

## Scalable Identity Wavelet in Hierarchical Image Codec

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**Abstract:** Wavelet based hierarchical image compression algorithms, used to omit unnoticeable details of the image at the receiver by decomposing the image into bands i.e., low band image is coded with high bits and high band image is coded with low or desired bits, encoded with the codec data rate on a communication channel and the reconstructed image is compared with the compressed image for its accuracy, have become rich in reconstruction of vast amount of degraded image under several types of artifact and to improve transmission and storage capability. We propose an alternative hierarchical image compression principle, so that the compression, codec design and reconstruction accuracy can be better understand than the previously proposed algorithms. These principles are identifying the high level low band reduction with a square-integrable scalable algorithm, allocating the level based bands under bit ordering with a bit identifier ordering image wavelet, parallel data rate reduction and exploitation of hierarchical image codec scheme in the generation of narrow wideband image wavelet. Moreover, a Scalable Identity Wavelet is presented to implement Spectrum Parallel Identification for Redundancy Encoding and Decoding (SPIRE), which provides better hierarchical image compression algorithm comparatively and image coding results comparatively. Apart from codec design and implementation, we state an evolutionary approach as well as visual image test results.

**Key Index terms:** Data rate, Image Codec, Image Compression, Image Wavelet, Hierarchical Tree.

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### I. Introduction

In recent years with use of abundant computing techniques, digitized equipment, rapid growing amount of data availability and increased flexible storage capacity: demand of evaluation of image compression algorithms, techniques and methods have given a great challenges to the researchers, in the multimedia image compression analysis, to forecast, develop, make and design an improved compression algorithms by meeting the hardware standards.

Introduction of [1] by Shapiro, was the first image coder to use zero-trees and wavelet coding algorithms has implemented significantly, which could not optimize the complexity of progressive data rate transmission and the principles of [2] improved upon the zero tree concept by sorting and coefficients and provided a better rate and PSNR comparative performance than [1]. An alternate to zero tree or significance-map coding is [10] [11], employing quad-tree partitioning and binary set splitting with k-d trees [12] algorithm are confined to reside in single sub band. [7], [13], [14], [15], [16], [17], introduced conditional-coding technique by coding the significance map of an image using neighbour coefficients, during the conceiving the final bit stream the distortion rate of received code-blocks bit stream will be heavy. Traditionally, the concept of Wavelet [3], [18], [19], [20] was entered in to image coding standard, success has got prevailed in 2-D discrete wavelet transform (DWT) on the basis of wavelet spaces LH, HL, HH and LL which can only limited to scan-lines but could not along the curves. Since JPEG2000 codes sub-blocks of sub bands, other methods [21] and [22] code sub bands independently and could not decode output bit stream completely resulting to quality degradation. This block coding paradigm [22], introduced to generate independent bit-streams which are packed in quality layers, which effects the rate-distortion property of overall image representation. Another category of image decomposition [8] and [9] are implemented on continuous, and polar space and scales and directional respectively, however their directional information outperform DWT but lost the scalable property of very low bit rate. In [4], preservation of lost data at the edges at low bit-rate compression is proposed, which results in increase in payload of redundancy bit-rates, [5] preservation of significant features is proposed, level and bit rate performance of the encoder need improvements In [6].

Thus, there is a demand and great need of an effective and efficient image compression algorithms to compress the multimedia image observations to improve the redundancy, design of an image compression codec to improve the channel efficiency and reconstruct the compressed multimedia image data without perceptual degradation to improve visual quality perceive.

The image compression schemes used in image data sets contain high resolution information, accessing these data sets and transporting or communicating will make existing processing techniques to fail under storage and transmission capabilities. For such applications, we want a compressed data to be coded and embedded and the same should be reconstructed without any loss. So, we need an efficient compression scheme for these kind of data sets for storage and transmission.

Image Compression methods are based on extracting image properties, and removing redundancy in the image and design a signal receiver by omitting details of the image that will not be noticed by the observer. These days, subband and wavelet based schemes outperform in image compression methods, also gained application in software codec designs and these schemes are suitable for implementation where scalability and image degradation are important issues.

## II. Problem Formulation

Foremost, the design of analysis and reconstruction filter bank with perfect reconstruction properties: splitting image filter bank in to low frequency and high frequency subband with good quality. Usage of tree structures in splitting the desired subbands, need to know bits selected under classified subbands further and their bit rates under different delay constraints in subbands design. In general image filter bank design, let's think  $H_0(c)$  and  $H_1(c)$  are the analysis low-pass and high-pass filters respectively and  $G_0(r)$  and  $G_1(r)$  are the synthesis low-pass and high-pass filters respectively.

Transformed Filter bank realisation is,

$$C_i(z) = \frac{1}{2} I(z) \{ H_0(cz) G_0(rz) + H_1(cz) G_1(cz) \} + \frac{1}{2} I(-z) \{ H_0(-cz) G_0(cz) + H_1(-cz) G_1(cz) \} \quad \text{Eq.(1)}$$

Equation 1, should be designed with

$$H_0(-cz) G_0(cz) + H_1(-cz) G_1(cz) = 0 \quad \text{Eq.(2)}$$

to remove distortion. Considering delays in filter bank, the condition to be satisfied is  $H_0(cz) G_0(rz) + H_1(cz) G_1(cz)$  should be exponent of delay  $n$  ie.,

$$H_0(cz) G_0(rz) + H_1(cz) G_1(cz) = C_i(cz)^{-n} \quad \text{Eq.(3)}$$

If the Equation 3 is satisfied, then we can have a better filter bank. To satisfy Equation 3,

$$H_0(cz) G_0(rz) + H_1(cz) G_1(cz) = C_i(cz)^{-n} = C_i(cz)^{-(N-1)n} (|H_0(cz)|^2 + |H_1(-cz)|^2) \quad \text{Eq.(4)}$$

where  $N$  is filter length, becomes

$$(|H_0(cz)|^2 = |H_1(-cz)|^2) \quad \text{Eq.(5)}$$

To design an accurate filter bank, we should choose

$$(|H_0(cz)|^2 + |H_1(-cz)|^2) = C_i(cz)^{-n} \quad (\text{for } n=2 \text{ only}) \quad \text{Eq.(6)}$$

