

Denoising of Rician noise in Magnitude MRI Images using wavelet shrinkage and fusion method

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Abstract: *Improving the signal-to-noise-ratio (SNR) of magnetic resonance imaging (MRI) using denoising techniques could enhance their value, provided that signal statistics and image resolution are not compromised. Here, a new denoising method based on wavelet based bayes shrinkage method of the measured noise power from each signal acquisition is presented. Bayes shrink method denoising assumes no prior knowledge of the acquired signal and does not increase acquisition time. Whereas conventional denoising/filtering methods are compromised in parallel imaging by spatially dependent noise statistics, wavelet based method is performed on signals acquired from MRI. Using numerical simulations, we show that proposed method can improve SNR in MRI reconstructed images without compromising image resolution. Application of Wavelet to MRI knee and DWI which achieved SNR improvements compared to conventional reconstruction. Comparison of Wavelet with standard filtering shows comparable SNR enhancement at low and high-SNR level and shows improved accuracy and retention of structural detail at a reduced computational load. The proposed methodology can be applied on final MRI reconstructed images. We have compared the performance of Bayes shrink combined with fusion to the normal thresholding techniques in order to enhance the visual quality of the image for proper diagnosis of disease.*

Keywords: *Haar transform, db3 transform, Bayes shrinkage, Fusion technique.*

I. Introduction

Medical imaging is a popular technique applied in the medical field where the internal organs can be viewed without incursion of human body. Medical image processing comprises of several important tasks such as noise suppression, registration, segmentation, reconstruction and compression. Over the years, various effective algorithms are formulated to solve the medical imaging problems. Noise occurs [1] in medical images during two phases acquisition and transmission. During the acquisition phase, noise can occur in an image due to two reasons firstly, the image acquisition devices induces noise to images, as they are susceptible to thermal noise and statistical randomness in emission of photons secondly, the physiological interference, which is the inability of a patient to manage his or her physiological processes and systems. It is difficult for the doctors to diagnose and to get useful information from these images. The noise in the images are inevitable, and hence, removing the noise is mandatory for improving the quality of the images so that the doctors can make use of these images to make accurate diagnosis. noise [2] occurs in medical images due to image quick record and transmission. This type of noise is very prevalent in image transmission process and especially takes place in the signal channels of medical imaging equipment. Noise in a medical image affects clinical visualization and making diagnostic interpretations. In general, there are two techniques to reduce noise in medical images, the first direction is to acquire a second image which results in a longer acquisition time and increased cost of the medical equipment. The second one is to apply some image processing technique to reduce noise in an acquired image which usually requires less acquisition time and can reduce the computational load and cost of the machine.

The modality MRI usually prone to this type of noise usually occurred in the transmission channels of multiple receiver coil in parallel MRI techniques (pMRI), in some protocols such as with diffusion weighted MRI imaging (DWI-MRI)sequence, these images in regions with low signal levels causes high noise in these areas. To increase the SNR, it is common practice to average several acquisitions in order to reduce noise variance. However this approach is time consuming in terms of acquisition and not adequate for typical clinical settings where patient cannot remain still for extended periods of time. Denoising techniques can be applied to improve the image quality as a post processing step, thereby not increasing the scan time of the machine.

In the current modalities of the MRI machine the signal to noise ratio depends mainly on the strength of magnetic fields of the system. In lower field systems such as Philips/Siemens Intera MRI 1.5T, the SNR is very low, and it is common practice to average the sequences to improve the quality of the Image, extending the time scan is expensive prone to motion artifacts, unacceptable in many MRI applications. And increase the strength of the magnetic field from 1.5T-3.0T to improve the SNR but will introduce radiofrequency-inhomogeneity artifacts, and demand high costs, because the noise attenuation requires high power supply devices to increase the super conduction effect [3,4]. The results suggest potential for practical application of the new method to boost SNR and hence reduce scan time in low magnetic field scanners.

