

Image Reconstruction Using Discrete Wavelet Transform

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Abstract : In the recent growth of data intensive and multimedia based applications, efficient image compression solutions are becoming critical. The main objective of Image Compression is to reduce redundancy of the data and improve the efficiency. The main techniques used are Fourier Analysis, Discrete Cosine Transform vector quantization method, sub-band coding method. The drawbacks in the above methods are, they cannot be used for real time systems. In order to overcome these problems, the Wavelet Transform method has been introduced. Wavelet Analysis is highly capable of revealing aspects of data like trends, breakdown points, discontinuities in higher derivatives and self similarity and can often compress or diagnose a signal without appreciable degradation. Here, we implement a lossy image compression technique using Matlab Wavelet Toolbox and Matlab Functions where the wavelet transform of the signal is performed, then calculated a threshold based on the compression ratio acquired by the user.

Keywords : CWT, DWT, Decomposition,, Haar Transform, Lossy Compression, Wavelet.

I. INTRODUCTION

In the last decade, there has been a lot of technological transformation in the way of communication. This transformation includes the ever present, ever growing internet, the explosive development in mobile communication and ever increasing importance of video communication. Data Compression is one of the technologies for each of the aspect of this multimedia revolution. Portable Devices would not be able to provide communication with increasing clarity, without data compression. Data compression is art and science of representing information in compact form. Uncompressed multimedia data requires considerable storage capacity and transmission bandwidth. Despite rapid progress in mass-storage density, processor speeds, and digital communication system performance, demand for data storage capacity and data-transmission bandwidth continues to outstrip the capabilities of available technologies. In a distributed environment large image files remain a major bottleneck within systems.

Image Compression is an important component of the solutions available for creating image file sizes of manageable and transmittable dimensions. Platform portability and performance are important in the selection of the compression/decompression technique to be employed. The basic model of image compression is shown in figure 1.1.

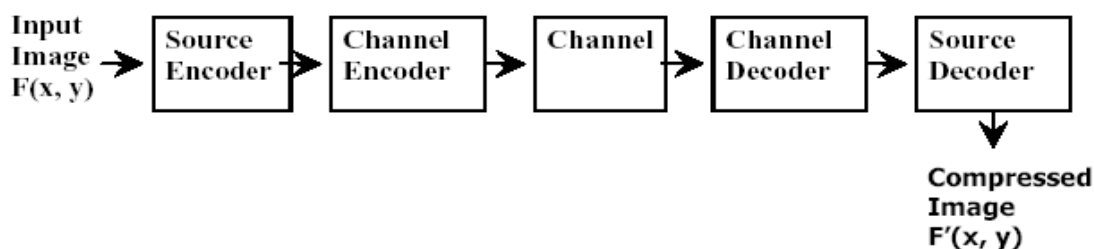


Figure 1.1: Block diagram of image compression

II. Image Compression

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space. It also reduces the time required for images to be sent over the Internet or downloaded from Web pages.

2.1 Principle

Images have considerably higher storage requirement than text; Audio and Video. This Data require more demanding properties for data storage. An image stored in an uncompressed file format, such as the popular BMP format, can be huge. An image with a pixel resolution of 640 by 480 pixels and 24-bit colour resolution will take up $640 * 480 * 24/8 = 921,600$ bytes in an uncompressed format.

The huge amount of storage space is not only the consideration but also the data transmission rates for communication of continuous media are also significantly large. An image, 1024 pixel x 1024 pixel x 24 bit, without compression, would require 3 MB of storage and 7 minutes for transmission, utilizing a high speed, 64 Kbits /s, ISDN line. Image data compression becomes still more important because of the fact that the transfer of uncompressed graphical data requires far more bandwidth and data transfer rate.

The figures in following table 2.1 show the qualitative transition

From simple text to full-motion video data and the disk space, transmission bandwidth, and transmission time needed to store and transmit such uncompressed data.

Table 2.1:Multimedia data types & uncompressed storage space, Transmission bandwidth & transmission time required

Multimedia Data	Size/Duration	Bits/Pixel or Bits/Sample	Uncompressed Size (B for bytes)	Transmission Bandwidth (b for bits)	Transmission Time (using a28.8K Modem)
A page of text	11" x 8.5"	Varying resolution	4-8 KB	32-64 Kb/page	1.1 - 2.2 sec
Telephone quality speech	10 sec	8 bps	80 KB	64 Kb/sec	22.2 sec
Gray scale Image 512 x 512	512	8 bps	262 KB	2.1 Mb/image	1 min 13 sec
Colour Image	512 x 512	24 bps	786 KB	6.29 Mb/image	3 min 39 sec
Medical Image	2048 x 1680	12 bps	5.16 MB	41.3 Mb/image	23 min 54 sec
SHD Image	2048 x 2048	24 bps	12.58 MB	100 Mb/image	58 min 15 sec
Full-motion Video	640 x 480, 1 min (30 frames/sec)	24 bps	1.66 GB	221 Mb/sec	5 days 8 hrs

III. Wavelet Analysis

A wavelet is a waveform of effectively limited duration that has an average value of zero. Comparing wavelets with sine waves, which are the basis of Fourier analysis. Sinusoids do not have limited duration they extend from minus to plus infinity and whereas sinusoids are smooth and predictable. Wavelets tend to be irregular and asymmetric.