Such a filter, which has all these coefficients is non-trivial, and require more design constraints. In the design of a filter bank, we need the detailed cut-off frequency, filter length, even analysis and synthesis filters, zero indexed coefficients and reversing the  $h_0(c)$  and  $h_1(c)$  by sign changing the odd coefficients of transformation requires a better filter design. This scenario made us to design a wavelet, providing efficient time and amplitude reversal approach of filter coefficients.

Second, image compression relates to two central concepts, among many: compressed image and reconstructed image. In general image compression framework, let's think of  $C_i$  representing a compressed image at the coder ( transmitter ) and  $R_i$  a reconstructed image at the decoder ( receiver ). The information between representing as  $I_{C_i}$  and  $I_{R_i}$ , with probable variation function  $p_{I_{C_i} I_{R_i}} ( C_i, R_i )$  and the data redundancy relation between these two central concepts is defined as,

$$R_d ( I_{C_i} : I_{R_i} ) = \int_{C_i} \int_{R_i} p_{I_{C_i} I_{R_i}} ( C_i, R_i ) \log p_{I_{C_i} I_{R_i}} ( C_i, R_i ) / p_{I_{C_i}} ( C_i ) p_{I_{R_i}} ( R_i ) d_{C_i} d_{R_i} \quad \text{Eq.(7)}$$

Data redundancy information entropy is given by,

$$E(I_{C_i}) = \int_i p_{C(i)} \log 1/p_{C(i)} d_i \quad \text{and} \quad E(I_{R_i}) = \int_i p_{R(i)} \log 1/p_{R(i)} d_i \quad \text{Eq.(8)}$$

and the differential entropy of  $C_i$  and  $R_i$ ,

$$R_d ( I_{C_i} : I_{R_i} ) = \int_{C_i} \int_{R_i} p_{I_{C_i} I_{R_i}} ( C_i, R_i ) \log 1 / p_{I_{C_i}} ( C_i ) p_{I_{R_i}} ( R_i ) d_{C_i} d_{R_i} \quad \text{Eq.(9)}$$

by

$$R_d ( I_{C_i} : I_{R_i} ) = p ( R_i ) - p ( R_i / C_i ) = p ( C_i ) - p ( C_i / R_i ) \quad \text{Eq.(10)}$$

Equation 10 is the probability of compressed image if reconstructed image is observed. Data redundancy information is given by

$$0 \leq R_d ( I_{C_i} : I_{R_i} ) = R_d ( I_{R_i} : I_{C_i} ) \quad \text{Eq.(11)}$$

Equation 11 is related to channel capacity,

$$0 \leq R_d ( I_{C_i} : I_{R_i} ) \leq C_c \quad \text{Eq.(12)}$$

The data redundancy measure between compressed and reconstructed image, as a poor measure

$$R_d ( I_{C_i} : I_{R_i} ) \geq 0 \quad \text{Eq.(13)}$$

to get equal quality consider  $C_i=R_i$ , The average data redundancy to be analysed is,

$$A_{rd} = \int_{C_i} \int_{R_i} p_{I_{C_i} I_{R_i}} ( C_i, R_i ) d ( C_i, R_i ), \quad \text{Eq.(14)}$$

Data redundancy between  $C_i$  and  $R_i$  with bit rate at codec is defined as,

$$B_{R(Ard)} = R_d(I_{Ci} : I_{Ri}) \leq C_c : A_{rd} \geq C_c \parallel E(I_{Ci} : I_{Ri}) \geq 0 : R_d(I_{Ci} : I_{Ri}) \leq 0 \quad \text{Eq.(15)}$$

In Equation 15,

if  $B_{R(Ard)}=0$ ,  $A_{rd} \geq C_c$  resulting in minimum distortion,

if  $B_{R(Ard)}$  is finite,  $E(I_{Ci} : I_{Ri}) \geq 0$  resulting in threshold distortion

and if  $B_{R(Ard)}$  is infinite,  $A_{rd} \geq C_c \parallel E(I_{Ci} : I_{Ri}) \geq 0 : R_d(I_{Ci} : I_{Ri}) \leq 0$ , resulting in maximum distortion. Eq.(16)

The bit data rate function is a performance metric of a source encoder, which should be optimal with the compression algorithm and encoding technique. Conversely, data redundancy distortion shows that bit data rate function optimises with large image data sets, requiring large memory, coding time and high omission in redundant data bits. This scenario might be impossible with the existing image compression algorithms and techniques, we suggest a better image coder jointly encoding low and high payload images with less data rate distortion and better image quality accuracy in compressed image and reconstructed image.

Third, if we take a 24-bit image containing 256 possible colors with file size of 50 Kb, after compression the file size should be below 50% of its original image file size and peak signal to noise ratio should be 100 dB, which is impossible in the domain of compression ratio. Compression involves partition of an image in to number of parts, to get better peak signal to noise ratio, higher compression ratio and to reduce file size beyond 50 %, we must employ a lossy compression. To accomplish this, we represent a bit rate allocation method by reducing the number of bits required to represent the compressed image through careful scaling and wavelet.

