IOSR Journal of Sports and Physical Education (IOSR-JSPE)
e-ISSN: 2347-6737, p-ISSN: 2347-6745, Volume 3, Issue 5 (Sep. – Oct. 2016), PP 81-85
www.iosrjournals.org

Fluttery Motion of Falling Plastic Pieces as a Weighted Random Walk

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Abstract : *We study plane motion of fluttery falling plastic pieces as a students' research activity in high school.We perform experiments and record its motion by using digital video camera. While its motion seems to be quite random due to air resistance, from 573 data of both turning angle and step of plastic pieces, we find that it can be well described by weighted random walk, in which turning angle and step are chosen at every step as random walk with definite weight. We quantitatively analyze its weight, and find that it varies as a function of time, that is, turning angle decreases and step increases as time passes. As for the educational benefit, this activity will be helpful to broaden students' horizons from solving Newtonian equations of motion with known forces to statistical analysis with unknown random forces.*
Keywords: *Physics Education, Random Motion, Random Walk*

I. Introduction

To describe fluttery motion of falling object such as disks [1–3], and motion of rising water bubbles [4,5] have been paid great attention by many scientists so far. Since these fluttery random motion are induced by random forces of air or water resistance, it can be, in principle, completely described by Newtonian equations of motion with corresponding random forces. In order to describe Brownian motion [6,7], the most important example of random motion, Paul Langevin [8] introduced random forces into equations of motion of tiny particles in the pollen of plants. However in practice, it is almost impossible to know precise information about such random forces.While fluttery motion in vertical direction of falling disks [1–3] has been well studied, its projective motion onto the horizontal plane is still worth studying more in detail. In fact, in previous works for plane motion of rising water bubbles [4,5] and falling snow [9], its motion is assumed (observed) to be spiral in theoretical (experimental) works. However, orbit of falling or rising motion of real objects is not always spiral. It can be separated into two patterns; stable spiral (or zigzag) motion and unstable random motion. In the former case, quantities of spiral motion such as radius and pitch depend on physical condition of the object and circumstances where the motion occurs. On the other hand in the case of random motion, which we concentrate on in this paper, one can statistically analyze the motion instead of solving equation of motion. Physical conditions of object and circumstances are reflected in statistical quantities such as average displacement of the object from the origin and its variance.

In this paper, we investigate fluttery motion projected onto the horizontal plane of falling plastic piece, with radius 3.0 cm and mass 0.16 g. The pieces are freely falling from 5.5 m height inside the building of our school, and we record its horizontal plane motion with digital video camera. As a plastic piece falls from the origin, it randomly changes direction of motion and speed every moment. From view point of horizontal plane motion, such random motion seems to be random walk, in which turning angle and step of the walk probably reflect physical conditions of object and circumstances. By recording 573 data of turning angle and step of the walk, we find that the fluttery motion can be well described not by pure random walk but by weighted random walk, in which turning angle and step are stochastically chosen at every step from distribution with definite weight. We quantitatively analyze its weight, and find that it varies as a function of time, that is, turning angle decreases and step increases as time passes.This paper is organized as follows. In the next section, we give brief review of random walk. In Section 3, we mention experiment of fluttery motion which we have performed. From 573 data of the measurement, we find average value and variance of both turning angle and step. In Section 4, after introducing three possible theoretical models by taking the average values and the variances, we verify that the fluttery motion can be well described by the model of weighted random walk. Section 5 is devoted to conclusions.