Fig 3.1: Sine Wave



Fig 3.2: Wavelet (db10)

The Wavelet Series is just a sampled version of CWT and its computation may consume significant amount of time and resources, depending on the resolution required. The Discrete Wavelet Transform (DWT), which is based on sub-band coding, is found to yield a fast computation of Wavelet Transform. It is easy to implement and reduces the computation time and resources required.

3.1 Analysis using digital filter banks

Filters are one of the most widely used signal processing functions. Wavelets can be realized by iteration of filters with rescaling. The resolution of the signal, which is a measure of the amount of detail information in the signal, is determined by the filtering operations, and the scale is determined by upsampling and down sampling (sub sampling) operations.

The DWT is computed by successive lowpass and highpass filtering of the discrete time-domain signal. This is called the Mallat algorithm or Mallat-tree decomposition. Its significance is in the manner it connects the continuous time multi-resolution to discrete-time filters.

In Continuous Wavelet Transform (CWT), the signals are analyzed using a set of basis functions which relate to each other by simple scaling and translation. In the case of DWT, a time-scale representation of the digital signal is obtained using digital filtering techniques. The signal to be analyzed is passed through filters with different cutoff frequencies at different scales.

IV. N-Stage Filtering: Approximations and Details

4.1 One stage Filtering

For many signals, the low-frequency content is the most important part. It is what gives the signal its identity. The high-frequency content, on the other hand, imparts flavor or nuance. In wavelet analysis, approximations and details are the most important terms. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. The filtering process, at its most basic level, looks like this:

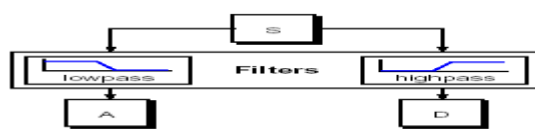


Figure 4.1: Basic level of filtering

The original signal, S , passes through two complementary filters and emerges as two signals. In the above figure A is the approximations and D is the details.

4.2 Multilevel filtering

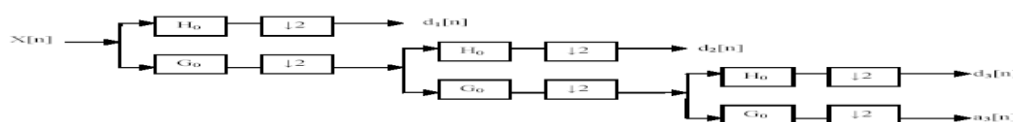


Figure 4.2: Three level wavelet decomposition tree

In the figure, the signal is denoted by the sequence $x[n]$, where n is an integer. The low pass filter is denoted by G_0 while the high pass filter is denoted by H_0 . At each level, the high pass filter produces detail information; $d[n]$, while the low pass filter associated with scaling function produces coarse approximations, $a[n]$. At each decomposition level, the half band filters produce signals spanning only half the frequency band. This doubles the frequency resolution as the uncertainty in frequency is reduced by half. In accordance with Nyquist's rule if the original signal has a highest frequency of ω , which requires a sampling frequency of 2ω radians, then it now has a highest frequency of $\omega/2$ radians. It can now be sampled at a frequency of ω radians thus discarding half the samples with no loss of information. This decimation by 2 halves the time resolution as the entire signal is now represented by only half the number of samples. Thus, while the half band low pass filtering removes half of the frequencies and thus halves the resolution, the decimation by 2 doubles the scale.

With this approach, the time resolution becomes arbitrarily good at high frequencies, while the frequency resolution becomes arbitrarily good at low frequencies. The time-frequency plane is thus resolved. The filtering and decimation process is continued until the desired level is reached. The maximum number of levels depends on the length of the signal. The DWT of the original signal is then obtained by concatenating all the coefficients, $a[n]$ and $d[n]$, starting from the last level of decomposition.