Finally, a fast algorithm is required to perform decomposition/representation, artifact features such as edges and shapes are preferred, orthogonal subbands stability should be satisfied and minimizing the quantization loss due to transmission errors by the overlapping of encoder and decoder parameters.

### III. Proposed Wavelet Design

Consider  $f(k)$ , our design system desired signal

$$f(k) = r(k) + c(k), \text{ Eq.(17)}$$

where  $r(k)$  denotes a row input vector with length  $R$  as follows

$$r(k) = [r(k-(R-1)) \dots r(k-2) \quad r(k-1) \quad r(k) \quad r(k+1) \quad r(k+2) \dots r(k+(R-1))], \quad \text{Eq.(18)}$$

and  $c(k)$  denotes a column input vector with length  $S$  as follows

$$c(k) = \begin{bmatrix} c(k-(S-1)) \\ \vdots \\ c(k-2) \\ c(k-1) \\ c(k) \\ c(k+1) \\ c(k+2) \\ \vdots \\ c(k+(S-1)) \end{bmatrix} \text{ Eq.(19)}$$

#### A. Design of Narrow Wideband Scalable Filter Bank (NWSF)

A general wavelet design requires filter scheme, used to filter input signal into small band signals and our filter design is given as,  $[H_0, H_1, \dots, H_{M-1}]$ , where  $M$  represent the number of analysis filters and each analysis filter output is a small band, haven down sampled in the number of  $N$ . Non uniform spaced filters are proposed, will result in to  $f_s/M$  total number of small bands, where  $f_s$  represents the sampling frequency, corresponds to  $f(k)$ .

In proposed ordering wavelet, the sampling is a variable terminology, like

- a. for critical down samples :  $M=N$  will be considered, in this case implementation complexity is reduced
- b. for under down sample :  $M < N$  will be considered, in this case processing delay is limited and
- c. for over down sample :  $M > N$  will be considered, in this case complexity is reduced. In above three, aliasing distortion will be considered as a major component.

Based on the traditional filter design and our filter design parameters, the forward filter bank is designed as,

$$h_m[k] = f(k) H \left[ \frac{\pi}{M} \left( m + \frac{1}{4} \right) \left( \frac{L_f - 1}{2} \right) + (-1)^m + \frac{\pi}{4} \right], \quad \text{Eq.(20)}$$

here  $m=0$  to  $M-1$  and  $k=0$  to  $L_f-1$ .

The filters are frequency shifted versions of each other and covers the whole scaling spectrum. Here in our proposed filter bank,  $M$  small bands are having good stop band rejection and the bandwidth of each passed small signals is approximately  $f_s/M$ , hence the small bands are correctly filtered in to the interval  $[-f_s/2N \quad f_s/2N]$ , with the condition  $M \geq N$ , avoiding aliasing distortions.

If  $f(k)$  passes through  $M$  small bands with  $N$  down samples, resulted each filter bank results to transform components:

$$H_m(z) = \sum_{n=0}^{-(N+1)} z^{-(n+1)} H_m(n:N)(z^{-(N+1)}) + \sum_{n=(N+1)}^{N-1} z^{+(n-1)} H_m(n:N)(z^{+(N-1)}), \text{Eq.}(21)$$

in which  $H_m(n:N)(z)$  is the  $n$  th order of  $N$  down samples component of  $m$ th subband filter  $h_m[k]$ .  $(m,n)$  in the above equation is related as

$$[H(z)]_{m,n} = [H_{m(0:N)}(zN) + \dots + H_{m(N-1:N)}(zN) : H_{m(0:N)}(zN) + \dots + H_{m(N+1:N)}(zN)] \\ = [H_m N-1:n(z) : H_m N+1:n(z)] = [H_m N:n(z)] \text{ for } m=0 \text{ to } M-1 \text{ and } n=0 \text{ to } N-1, \text{Eq.}(22)$$

For the synthesis part,

$$G_m(z) = \sum_{n=0}^{-(N+1)} z^{-(n+1)} G_m(n:N)(z^{-(N+1)}) + \sum_{n=(N+1)}^{N-1} z^{+(n-1)} G_m(n:N)(z^{+(N-1)}) \\ = [G_m N-1:n(z) : G_m N+1:n(z)] = [G_m N:n(z)] \text{ for } m=0 \text{ to } M-1 \text{ and } n=0 \text{ to } N-1, \text{Eq.}(23)$$

In case synthesis part we are using an Identifier matrix  $I_d$  with ones along its cross diagonals and zero in the remaining ie.,  $I_d G_m(z)$ .

By analysis and synthesis bank, the small band system which holds the property

$$H_m(z) I_d G_m(z) = F_{N:M}, \text{Eq.}(24)$$

In the above equation if  $I_d$  is absent, overall subband system is a pure delay and system reduces to combined analysis and synthesis tapped delay line, which leads to imperfect reconstruction.

To reconstruct a perfect filter characteristics, we propose a  $I_d$  matrix presenting which eliminates one or more artifacts, as small amplitude and phase distortion, tolerable quantization errors and oversampled modulated filter bank or reduce them at an acceptably low level by considering frequency selectivity and implementation cost.