II. Brief Review of Random Walk

In this section, we give a brief review on random walk in two- dimensions, specified as 2DRW. Let us consider 2DRW on (x, y) plane. If step of the walk varies, saying a_j for the j -th step, the location $P_N = (x_N, y_N)$ of a particle after N steps is given by

$$(x_N, y_N) = \sum_{j=1}^{N} a_j \left(\cos \sum_{i=1}^{j} \theta_i , \sin \sum_{i=1}^{j} \theta_i \right), \tag{1}$$

where the turning angle θ_i is measured from the direction of the i-1 th step as shown in Figure 1.
The distance from the origin after N steps, r_N , is defined as

$$r_N = \sqrt{x_N^2 + y_N^2}. \tag{2}$$

In order to study fluttery motion of falling plastic pieces, we suppose that step a and turning angle θ are randomly chosen in each step with corresponding probability density, referring to such motion as weighted random walk. While it is well known that one-dimensional random walk corresponds to the binomial distribution, we will adopt the normal distribution defined as

$$N\left(\mu, \sigma^2\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\left(x-\mu\right)^2}{2\sigma^2} \right], \tag{3}$$

where μ and σ^2 are the expectation value and the variance. We determine μ and σ for both a and θ from experiments and show that model of weighted random walk can well describe plane motion of falling plastic piece.

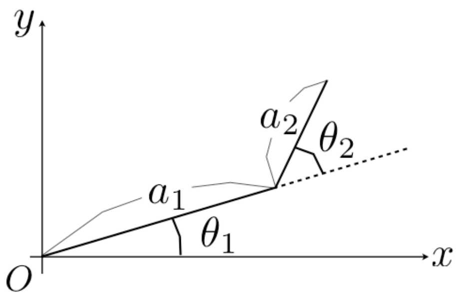

Figure 1: Definition of turning angle θ_i and step a_i for $i = 1,2$.

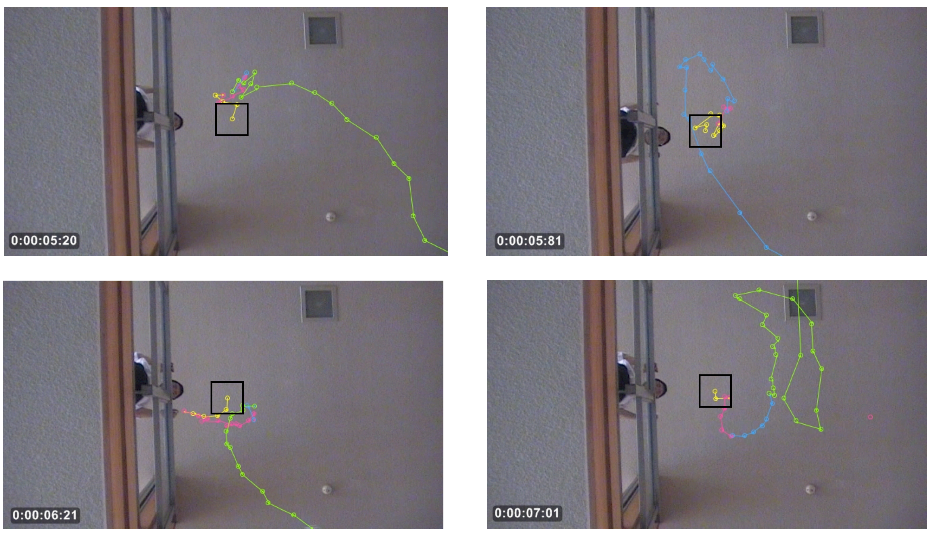

Figure 2: Typical examples of orbit for falling plastic piece, with plotting points in each 0.20 sec. After the piece starts falling, it goes out from the picture in 5.20, 5.81, 6.21 and 7.01 sec, respectively. One can see that the object differently behaves in the first and the second half of motion. See Sec. 4 for the meaning of the square around the origin.

III. Experiment

In order to analyze orbit of freely falling plastic pieces, we first mention experimental method. After that, we obtain statistical quantities of the motion such as average value and variance of turning angle θ and step a.We set a video camera on the floor of a building of our school, pointing upward direction. Plastic pieces are released from hand 5.5 m above the camera and it records the plane orbit of fluttery falling pieces. We plot position of the piece in each 0.20 sec. The piece is of radius 3.0 cm and of mass 0.16 g, since too small pieces will be immediately put in stable spiral orbit and too large pieces will be easily deformed.