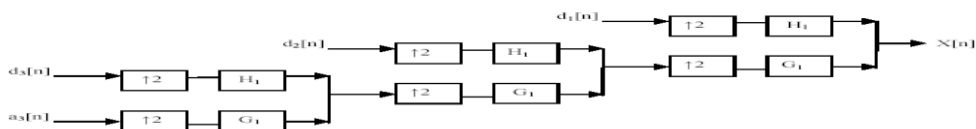


Figure4.3: Three level wavelet reconstruction tree

Figure4.3 shows the reconstruction of the original signal from the wavelet coefficients. Basically, the reconstruction is the reverse process of decomposition. The approximation and detail coefficients at every level are upsampled by two, passed through the low pass and high pass synthesis filters and then added. This process is continued through the same number of levels as in the decomposition process to obtain the original signal. The Mallat algorithm works equally well if the analysis filters, G_0 and H_0 , are exchanged with the synthesis filters, G_1 and H_1 .

Conditions for Perfect Reconstruction

In most Wavelet Transform applications, it is required that the original signal be synthesized from the wavelet coefficients. To achieve perfect reconstruction the analysis and synthesis filters have to satisfy certain conditions. Let $G_0(z)$ and $G_1(z)$ be the low pass analysis and synthesis filters, respectively and $H_0(z)$ and $H_1(z)$ the high pass analysis and synthesis filters respectively. Then the filters have to satisfy the following two conditions given as:

$$G_0(-z)G_1(z) + H_0(-z)H_1(z) = 0 \tag{4.1}$$

$$G_0(z)G_1(z) + H_0(z)H_1(z) = 0 \tag{4.2}$$

V.STEPWISE IMPLEMENTATION

When digital images are to be viewed or processed at multiple resolutions, the discrete Wavelet Transform (DWT) is the mathematical tool of choice. In addition to this, being an efficient, highly intuitive framework for the representation and storage of multi-resolution images, the DWT provides powerful insight into an image’s spatial and frequency characteristics. Our project is a lossy compression since it uses a transform coding technique (by using DWT) and to perform the compression and reconstruction, MATLAB is used as a tool. In MATLAB we use some functions and decompose, compress and reconstructed image.

Firstly we load an image and the image is transformed into matrix form and then decomposition is done. Now the results of the decomposition are displayed. Now by using Huffman coding the redundancy bits are removed and then reconstruction is done.

V. Result Analysis

5.1 Output Screens



Fig 5.1: Input Image

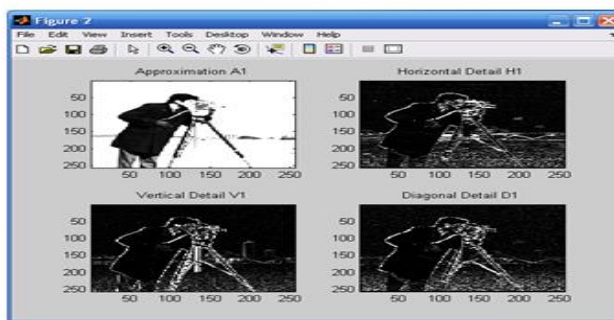


Fig 5.2: Horizontal, Vertical & Diagonal details at level 1

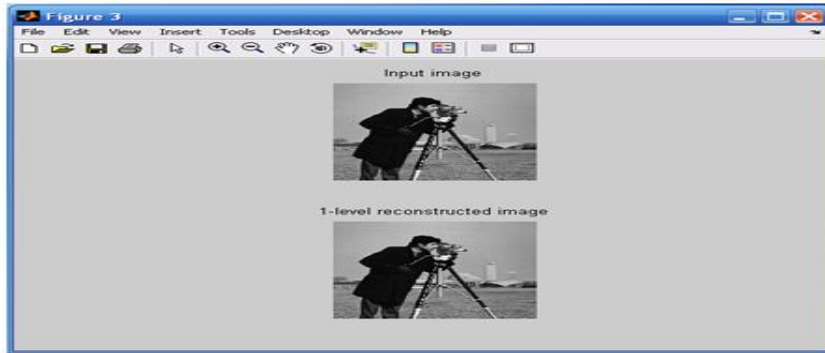


Fig 5.3: 1-Level Reconstructed Image.