Proposed simple relational filter design with the above proposed analysis and synthesis bank is Narrow Wideband Scalable Filter Bank (NWSF), by choosing

$$H_m(z) I_d G_m(z) = F_{N:M}, \text{Eq.}(25)$$

$$G_m(z) = H_m(z)^{-T} I_d = \frac{\sigma}{\vartheta} \int_{n=0}^{N-1} H(z) Id., \text{Eq.}(26)$$

where  $\sigma = z^\lambda$  is scalable filter matrix and the above equation is Square Integrable Scalable Matrix, taking the form related to  $H_m(z)$ , as  $\vartheta$ , with  $\vartheta \in G$  and  $\sigma \in H$

Above is the case for  $2M$  small band are combined to obtain  $M$  modulated filters. In the proposed filter design NMSF, an optimization procedure is introduced to result in eliminating amplitude distortions by limiting aliasing effects and improving bandwidth characteristics. These can be modelled by  $I_d$  matrix,

$$I_d(k) = p(k) [x(k) + \Omega y(k)], \text{Eq.}(27)$$

with  $p(k)$  is the column row vector representing,

$$p(k) = \begin{bmatrix} p[k - M + 1] & \dots & p(k) \\ \vdots & \ddots & \vdots \\ p[k - L - Lf + 2] & \dots & p(k - Lf + 1) \end{bmatrix} 0 - Lf(k), \text{Eq.}(28)$$

with  $x(k)$  and  $y(k)$  as complex valued vectors representing,

$$x(k) = \begin{bmatrix} x^\sigma[k] \\ \vdots \\ x^\sigma[k - L - Lf + 2] \end{bmatrix} 0 - Lf(k), \text{Eq.}(29)$$

and

$$y(k) = \begin{bmatrix} y^\sigma(k)[0] \\ \vdots \\ y^\sigma(k)[Lf - 1] \end{bmatrix} 0 - Lf(k), \text{Eq.}(30)$$

and  $\Omega$  is the scaling factor for square-integrable parameter. If  $\Omega$  is larger, making the larger error presence in  $I_d$  and becomes unstable filter design.

**B. Design of Scalable Identity Wavelet**

$$s[k]=f[k]^T w[k], \text{ Eq.(31)}$$

where  $s[k]$  is the response of the designed filter,  $w[k]$  is filter weigh factor representing at time  $t$ ,

$$w[k]=\begin{bmatrix} w[nL-1] & \cdot & w[(n+1)L] \\ \cdot & \cdot & \cdot \\ w[nL-Lf+2] & \cdot & w[(n+1)L-Lf+1] \end{bmatrix} 0-Lf(t), \text{ Eq.(32)}$$

and re-representing

$$f[k]=\begin{bmatrix} f[nL+1] & \cdot & f[(n+1)L] \\ \cdot & \cdot & \cdot \\ f[mL+1] & \cdot & f[nL+mL-Lf+2] \end{bmatrix} 0-Lf(t), \text{ Eq.(33)}$$

The error cost function for the proposed wavelet design is formulated as,

$$e[k]=f_k-I_d^T w_k s_k, \text{ Eq.(34)}$$

where  $e[k]$  is the error cost function, representing

$$\min_{w(k)} \sum_{l=0}^{L-1} |e[n-m-2]|^2 = \min_{w(k)} \|f[k(n)] \cup f[k(m)]\|^2 - \min_{w(k)} \|I_d - s[k]^T \times w[k-n-m]\|^2, \text{ Eq.(35)}$$

with

$$I_d = \begin{bmatrix} I_{df} & 0 \\ 0 & 0_{N-M-Lf} \end{bmatrix} \updownarrow L, \text{ Eq.(36)}$$

The main design of NWSF includes a linearity and a overlapping filtering operation, by utilizing elliptical and circular convolution in frequency domain and also restoration operations based on circular overlapping of small bands in filter frequency response to eliminate the error cost function. To implement these, we are proposing a technique called Filter Continuity-provides an linear small bands filter non-overlapping error free response.

The filter response,

$$s[k]=\begin{bmatrix} f[nL+1] & \cdot & f[(n+1)L] \\ \cdot & \cdot & \cdot \\ f[mL+1] & \cdot & f[nL+mL-Lf+2] \end{bmatrix} 0-Lf(t)^T$$

$$\begin{bmatrix} w[nL-1] & \cdot & w[(n+1)L] \\ \cdot & \cdot & \cdot \\ w[nL-Lf+2] & \cdot & w[(n+1)L-Lf+1] \end{bmatrix} 0-Lf(t), \text{ Eq.(37)}$$

$$= \begin{bmatrix} s_{0(0n)}(z^l) & 0 & s_{0(0n-m)}(z^{l-m}) & 0 & \cdot & s_{0(0n-m)}(z^{l-m-n}) \\ 0 & (z^{l-m-1})s_{0(0n-m-1)} & 0 & (z^{l-m-2})s_{0(0n-m-2)} & \cdot & (z^{l-m-n})s_{0(0n-m-n)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ s_{0(0n-n)}(z^{l-n}) & 0 & s_{0(0n-m)(0n-n)}(z^{l-m)(l-n}) & 0 & \cdot & s_{0(0n-m)}(z^{l-m)(l-n}) \\ 0 & (z^{l-n-1})s_{0(0n-n-1)} & 0 & (z^{l-n-1})s_{0(0n-n-1)} & \cdot & (z^{l-m-n})s_{0(0n-m)} \end{bmatrix} 0-Lf(k), \text{ Eq.(38)}$$

is obtained, now if  $h[k]$  and  $g[k]$  are defined as

$$h_m[k]=\begin{bmatrix} n+m+1 & \text{if } 0 \leq n \leq l \ \& \ 0 \leq m \leq l \\ 0 & \text{if } n \leq l \ \& \ m \leq l \end{bmatrix} 0-Lf(k), \text{ Eq.(39)}$$

and

$$g_m[k]=\begin{bmatrix} n^\sigma + m^\sigma & \text{if } 0 \leq n \leq l \ \& \ 0 \leq m \leq l \\ 0 & \text{if } n \leq l \ \& \ m \leq l \end{bmatrix} 0-Lf(k), \text{ Eq.(40)}$$

From the above, the response filter weight is as follows:

$$w_k = \begin{bmatrix} w^\sigma(n)[0] \\ w^\sigma(n-1)[m] \\ w^\sigma(n-2)[m-1] \\ \vdots \\ w^\sigma(n-l)[m-l] \end{bmatrix}, \text{ Eq.(41)}$$

and its frequency transform as follows:

$$w_l(n) = W(z) = F \begin{bmatrix} wk \\ 0 \end{bmatrix}^{0-L_f(k)}_{M-L_f(k)}, \text{ Eq.(42)}$$

Above equation results to 2M value decomposition, to further decompose the image we have used a method called high level low band reduction.