To mitigate the effect of above limitations, the manufacturers design parallel magnetic resonance imaging (pMRI) techniques in the current modalities can speed up MRI scan through a multi-channel coil array[4,5]. Nevertheless, noise amplification is serious in pMRI reconstructed images at high acceleration factor(R). Therefore it is necessary to introduce denoising procedure to improve the quality of pMRI image. These techniques have been well adopted with Multi-echo sequences and dynamic imaging protocols, and are frequently used clinically in applications throughout the body. Parallel imaging works by acquiring a reduced amount of k-space data with an array of multiple receiver coils. These under sampled data can be acquired more quickly, but the under sampling leads to aliased and noise images. One of several parallel imaging algorithms can be used to reconstruct noise or artifact free images from either the aliased images (SENSE-type reconstruction) or from the under sampled data (GRAPPA-type reconstruction)[6].The advantages of parallel imaging in a clinical setting include faster image acquisition, which can be used, for instance, to shorten breath-hold times resulting in fewer motion-corrupted examinations. The number of receiver channels in the coil array limits the maximum acceleration factor(R). In general, the acceleration factor cannot be higher than the number of coils in the array, although this parameter is usually chosen to be much smaller in order to generate images of clinical quality. When $R > 2$, it causes noises to be amplified hence need of denoising filters to enhance the quality of images for clinical purposes. The most parallel imaging reconstruction algorithms filters in the current MRI scanners in the clinical practice are sensitivity encoding (SENSE) and Generalized auto calibrating partially parallel acquisitions (GRAPPA).The coil sensitivity [6,7] describes how sensitive a given channel is to a specific point in space. The sensitivity is often dependent on the object in the receiver coil and therefore can vary from patient to patient. Because parallel imaging relies on these coil sensitivity differences, acceleration can only practically take place in directions with coil sensitivity variations. The only major drawback to the current parallel imaging reconstruction algorithm in MRI scanner (SENSE /GRAPPA) reconstruction is the need for an accurate coil sensitivity map. Errors in the coil sensitivity map will cause artifacts or noise in the form of residual aliasing in the reconstructed full field of view (FOV) image. There are many factors that can cause the sensitivity maps to be inaccurate. The coil sensitivity profiles depend on the placement of the coils relative to the anatomy being imaged. If the patient moves during the course of the examination, the coil sensitivities may change, and the resulting images can contain artifacts or noise. These artifacts or noise can be mitigated by reacquiring the information needed to calculate the sensitivity map and using these new maps in the reconstruction. Additionally, in regions with low signal levels, for instance, the brain or sinusitis images, it can be difficult to determine the sensitivity map due to the high noise in these areas. To mitigate these limitations an efficient Denoising algorithm is proposed to remove the noise and artifacts from the current available scanners. Artifacts such as residual spatial aliasing and noise enhancement can be mitigated by choosing an appropriate coil array and reconstruction algorithm and by optimizing the parallel imaging parameters (such as the acceleration construction factor(R), FOV, number of ACS lines, or GRAPPA kernel type).

Choosing appropriate coil array becomes the overhead of cost and the additional hardware requirement. There is improvement in reconstruction algorithm, and to develop robust denoising filtering algorithm to improve the signal to noise ratio and resolution of the image.

Several methods to measure the SNR have been described [8].They can be differentiated into methods based on a single image on a pair of images or on a series of many images. SNR measurements based on two ROIs in a single image (one in the tissue of interest, the other in the image background, i.e., in air, outside the imaged object) can be subdivided into methods using the standard deviation of the background intensity and those using the mean value of the background intensity. We refer to these “two region methods” as SNR standard deviation and SNR mean respectively. With the appropriate conversion factors derived from the noise statistics, both methods yield identical results.

NOISE MODELLING IN MRI

The main source of noise in MRI images is the thermal noise in the patient [9]. The MRI image is commonly reconstructed by computing the inverse discrete Fourier transform of the raw data [63]. The signal component of the measurements is present in both real and imaginary channels each of the two orthogonal channels is affected by additive white Gaussian noise. The noise in the reconstructed complex-valued data is

thus complex white Gaussian noise. Most commonly, the magnitude of the reconstructed MRI image is used for visual inspection and for automatic computer analysis. Since the magnitude of the MRI signal is the square root of the sum of the squares of two independent Gaussian variables, it follows a Rician distribution. In low intensity (dark) regions of the magnitude image, the Rician distribution tends to a Rayleigh distribution [9] and in high intensity (bright) regions it tends to a Gaussian distribution. A practical consequence is a reduced image contrast noise increases the mean value of pixel intensities in dark image regions. Due to the signal-dependent mean of the Rician noise, both the wavelet and the scaling coefficients of a noisy MRI image are biased estimates of their noise-free counterparts. In [9] it was shown that one can efficiently overcome this problem by filtering the square of the MRI magnitude image in the wavelet domain. In the squared magnitude image, data are non-central chi-square distributed, and the wavelet coefficients are no longer biased estimates of their noise-free counterparts. The bias still remains in the scaling coefficients, but is not signal-dependent and it can be easily removed.