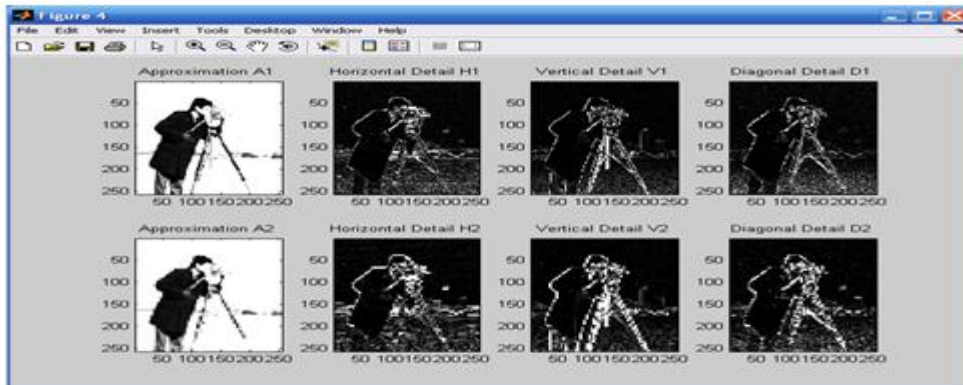


Fig 5.4: Horizontal, Vertical & Diagonal details at level 2.

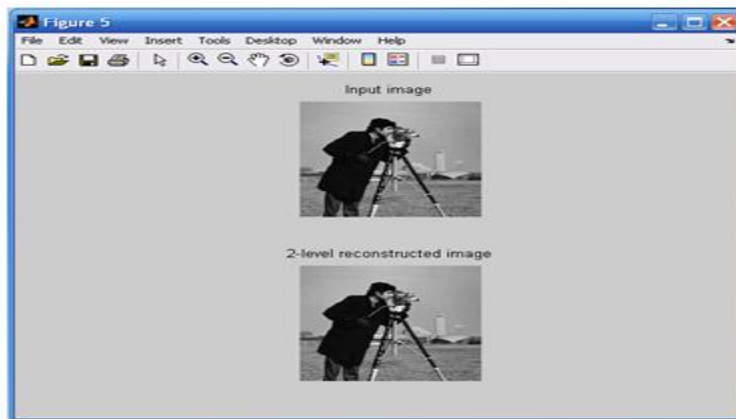


Fig 5.5: 2-Level Reconstructed Image.



Fig 5.6: Output with Decomposition level 1



Fig 5.7: Output with Decomposition level 2

Outputs at different Decomposition levels:

DECOMPOSITION LEVEL: 1

Enter the decomposition level: **1**
 Decomposition vector of size 1*524288 is stored in c
 Level-dependent thresholds: **1.0000**
 The entropy used: **threshold**
 The type of thresholding : **Hard Thresholding**
 Approximation coefficients: **1**
 Wavelet Packet Object Structure

=====
 Size of initial data: **[256 256]**
 Order : **4**
 Depth : **2**
 Terminal nodes : **[5 6 7 8 9 10 11 12 3 4]**

 Wavelet Name : **haar**
 Low Decomposition filter: **[0.7061 0.7061]**
 High Decomposition filter: **[-0.7061 0.7061]**
 Low Reconstruction filter: **[0.7061 0.7061]**
 High Reconstruction filter: **[0.7061 -0.7061]**

 Entropy Name : **threshold**
 Entropy Parameter : **1**

 The L² recovery : **97.8426**
 The compression scores in percentages : **25.7862**
 The number of rows in compressed image : **256**
 The number of columns in image : **256**

DECOMPOSITION LEVEL: 2

Enter the decomposition level:
 Decomposition vector of size 1*524288 is stored in c
 Level-dependent thresholds: **1.0000**
 The entropy used: **threshold**
 The type of thresholding : **Hard Thresholding**
 Approximation coefficients: **1**
 Wavelet Packet Object Structure

=====
 Size of initial data: **[256 256]**
 Order : **4**
 Depth : **2**
 Terminal nodes : **[5 6 7 8 9 10 11 12 3 4]**

Wavelet Name : **haar**
Low Decomposition filter: [**0.7071 0.7071**]
High Decomposition filter: [**-0.7071 0.7071**]
Low Reconstruction filter: [**0.7071 0.7071**]
High Reconstruction filter: [**0.7071 -0.7071**]

Entropy Name : **threshold**
Entropy Parameter : **1**

The L² recovery : **99.9985**
The compression scores in percentages : **26.6602**
The number of rows in compressed image : **256**
The number of coloums in image : **256**

VII. Conclusion & Future Scope

Wavelet analysis is very powerful and extremely useful for compressing data such as images. Its Power comes from its multi-resolution. Although other transforms have been used, for example the DCT was used for the JPEG format to compress images; wavelet analysis can be seen to be far superior. This is because the wavelet analysis is done on the entire image rather than sections at a time. A well known application of wavelet analysis is the compression of fingerprint images by the FBI. Changing the decomposition level changes the amount of detail in the decomposition. Thus, at higher decomposition levels, higher compression rates can be gained. The different images will have different compressibility. There are many possible extensions to this work. These include finding the best thresholding strategy, finding the best wavelet for a given image, investigating other wavelet families, the use of wavelet packets and image de-noising.

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