**C. High level low band reduction wavelet design :**

For each block of L samples, the low level band reduction is performed as,

$$\underline{W}^{[n]} = F \begin{bmatrix} w[(n+1)L - M] \\ \vdots \\ w[(n+1)L] \end{bmatrix} \quad 0 - L_f(k) : \text{ is for } 0 \text{ to } L_f \text{ samples, , Eq.(43)}$$

$$\underline{W}^{[n]} = F \begin{bmatrix} 0_{M-L} & 0 \\ 0 & I_L \\ w_{L_f} \\ \vdots \\ w_{M-L_f} \end{bmatrix}^{L_f}_{M-L_f} \quad : \text{ is for } M-L_f \text{ to } L_f \text{ samples, , Eq.(44) and}$$

$$\underline{W}^{[n]} = F \begin{bmatrix} I(z)_{(M-N) \times N} \\ w(n)_{(M-N)} \\ e(n)_{(M-N)} \\ 0 \end{bmatrix}^L_0 \quad : \text{ is for total samples } 0 \text{ to } L. , \text{ Eq.(45)}$$

- a. if L=0, the NWSF, here where the importance of narrow wideband scalable design results to low level small band filter responses containing (L) samples,
- b. if L=[L<sub>f</sub>], the NWSF results to high level small band filter responses containing (L-M) samples and
- c. if L=[M-L<sub>f</sub>], the NWSF results to high level small band filter responses will be resulted with ( L-M-l) samples.

Often, the value of L<sup>σ</sup> is considered for samples prediction, which is added with λ resulting in prevention of overflow samples between small bands.

**IV. Spectrum Parallel Identification for Redundancy Encoding and Decoding (SPIRE)**

Approach to perfect reconstruction system include encoding and decoding process, by the performance of proposed small bands, analysis and synthesis bank analysis. Consider, N sampling state, M small bands, f as system input results in to s as system output, the s wavelet response is, we have identified few parallel spectrum encoding relations, they are

Parallel Identifier 1 : s<sub>H</sub> = w<sub>H</sub>f, , Eq.(46)

where  $s_H$  is the filter response for analysis filters, where

$$w_H = \begin{bmatrix} w_0(Lf - n) & \dots & w_0(0) \\ w_1(Lf - n - 1) & \dots & w_1(0) \\ \dots & \dots & \dots \\ w_m(Lf - n - m) & \dots & w_m(0) \end{bmatrix}, \text{ Eq.(47)}$$

Parallel Identifier 2 :  $s_G = w_G f$ , Eq.(48)

where  $s_G$  is the filter response for decimation filter, where

$$w_G = \begin{bmatrix} w_0(Lf - n - m) \\ w_0(Lf - n) \\ w_0(Lf) \\ w_0(Lf + n) \\ w_0(Lf - n + m) \end{bmatrix}, \text{ Eq.(49) and}$$

the structural content of  $s$  is related as,

$$S = \begin{bmatrix} s_0(0) & \dots & s_0(Lf - n) \\ s_1(0) & \dots & s_0(Lf - n - m) \\ s_2(0) & \dots & s_0(Lf) \\ \dots & \dots & \dots \\ s_n(0) & \dots & s_0(Lf + n + m) \\ s_m & \dots & s_0 \end{bmatrix}, \text{ Eq.(50)}$$

By combining the above equations, we get

$$s = \tau H^T \tau G f, \text{ Eq.(51)}$$

where  $\tau H^T$  and  $\tau G$  are inner matrices of  $H_m$  and  $G_m$ .

Progressive transmission of the encoded bits will be resulted as,

$$S = \begin{bmatrix} s_0(0) & s_0(k_0) & s_0(k_1) & \dots & s_0(0) & s_0(k_0) & s_0(k_1) & \dots \\ s_0(-1) & s_0(k_0 - 1) & s_0(k_1 - 1) & \dots & s_0(-1) & s_0(k_0 - 1) & s_0(k_1 - 1) & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ s_0(2) & s_0(k_0 - (N - 2)) & s_0(k_1 - (N - 2)) & \dots & s_0(2) & s_0(k_0 - (N - 2)) & s_0(k_1 - (N - 2)) & \dots \\ s_0(1) & s_0(k_0 - (N - 1)) & s_0(k_1 - (N - 1)) & \dots & s_0(1) & s_0(k_0 - (N - 1)) & s_0(k_1 - (N - 1)) & \dots \end{bmatrix}, \text{ Eq.(52)}$$

Based on the  $s$ , the computation of encoder results in,

$$C_d = [H_0^T \ H_1^T \ \dots \ H_{N-1}^T] * [G_0^T \ G_1^T \ \dots \ G_{M-1}^T], \text{ Eq.(53)}$$

ie., ( by assuming the pre-stored bits are zero) if  $0 \leq N \leq l, 0 \leq M \leq l$ .

$$\text{Parallel Identifier 3 : } C_d = \sum_{p=0}^{=-n-m-l} H_{N-l-p}^T G_{M-l-p}^T, \text{ Eq.(54)}$$