II. Materials and Methods

Materials

The experiments were conducted on MRI datasets. The dataset consists of clinical MRI[10] collected from JSS hospital Mysuru Karnataka, India. The proposed approach was evaluated with images acquired using Spin Echo Sequences with long repetition time (TR) and short echo time (TE) by Philips 3.0T scanner. The detailed information of the imaging scanners is as follows:

Philips 3.0Tscanner: PD (proton density) weighted sequence of MR knee image with ligament, with the acquisition parameters are TR=4.6sec, TE=30ms, slice thickness=2.5mm, Resolution of 672x672. Image Reconstruction: Sense, Reduction factor(Under sampling=6)

Philips 3.0Tscanner: DWI data set (60 gradients, 1 baseline, matrix: 128 × 128 × 66, isotropic resolution 2 × 2 × 2 m3) with a dual b value of 800 s/mm² and 1000 s/m² parallel MRI reconstruction: SENSE

Philips 3.0Tscanner: MRI T1 weighted Brain image with acquisition parameters are TR=5.3sec, TE=20ms, slice thickness=3.5mm, Resolution of 512x512. Parallel Image Reconstruction: SENSE

Methods

A. Image Denoising Methods.

De-noising plays a very important role in the field of the medical image pre-processing. It is often done before the image data is to be analyzed. Denoising is mainly used to remove the noise that is present and retains the significant information, regardless of the frequency contents of the signal. It is entirely different content and retains low frequency content.

There are two basic methods are used in image denoising purpose.

- 1) Filtering method.
- 2) Transform method.

In Filtering method, a traditional way to remove noise from image data is to employ filters[11,12]. With filters, the noise is removed without any attempts to explicitly identify it. Spatial filters employ a low pass filtering on groups of pixels with the assumption that the noise occupies the higher region of frequency spectrum. Generally spatial filters remove noise to a reasonable extent but at the cost of blurring images which in turn makes the edges in pictures invisible, tend to blur sharp edges, destroy lines and other fine image details, and perform poorly in the presence of signal-dependent noise.

For de-noising there are two transforms methods are available.

- 1) Fourier Transform: This method refers use of low pass filters using Fast Fourier Transform (FFT). In frequency smoothing methods the removal of the noise is achieved by designing a frequency domain filter and adapting a cut-off frequency when the noise components are de-correlated from the useful signal in the frequency domain. These methods are time consuming and depend on the cut-off frequency and the filter function behaviour. Furthermore, they may produce artificial frequencies in the processed image.
- 2) Wavelet Transform: A brief introduction to wavelet transform is given subsequent sections.

B. Haar wavelet transform:

In our method we use the Haar wavelet to perform the Wavelet transform [11,12]. It is the simplest of all wavelets and its operation is easy to understand. Haar wavelets have their limitations too. They are piecewise constant and hence produce irregular blocky approximations. However these wavelets are not easy to comprehend and are also computationally intensive.

C. Daubechies wavelet transform: Daubechies wavelet is the first wavelet family which has set of scaling function [13,14] which are orthogonal. This wavelet has finite vanishing moments. Daubechies wavelets have balanced frequency responses but non linear phase responses. Daubechies wavelets are useful in compression and noise removal of audio signal processing because of its property of overlapping windows and the high frequency coefficient spectrum reflect all high frequency changes.

D. Bayes shrinkage

The Bayes Shrink wavelet thresholding adopts the Bayesian approach [15,16] which assumes the knowledge of the probability distribution of the original signal and seeks to optimize the threshold operator for the purpose of minimizing the expected risk. In particular, it is assumed that for the various sub-bands and decomposition levels, the wavelet coefficients of the original image follow approximately a Generalized Gaussian Distribution (GGD).

In the wavelet domain, we have the relationship

$$Y_j^{sub} = X_j^{sub} + W_j^{sub} \tag{1}$$

Due to the independence assumption between the original signal ‘x’ and the noise ‘w’, the joint distribution of X_j^{sub} and W_j^{sub} is the product distribution of X_j^{sub} and W_j^{sub} . The conditional probability distribution of X_j^{sub} , given the observed noisy wavelet coefficients Y_j^{sub} , is called the posterior distribution. This posterior distribution can be used to construct a decision soft thresholding operator that computes a de-noised estimate $\hat{X}_j^{sub} = T(X_j^{sub}, \lambda)$ of X_j^{sub} from the noisy data Y_j^{sub} by minimizing the bayes risk.

This section focuses on the estimation of the GGD parameters σ_X and β which in turn yields a data-driven estimate of $T_B(\sigma_X)$ that is adaptive to different sub band characteristics. The noise variance σ^2 needs to be estimated first. In some situations, it may be possible to measure σ^2 based on information other than the corrupted image. If such is not the case, it is estimated from the sub band HH_1 by the robust median estimator

$$\hat{\sigma} = \frac{\text{Median}(|Y_{ij}|)}{0.6745}, Y_{ij} \in \text{subband } HH_1 \tag{2}$$

The parameter β does not explicitly enter into the expression of $T_B(\sigma_X)$, only the signal standard deviation, σ_X , does. Therefore it suffices to estimate directly σ_X or σ_X^2 .

