1)d_x u_x + (n - 1)d_y u_y, \quad (4)$$

If these arrays in N quantity are placed at an even intervals y -axis a rectangular array can be formed. Let the progressive shifts be β_x and β_y along x -directions and y directions. It is assumed in this case that the current distribution denoted as (I_0) is uniform for all the elements of array. The Array Factor (AF) of a linear array of M elements along the x -axis is:

$$AF = \sum_{m=1}^M I_{m1} e^{j(m-1)(k d \sin \theta \cos \Phi + \beta_x)} \quad (5)$$

Where $\sin \theta \cos \Phi = \cos \gamma_x$ is the directional cosine with respect to the x -axis. It is assumed that all the elements are equispaced with an interval of d_x and a progressive shift of β_x .

The I_{m1} denotes the excitation amplitude of the elements at the points with coordinates $x=(m-1)d_x$, $y=0$. This is the element of the m^{th} row and the 1^{st} column of the array matrix.

Then, the Array Factor (AF) of the complete $M \times N$ array can be given by:- [1]-[3]

$$AF = I_0 \sum_{m=1}^M e^{j(m-1)(k d \sin \theta \cos \Phi + \beta_x)} \cdot \sum_{n=1}^N e^{j(n-1)(k d \sin \theta \cos \Phi + \beta_y)} \quad (6)$$

The radiation pattern of a rectangular array can be found by the product of the array factors of both the linear arrays in the directions x and y . The normalized value of array factor is obtained by:-

$$AF_n(\theta, \phi) = \begin{bmatrix} \sin \left[M \frac{\Psi_x}{2} \right] \\ (1/M) \frac{\sin \left[\frac{\Psi_x}{2} \right]}{\sin \left[\frac{\Psi_x}{2} \right]} \end{bmatrix} \begin{bmatrix} \sin \left[N \frac{\Psi_y}{2} \right] \\ (1/N) \frac{\sin \left[\frac{\Psi_y}{2} \right]}{\sin \left[\frac{\Psi_y}{2} \right]} \end{bmatrix} \quad (7)$$

Where-

$$\Psi_x = k d_x \cos \theta + \beta_x \quad \text{and} \quad \Psi_y = k d_y \cos \phi + \beta_y \quad (8)$$

The normalized array factor can be calculated from the equation (7) and is used for designing of thinned planar antenna array using Hadamard matrix arrangement for the various elements of antenna array and for array optimization by lowering down the SLL.

IV. Computed Results

In calculation, for all the various cases the value of normalized array factor (7) is used in designing main lobes and side lobes in the pattern. In fully filled array, all the elements of the array are assumed to be excited with energy i.e. in 'on' state whereas in the case of thinned array some or the other elements are in 'off' state according to arrangement given by Hadamard matrix.

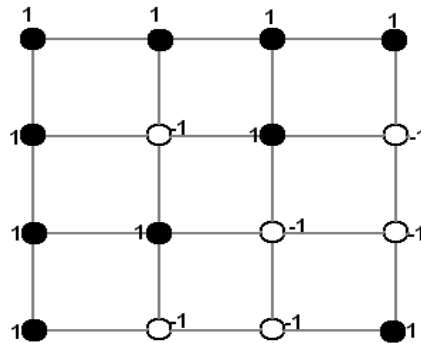


Figure 02. Hadamard Arrangement of a thinned planar array of 4x4

The 'on' and 'off' states of antenna array elements in a 4X4 planar thinned array, using Hadamard matrix arrangement of (3), is shown in Fig. 02. Here '1' denotes 'on' position and '-1' denotes 'off' position. The d_x spacing along x-axis and d_y along y-axis are changed from 0.50λ to the value 0.75λ to achieve lowest value of side lobe level. Using the array arrangement as given in Fig.02 is compared in the Fig.03 with fully filled array with the same size.

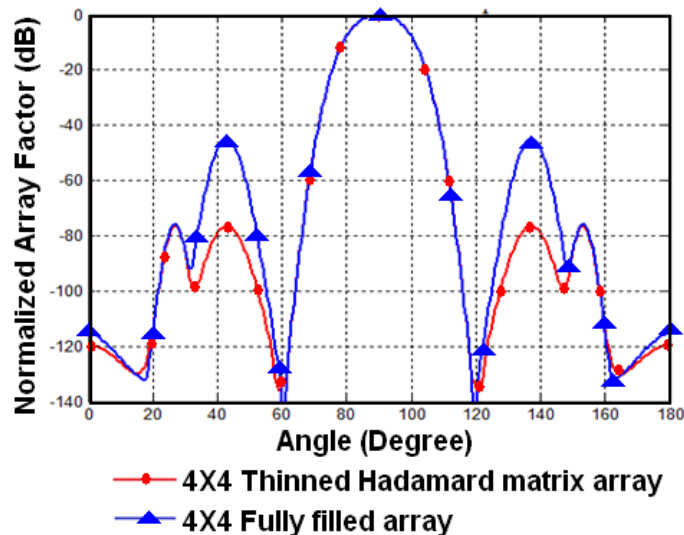


Figure 03. Comparison between 4x4 thinned planar array using Hadamard arrangement and fully filled array