Progressive reception of the decoded bits will be resulted as,

$$R_d = \begin{bmatrix} R_0(0) & R_0(k_0) & R_0(k_1) & \dots & R_0(0) & R_0(k_0) & R_0(k_1) & \dots \\ R_0(+1) & R_0(k_0 + 1) & R_0(k_1 + 1) & \dots & R_0(+1) & R_0(k_0 + 1) & R_0(k_1 + 1) & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ R_0(2) & R_0(k_0 + (N + 2)) & R_0(k_1 + (N + 2)) & \dots & R_0(2) & R_0(k_0 + (N + 2)) & R_0(k_1 + (N + 2)) & \dots \\ R_0(1) & R_0(k_0 + (N + 1)) & R_0(k_1 + (N + 1)) & \dots & R_0(1) & R_0(k_0 + (N + 1)) & R_0(k_1 + (N + 1)) & \dots \end{bmatrix}, \text{ Eq.(55)}$$

these results depends on  $I_d$  and  $\sigma$  selected.

To result the above equation with no resultant artifacts, the condition to be satisfied is,

$$\min_{H_N, G_N} \int_{fs/2M}^{\tau} (\tau |H_N(z)| + \gamma |G_N(z)|) df + \begin{bmatrix} w_{n-l}^{m-l}(z) & 0 \\ 0 & I_d^{m-l}(z) \end{bmatrix}, \text{ Eq.(56)}$$

here  $\tau$  and  $\gamma$  are stop band weighing factors.

Based on the error redundancy in proposed method,

Parallel Identifier 4 :

$$E_{\text{redundancy}} = \min_{(e_0, e_1, \dots, e_{(N-M-1)})} \frac{\sum_{n=0}^{l-1} s(z)_{(N-M-1)}}{\sum_{n=0}^{l-1} f(z)_{(N-M-1)}} \cong \min_{(e_0, e_1, \dots, e_{(N-M-1)})} \frac{s(z) \left[ \sum_{n,m=0}^{(n-1)(m-1)} I_d(z)^{S_k^{(n-1)}(e_0, e_1, \dots, e_{(N-M-1)})} W_k(z)^{S_k^{(m-1)}(e_0, e_1, \dots, e_{(N-M-1)})} \right]}{f(z) \left[ \sum_{n,m=0}^{(n-1)(m-1)} I_d(z)^{S_k^{(n-1)}(e_0, e_1, \dots, e_{(N-M-1)})} W_k(z)^{S_k^{(m-1)}(e_0, e_1, \dots, e_{(N-M-1)})} \right]}, \text{ Eq.(57)}$$

**V. Experiments And Results**

We have constructed a separable three ways eight-band decomposition using cascaded 24-NWSF taps and then coded the bands using SIW wavelet modulation, scheme utilizes hierarchical pyramid of NWSF tap filters and applied to images. These pyramids were derived from wavelet theory to subband coding of images, as follows,

$$P_r = R_p + \frac{1}{3} \log_2 \left[ \frac{\sigma_n^2}{\left[ \prod_{n=0}^{N-1} s_n^l \right]^{1/N}} \left( \frac{\sigma_m^2}{\left[ \prod_{m=0}^{M-1} s_m^l \right]^{1/M}} \right) \right]^{L=L_f}, \text{ Eq.(58)}$$

where,  $P_r$  is pyramid reconstruction parameter,  $R_p$  is reconstruction bit rate,  $\sigma_n^2$  and  $\sigma_m^2$  are square-integrable scalable weigh parameters, and  $s_n^l$  and  $s_m^l$  are system responses under pyramid construction. Here, negative values of  $R_p$  were set to zero and the other bit rates raised to maintain proper bit rate to  $P_r$  reconstruction.

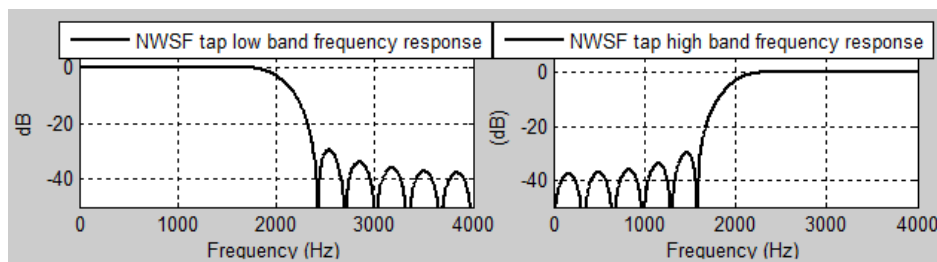
The above equation is processed to resample the image under hierarchical tree structure by the function as,

$$R_{es} = (-1)^{(k_n+k_m)}, \text{ Eq.(59)}$$

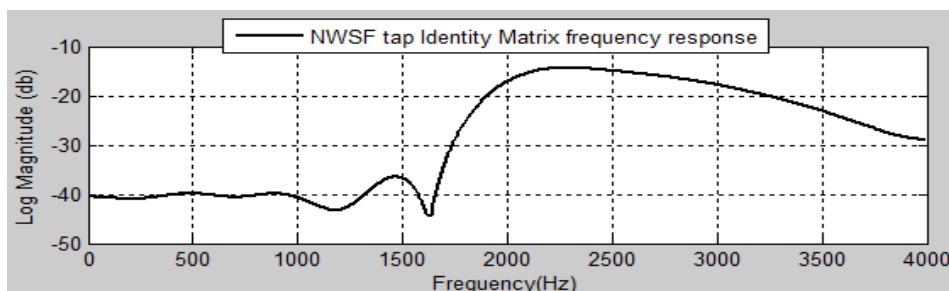
This method gives a reasonable approximation to compressed image. The coefficients for  $\sigma_n^2$  and  $\sigma_m^2$  are same for small band images of the filter proposed, and the quantizer implemented for the proposed encoder is using uniform quantization for high payload bits rates and considering image probability even distribution is constant evenly over each small band filter taps. Our proposed method, considered a relatively low bit rates, uniform quantization, simple design and entropy order is equal to optimal nit rate for the image sub bands.