Recall the observation model is $Y = X + V$, with X and V independent of each other, hence

$$\sigma_Y^2 = \sigma_X^2 + \sigma^2 \tag{3}$$

Where σ_Y^2 is the variance of Y . Since Y is modeled as zero mean, σ_Y^2 can be found empirically by

$$\hat{\sigma}_Y^2 = \frac{1}{n^2} \sum_{i,j=1}^n Y_{i,j}^2 \tag{4}$$

Where $n \times n$ is the size of the subband under Consideration. Thus

$$\hat{T}_B(\hat{\sigma}_X) = \frac{\hat{\sigma}^2}{\hat{\sigma}_X} \tag{5}$$

Where

$$\hat{\sigma}_X = \sqrt{\max(\hat{\sigma}_Y^2 - \hat{\sigma}^2, 0)} \tag{6}$$

In the case that $\hat{\sigma}^2 \geq \hat{\sigma}_Y^2$, $\hat{\sigma}_X$ is taken to be 0. That is $\hat{T}_B(\hat{\sigma}_X)$, is ∞ , or, in practice, $\hat{T}_B(\hat{\sigma}_X) = \max(|Y_{i,j}|)$, and all coefficients are set to 0. This happens at times when σ is large (for example, $\sigma > 20$ for a grayscale image).

To summarize, we refer to our method as Bayes Shrink which performs soft thresholding, with the data-driven, subband dependent threshold,

$$\hat{T}_B(\hat{\sigma}_X) = \frac{\hat{\sigma}^2}{\hat{\sigma}_X} \tag{7}$$

E. Image fusion

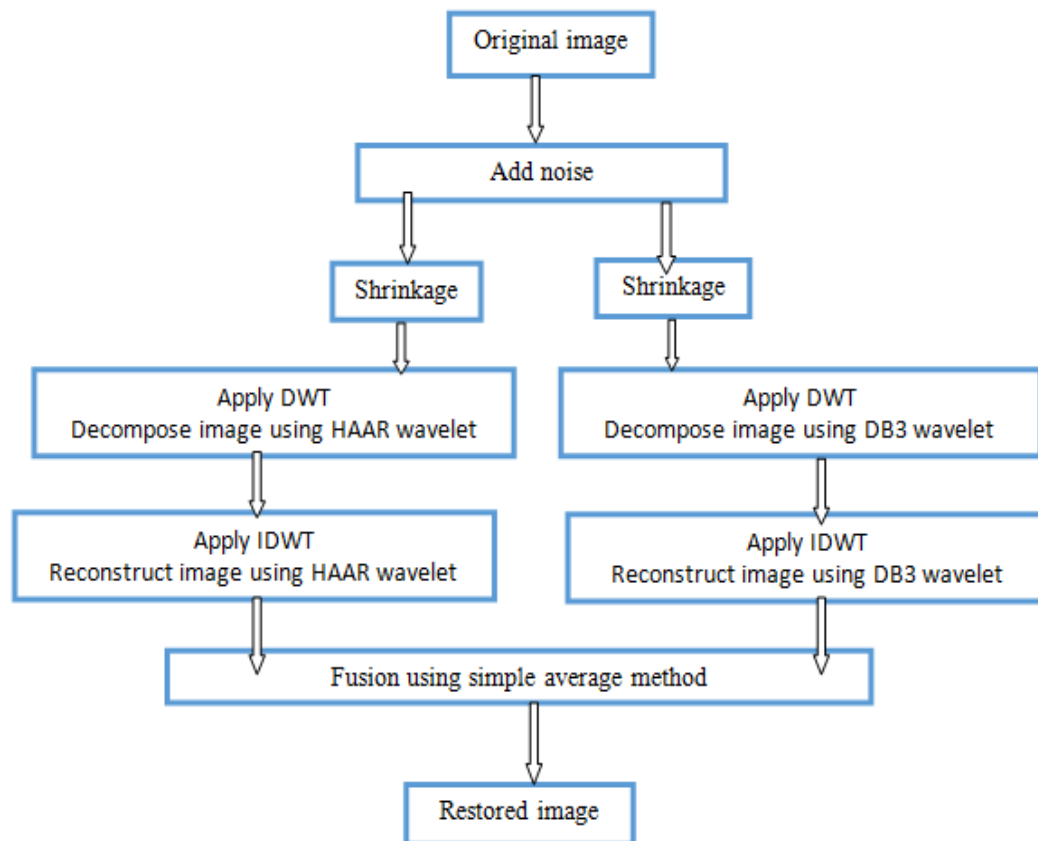
Image fusion is the process [17, 18] of combining information of interest in two or more images of a scene into a single highly informative image. Information of interest depends on the application under consideration. The aim of image fusion is to integrate complementary as well as redundant information from multiple images to create a fused image output. Therefore, the new image generated should contain a more accurate description of the scene than any of the individual source images and is more suitable for human visual and machine perception or further image processing and analysis tasks. For medical image fusion, the fusion of images can often lead to additional clinical information not apparent in the separate images. Another advantage is that it can reduce the storage cost by storing just the single fused image instead of multisource images.