Sl. No.	Inter- element spacing in x-axis (d_x)	Inter- element spacing in y-axis (d_y)	SLL obtained for 4x4 Hadamard Matrix Planar Array (dB)	SLL obtained for fully filled 4x4 Planar Array (dB)
1	0.51λ	0.51λ	-51	-31
2	0.52λ	0.52λ	Grating lobe	Grating lobe
3	0.53λ	0.59λ	-45	-50
4	0.56λ	0.50λ	-56.5	-30
5	0.64λ	0.64λ	Grating lobe	Grating lobe
6	0.65λ	0.55λ	-66	-41
7	0.70λ	0.51λ	-70	-42
8	0.75λ	0.56λ	-75	-45
9	0.76λ	0.76λ	Grating lobe	-15

Table 01. SLL values for different inter element spacing

The optimized results using Hadamard matrix of 4x4 thinned array can be tabulated in the table-01. The best result for lowest value of SLL is -75dB when $d_x=0.75\lambda$, $d_y=0.56\lambda$ for a working frequency of 34.88 GHz. At the same value of inter-element spacing the value of SLL for fully filled antenna array is -45 dB.

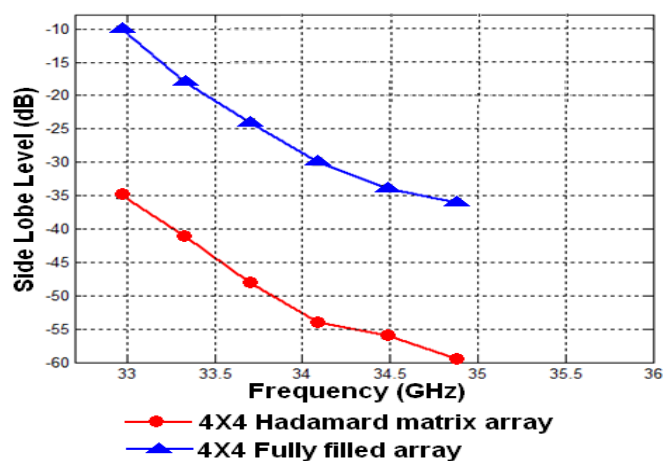


Figure 04. Variation of optimized lowest value of SLL for 4x4 thinned planar linear array based on the Hadamard matrix and fully populated array with different frequency

The lowest value of SLL are compared with that of fully populated array in Fig.04 the array arrangement is fixed according to Fig.02 but the inter-element spacing are different in all the mentioned frequencies.

V. Conclusion

The Optimization of thinned planar array antenna can be seen based on Hadamard matrix arrangement. In optimization, it is required to optimize the various ‘on’ and ‘off’ positions of the antenna array also, for which lowest SLL can be obtained, and in addition with the optimization for inter-element spacing.

References

- [1]. R. S. Elliott, Antenna Theory and Design, Revised Edition, John Wiley, New Jersey, 2003.
- [2]. C. A. Balanis, Antenna Theory and Design, 3rd Edition, John Wiley.
- [3]. W. L. Stutzman and G. A. Thiele, Antenna Theory and Design, Wiley & Sons, 1998.
- [4]. S. J. Blank and M. F. Hutt, “On the empirical optimization of antenna arrays,” IEEE Antennas and Propagation Magazine, vol. 47, no. 2, pp.58-67, April 2005.
- [5]. M. Skolnik, J. Sherman and F. Ogg, “Statistically designed density- tapered array,” IEEE Transactions on Antennas and Propagation, vol.12, no. 4, pp. 408–417, 1964.
- [6]. R. Bera and J. S. Roy, “Thinning of elliptical and concentric elliptical antenna arrays using particle swarm optimization”, Microwave Review, vol. 19, No.1, pp. 2-7, Sept. 2013
- [7]. M. Donelli, A. Martini, and A. Massa, “A hybrid approach based on PSO and Hadamard difference sets for the synthesis of square thinned arrays,” IEEE Trans. Antennas & Propagation, vol. 57, no. 8, pp. 2491-2495, Aug. 2009.
- [8]. Oliveri,G.; Caramanica, F.; Fontanari, C.; Massa, A. "Rectangular thinned arrays based on McFarland difference sets", IEEE Transactions on Antennas and Propagation, vol. 59, no. 5, pp. 1546–1552, May 2011.
- [9]. S.S.Agaian, Hadamard Matrices and Their Applications, Springer, 1985.
- [10]. W. P. Du Plessis, "Efficient synthesis of large-scale thinned arrays using a density-taper initialized genetic algorithm", International Conference on Electromagnetics in Advanced Applications (ICEAA), pp. 363–366, 2011.

- [11]. T. B. Chen, Y. L. Dong, Y. C. Jiao and F. S. Zhang, "Synthesis of circular antenna array using crossed particle swarm optimization algorithm," *Journal of Electromagnetic Waves and Applications*, vol. 20, No. 13, pp. 1785-1795, 2006.
- [12]. W.-B. Wang, Q. Feng and D. Liu, "Synthesis of thinned linear and planar antenna arrays using binary PSO algorithm," *Progress In Electromagnetics Research*, vol. 127, pp. 371-387, 2012.
- [13]. R. L. Haupt, "Interleaved thinned linear arrays," *IEEE Trans. Antennas and Propagation*, vol.53, no. 9, pp. 2858-2864, 2005
- [14]. L. E. Kopilovich, "Square array antennas based on Hadamard difference sets", *IEEE Transactions on Antennas and Propagation*, pp. 263 - 266 vol. 56, no. 1, Jan. 2008.