The Scalable Identity Wavelet generally offer coding performance superior to the discrete wavelets and the aliasing errors are not available as NWSF taps are distributed in the separable system in scalable form in its design.

Evaluation of the proposed principles are shown in below figures. Figure 1 shows the designed NWSF low band and high band frequency response, which shows the perfect filter construction without any aliasing effect. Figure 2 shows the designed NWSF taps, implemented for Id matrix frequency response which shows the performance of 2M based filter response to identify the two level low band reduction wavelet frequency response. Figure 3 shows NWSF square-integrable  $\sigma = z^\lambda$  parameter with varying step-size  $\lambda$  value, which shows a liner integrable function implementation for equal response for all level of hierarchical coding. Figure 4 shows Scalable Identity Wavelet response under 2M based filter response, which shows the wavelet response at low level frequency channel.

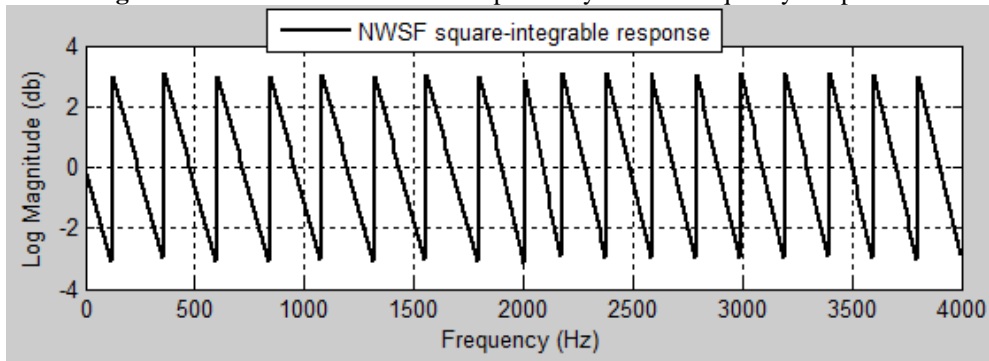


**Fig 1:** Matlab Simulation : NWSF Low Band and High Band Frequency Response

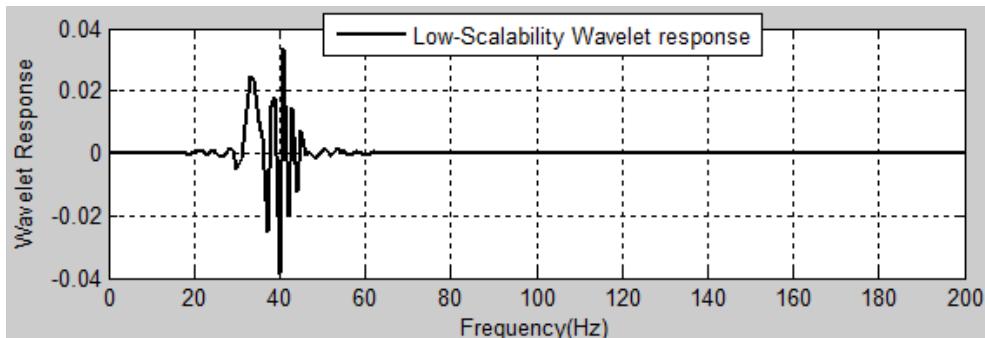




**Fig 2:** Matlab Simulation : NWSF Tap Identity Matrix Frequency Response



**Fig 3:** Matlab Simulation : NWSF Square-Integrable Frequency Response

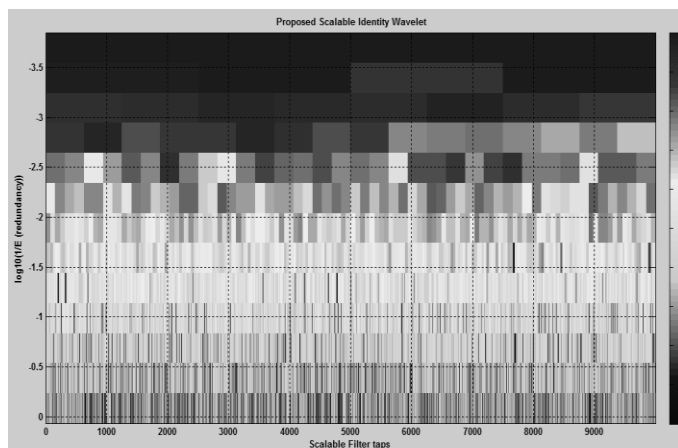


**Fig 4:** Matlab Simulation : NWSF Low-Scalability Wavelet Response

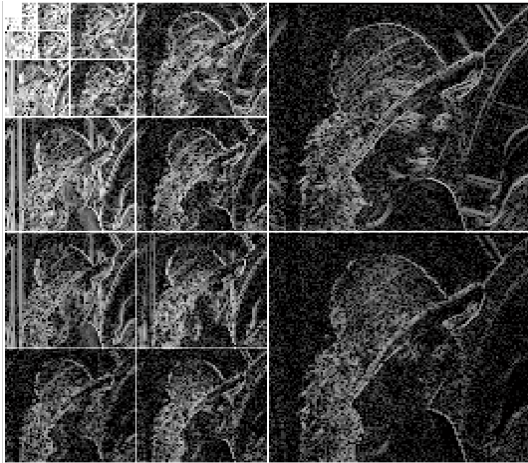
Figure 5 shows the input image LENA taken for proposed analysis on an image. Figure 6 shows the NWSF response for proposed SI Wavelet, which shows an analysis has been drawn for high level low band reduction is possible. Figure 7 shows the proposed Scalable Identity Wavelet Band Spectrum , which shows the hierarchical scalable level decomposition for 2M NWSF taps design. Figure 8 shows the encoding pattern of the proposed wavelet, which provides a reliable codec design for an efficient image compression algorithm. Figure 9 shows the reconstructed image with rate = 1 bpp, which shows the obtained image is approximate to original image.