F. Simple Average method

The value of the pixel of each image is taken and added. This sum is then divided by 2 to obtain the average. The average value is assigned to the corresponding pixel of the output image. The pixels in the resultant fused image are obtained by taking average of the every corresponding pixel in the input images.

G. Proposed Algorithm Implementation

- Add noise to the MRI image.
- Noises added here is Rician, which is the magnitude of the real and complex value of the Gaussian noise.
- Decompose the noisy image using forward discrete HAAR wavelet transform.
- Level of decomposition selected is $L=4$.
- Threshold the horizontal, vertical and diagonal coefficients using soft thresholding, hard thresholding and bayes shrink separately.
- Reconstruct the image using inverse discrete HAAR wavelet transform applying level 4 reconstruction.
- Decompose the noisy image using forward discrete DB3 wavelet transform.
- Level of decomposition selected is $L=4$.
- Threshold the horizontal, vertical and diagonal coefficients using soft thresholding, hard thresholding and bayes shrink separately.
- Reconstruct the image using inverse discrete DB3 wavelet transform applying level 4 reconstruction.
- Fuse the two restored images obtained using HAAR and DB3 wavelets using simple average method



III. Results And Discussion

Experiments and Results

Numerical simulations for MRI reconstructed images are based on the 512×512 low-SNR DWI image. DWI images are prone to low signal to noise ratio and introduce bias in the reconstructed image. It is clear from the denoised results in Figs. 2 and 3 that the proposed filtering achieves much closer result to the original image than the image filtered with other standard filter. Qualitatively, the proposed filter provides better preservation of fine structures, good contrast between tissues and fewer oscillations over homogeneous areas and better noise removal over the other methods under considerations. proposed filtering method can contribute significantly in the noise removal as well as in the image structure preservation. It is clear from the results in figure that the proposed filter achieves closer result to the original image in which it is effectively estimate the diffusion tensor parameters like fractional Anisotropy (FA) and tractography for the brain connectivity required for the Alzheimer disease detection for diffusion weighted images. The proposed filtering parameters depends upon the standard level of noise (σ) here DWI images are corrupted by Rician noise and the proposed filter is better edge preservation. The proposed filter works well for both low SNR and high SNR data and to avoid over smoothing over the tissue boundaries to preserve the edge information corrupted by Rician noise required further segmentation to classify whether the tissue is benign or malignant. The performance of the proposed filter is better understood by PSNR versus noise level as shown in table1 and graph1. From these analysis the proposed Bayes shrink with fusion improve its visual quality as well as edge preservation. From graph1 it is observed that proposed filter is good choice for removing Rician noise compared to Gaussian noise is judged by the PSNR values. The performance of the image quality index versus noise level as shown in graph2. From these it is observed that the proposed Bayes shrink fusion method which enhances the image quality index for Rician noise compared to additive white Gaussian noise most of the papers from the literature are based on filtering of Gaussian noise (high SNR data) but the proposed filter is robust for low SNR data especially for HARDI-DWI data. Fig.1 shows the performance of reconstruction filter in which MRI knee image with ligament is corrupted by Rician noise the proposed approach shows better preservation of the edge details with minimal blurring. The application of proposed technique can be applied to DWI image as shown in fig.2. In which it is prone to low SNR corrupted by non-Gaussian distribution called Rician noise distribution it restores the original image and effectively finds the diffusion parameters like ADC(antitriophic diffusion coefficient) and fractional anstriophy(FA) required for the fiber tracking and the detection of alzameir disease. The restoring performance of the proposed filter for MRI T1 weighted image is as shown in fig.3. Better edge preservation and enhancement of the anatomical details compared to other thresholding methods.