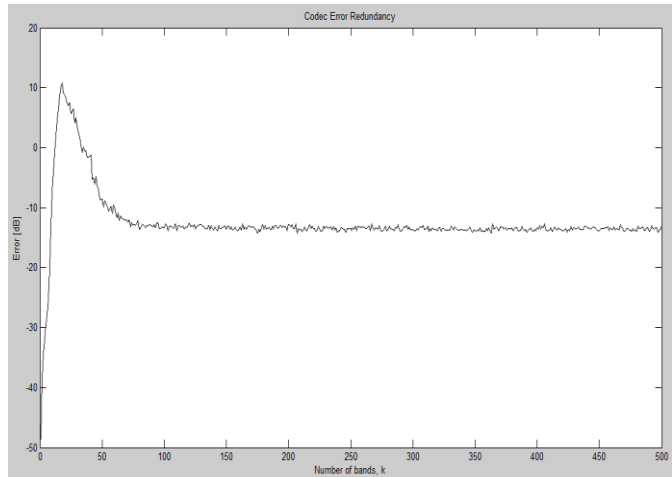
**Fig 5 :** Original LENA



**Fig 7 :** Matlab Simulation : Scalable Identity Wavelet Spectrum



**Fig 6:** Matlab Simulation : Narrow Wideband Scalable Filter Bank (NWSF)



**Fig 10 :** Matlab Simulation : Image Codec Error Redundancy Plot



**Fig 8 :** Matlab Simulation : Image encoding with proposed coding scheme



**Fig 9 :** Matlab Simulation : Image obtained with proposed coding scheme

From the figure 6, the decomposition of image LENA is shown, where the LENA image is filtered in to high and low bands coefficients initially and low bands are further decomposed into a very high level bands coefficients. This example figure is simulated for 5M samples to extract the 5 level 2 band decomposition hierarchical pyramid construction. At 5M samples level the maximum extraction low band coefficients are shown in white pixels, representing the decomposition at optimal error presence. With the use of present and traditional hierarchical and wavelet based image compression algorithms, the reconstruction of these types of high level low band images is error presence, so remove these error redundancies at these types of 5M sample 2 level band decompositions, we are proposing a new method under the proposed compression principle by making use of  $I_d$  structures at synthesis banks reconstruction ie., at reconstruction of the compressed image. Our proposed method is capable to construct these high level low band coefficients with less error redundancy comparatively as shown in figure 9, thus making the proposed SPIRE compression technique to future error redundant image compression technique.

### VI. Numerical Results

Practical tests have shown that the image transformation used five-level pyramids with the 9/7- tap filters of [1][2], by using the progression transmission channel schemes, compressed image will be reconstructed at the decoding file, based on the data rate of the compressed data will be reconstructed, comparison of these were computed by compression ratio numerical metric analysis. The progressive transmission bits redundancy is measured by the compression ratio

$$\text{Compression Ratio (CR)} = \frac{\text{CompressedFileSize(NumberOfbit sin finalfile)}}{\text{UncompressedFileSize(NumberOfbit sin originalfile)}}, \text{ Eq.(60)}$$

The time taken to compress the original image and time taken to reconstruct the compressed image is compression time and decompression time of the codec designed. Table 1 shows the comparative results of the compression time and compression ratio for the rate of proposed image compression technique. Table 2 shows the comparative results for PSNR values, for different rate values.

**Table 1:** Image LENA 512 x 512 code and decode time comparison

Rate(bpp)	Binary Uncoded[2]		SPIHT [2]		Proposed Coding Scheme		
	Code	Decode	Code	Decode	Code	Decode	CR
0.25	0.07	0.04	0.18	0.14	0.22	0.20	59.95
0.50	0.14	0.09	0.33	0.29	0.29	0.26	46.28
1.00	0.27	0.17	0.64	0.57	0.35	0.39	25.35

The distortion of compressed image is measured by the peak signal to noise ratio

$$\text{PSNR} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right) \text{dB}, \text{ Eq.(61)}$$

where MSE denotes the mean squared-error between the original and reconstructed images.

**Table 2:** Image LENA 512 x 512 PSNR value comparison.

Rate(bpp)	SPIHT [2]	Proposed
0.15	31.9 dB	46.2 dB
0.25	34.1 dB	50.8 dB
0.5	37.2 dB	52.4 dB

### VII. Conclusions and Discussions

We have analysed an image compression algorithm which has presented a Scalable Identity Wavelet and accomplished a Spectrum Parallel Identification for Redundancy Encoding and Decoding scheme. This SPIRE algorithm uses the principles of defining the filter analysis and synthesis banks, designing the wavelet based on identifiers decomposed wavelet, hierarchical implementation through ordering the low band level at high scalable integrity, spectral band decomposition at level-independence hierarchy and codec design with proper error redundancy. The implementation of these principles have shown better comparative results in coding and decoding is a new one and shown more effective and efficiency in previous implementations of EZW [1] and SPIHT [2]. Proposed Hierarchical Image Codec results shown better results comparatively with DWT technique and Arithmetic Coding scheme.

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