PERFORMANCE PARAMETERS

The quality of the denoising filter is evaluated using the performance metrics[19] are defined as follows

1) Peak Signal to Noise Ratio (PSNR):

PSNR values can be calculated by comparing two images one is original image and other is distorted image .The PSNR has been computed using the following formula:

$$PSNR = 10 \log_{10} \left(\frac{R^2}{MSE} \right) \quad (8)$$

Where R is the maximum fluctuation in the input image data type. If the input is an 8-bit unsigned integer data type, R is 255, etc.

2) Mean Squared Error (MSE):

One obvious way of measuring this similarity is to compute an error signal by subtracting the test signal from the reference, and then computing the average energy of the error signal. The mean-squared-error (MSE) is the simplest, and the most widely used for image quality measurement.

$$MSE = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (x(i, j) - y(i, j))^2 \quad (9)$$

Where x(i, j) represents the original image and y(i, j) represents the denoised (modified) image and i and j are the pixel position of the M×N image. MSE is zero when x(i, j) = y(i, j) .

3) Image quality index (Q)

It is mathematically defined by modeling the image distortion relative to the reference image as a combination of three factors: loss of correlation, luminance distortion, and contrast distortion.

$$Q = \frac{\sigma_{fg}}{\sigma_f \sigma_g} \frac{2 \bar{f} \bar{g}}{(\bar{f})^2 + (\bar{g})^2} \frac{2 \sigma_f \sigma_g}{\sigma_f^2 + \sigma_g^2} \quad (10)$$



Original image



Noisy image (Rician)

Restored image using HAAR wavelet



Soft thresholding



hard thresholding



bayes shrink

Restored image using DB3 wavelet(rician)



Soft thresholding



hard thresholding



Bayes shrink

Restored image using fusion (rician)



Soft thresholding



hard thresholding



bayes shrink

Fig.1. MRI knee image corrupted by Rician noise

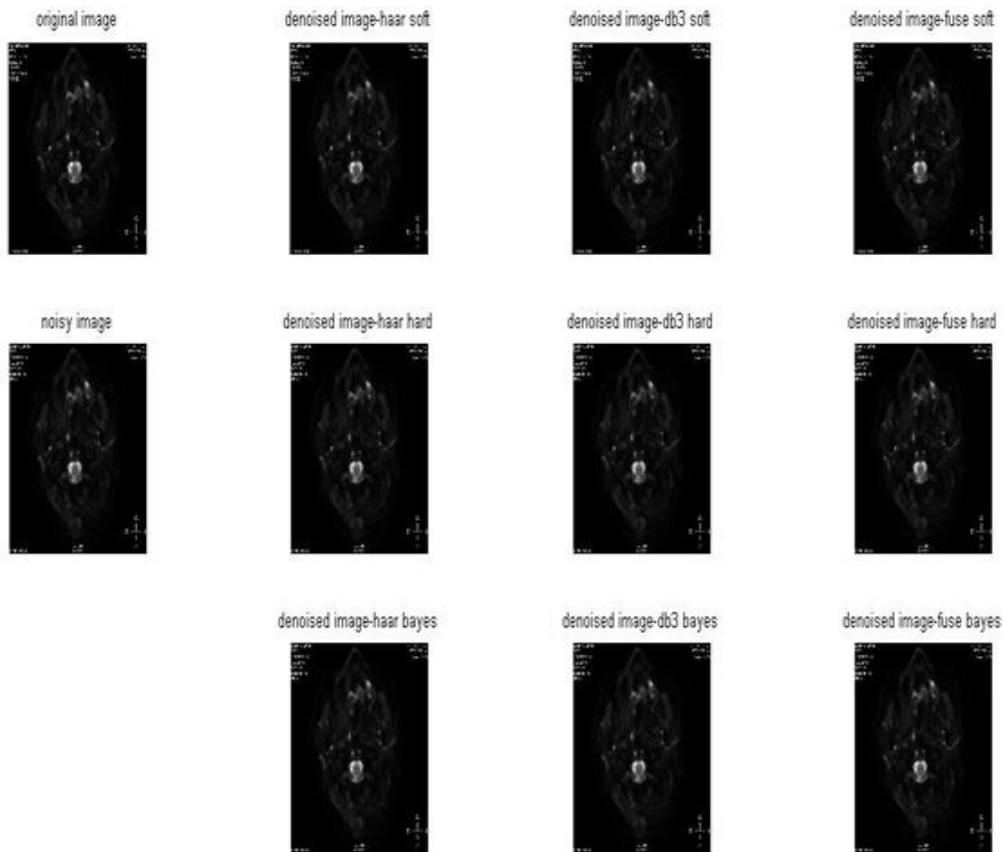


Fig.2. MRI Diffusion weighted image corrupted by Rician noise

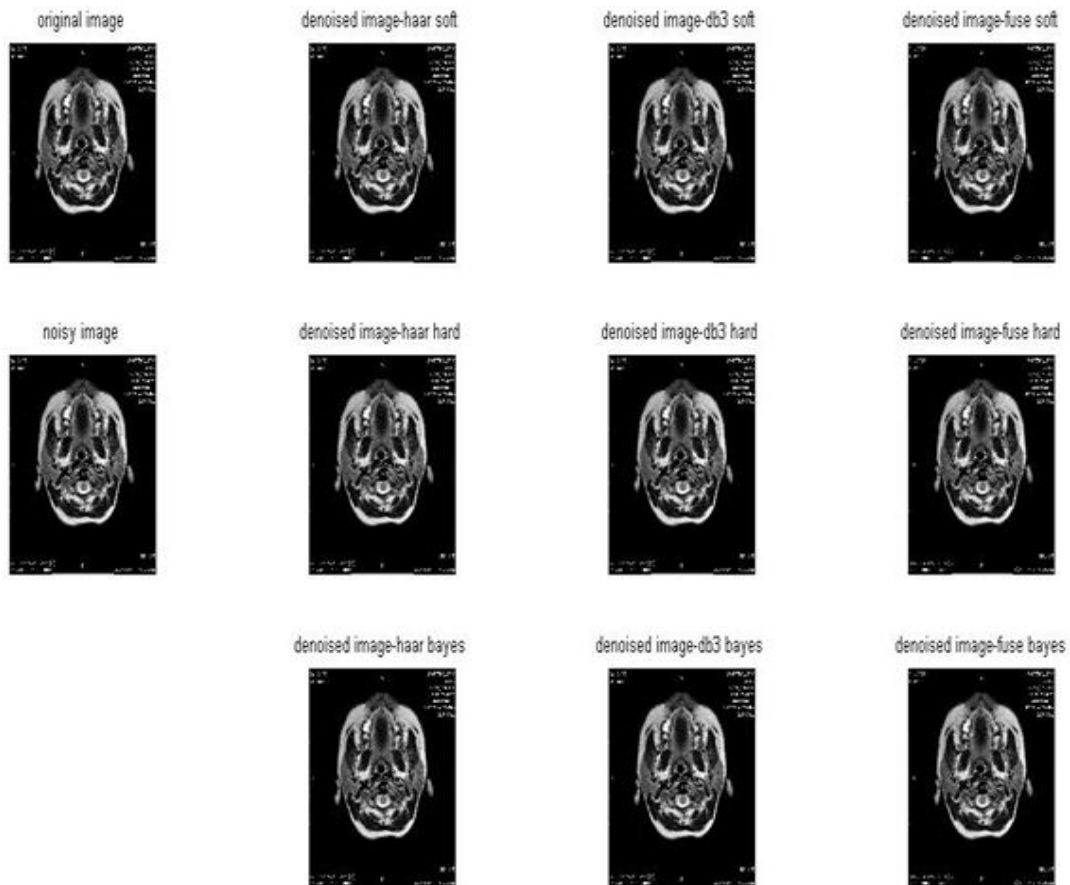
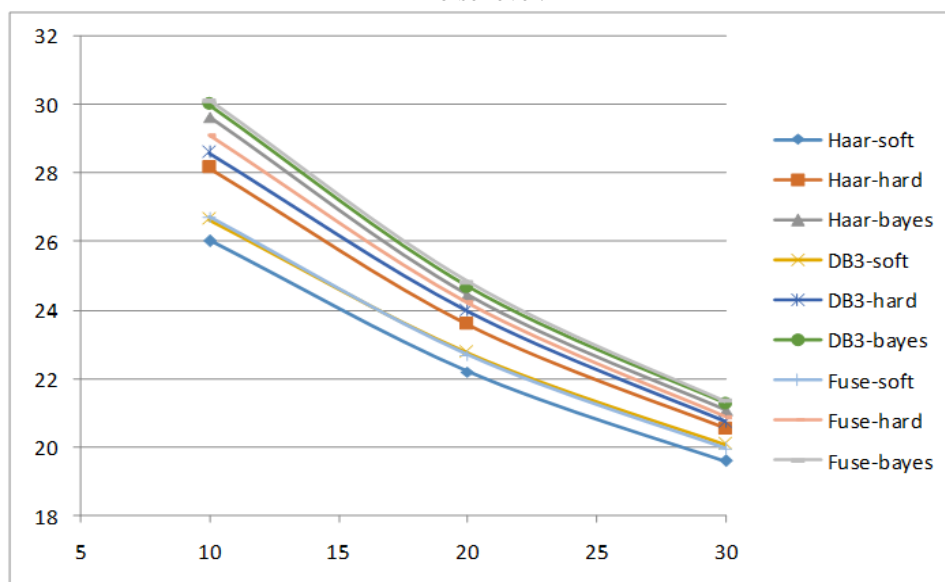


Fig.3. MRI T1 weighted Brain image corrupted by Rician noise

Table 1: Performance parameters of MRI knee image corrupted by Rician noise –PSNR v/s noise level
Rician noise:

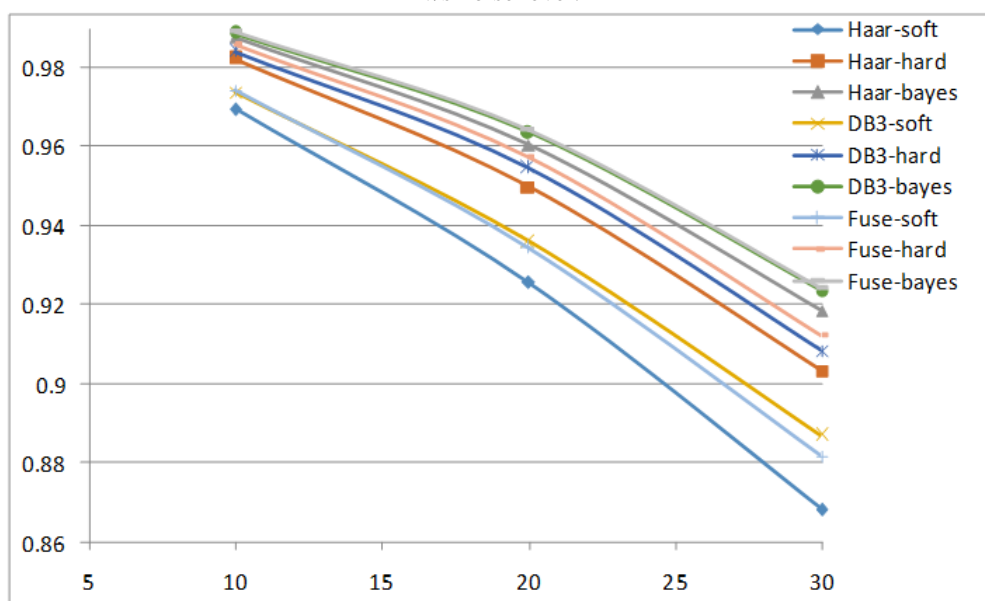
Noise Level (σ)	Haar wavelet			DB3 wavelet			Fusion		
	λ_{UNIV}		Bayes Shrink	λ_{UNIV}		Bayes Shrink	λ_{UNIV}		Bayes shrink
	Soft	hard		Soft	hard		Soft	hard	
10	26.0397	28.1698	29.6475	26.6462	28.6039	30.0209	26.7381	29.1061	30.1421
20	22.2134	23.5839	24.4402	22.7524	23.9805	24.6937	22.6999	24.2298	24.8199
30	19.5907	20.5457	21.0913	20.0812	20.7380	21.2678	19.9622	20.8978	21.3484

Graph 1: Performance parameters of MRI knee image corrupted by Rician noise –PSNR v/s noise level.



X-axis → Noise level in terms of standard deviation(σ)
 Y-axis → PSNR in dB

Graph 2: Performance parameters of MRI knee image corrupted by Rician noise –Image quality index v/s noise level.



X-axis → noise level in terms of standard deviation(σ)
 Y-axis → Image quality index (Q)

IV. Conclusion

The denoising of MRI images is performed using Bayes shrink with fusion method and compared with standard thresholding methods. The peak signal-to-noise ratio (PSNR) and image quality index are calculated and it is found that our proposed filter achieves better edge preservation and enhancement of anatomical details and is best suited for low and high corrupted level of noise. Comparison of Wavelet with standard filtering shows comparable SNR enhancement at low-SNR levels and high level of noise, but improved accuracy and retention of structural detail at a reduced computational load. Our proposed filter shows significant improvement in case of DWI images for fiber tracking and diagnosis of alzemeir disease. This will be beneficial for various applications, such as q-space imaging and diffusion kurtosis imaging.

V. Future scope

The above methods are being performed on image resolution of 512x512 and work is being done to remove Rician noise from MRI images and future plan is to make it valuable for high resolution of images and to develop adaptive shrinkage technique to remove high level of noise and to improve PSNR from medical images. In this paper we have provided an experimental validation of the proposed method to deal with Rician noise perturbed images. A mathematical validation of the proposed technique for the given noise distribution remains to be performed. Further, as mentioned earlier, some of the current generation MR images no longer satisfy the assumption of Rician noise.

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