Now we make a plot of the orbit, and measure the turning angle θ and step a of each step, defined in Figure 1. Since we measure these quantities from print-out picture with reduced scale, the value of a [cm] does not have any physical meanings, but its distribution does. Figure 2 shows typical examples of the orbit, where the piece starts falling at the center of the picture. Points are plotted in each 0.20 sec. One can see that the piece stays near the center for a while, it eventually move apart from the origin with larger steps. In the case of Figure 2, the piece goes out from the picture in 5.20, 5.81, 6.21 and 7.01 sec, respectively. As can be shown in Figure 2, in the first half of the fall, the piece moves around the origin with larger turning angles and smaller steps, while it moves with smaller turning angles and larger steps in the second half. Thus the piece behaves differently during the fall. Therefore for the data of turning angle and step, we separately analyze those of the first half (refer to as “1st”) and of the second half (refer to as “2nd”). One reason for larger steps in “2nd” is that the piece comes closer to the camera as it falls. In our analysis, we have not discriminated such effect of vertical motion from purely horizontal motion. One may concern that only purely horizontal motion should be extracted from three dimensional motion. However in educational point of view of this study, to statistically treat random motion by weighted random walk and not to describe it by Newtonian equations of motion, we simply perform analysis including both motions.

Figure 3 shows the relation between turning angle and step made from 573 data (24-times experiments) for both turning angle and step. Gray circles and “+” denote “1st” and “2nd” data, respectively. One can see that in the “1st” data, turning angles distribute to wide range of θ with smaller step a comparing to the “2nd” ones. Figure 4 shows frequency of angle (left panel) and that of step (right panel). Light-gray and dark-gray parts correspond to “1st” and “2nd” data, respectively. One can see that the histogram of turning angle (left) has a peak about $0 < \theta < 20$ [deg.] and that of step about $0.4 < a < 0.6$ [cm]. From those data, one can estimate the average value $\langle \cdots \rangle$ and the variance $\sigma(\cdots)$ for both θ and a, as shown in Table 1. The “1st” (“2nd”) data show relatively large (small) turning angle and small (large) step, respectively, as expected from Figure 2. The “total” represents values from all data. We suppose that fluttery motion of falling piece is described by weighted random walk, with the weight of normal distribution Eq. (3) and the values of Table 1.

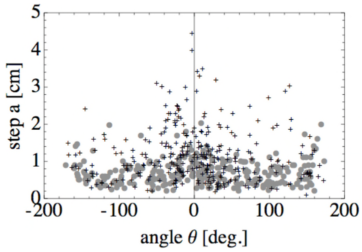

Figure 3: Relation between turning angle θ and step a. Gray circles and “+” denote the “1st” and “2nd” values, respectively.

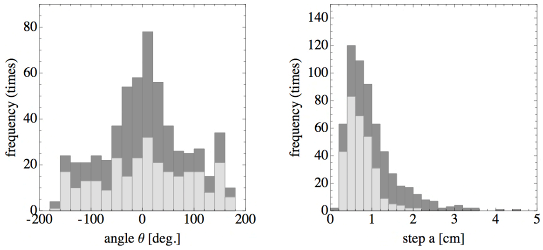

Figure 4: Histogram for turning angle θ (left) and step a (right). Light-gray and dark-gray parts correspond to “1st” and “2nd” values, respectively.

Table 1: Average value $\langle\cdots\rangle$ and variance $\sigma(\cdots)$ for turning angle θ [deg.] and step a [cm].

	$\langle\theta\rangle$	$\sigma(\theta)$	$\langle a\rangle$	$\sigma(a)$
total	4.49	81.6	0.95	0.64
1st	8.33	89.9	0.71	0.33
2nd	1.08	73.9	1.19	0.76

IV. Models and verification

In this section we give possible theoretical models to describe fluttery motion of plastic pieces, and discuss validity of models. While, in principle, such motion is predictable because it is described by Newtonian equations of motion, it is difficult to know exact magnitude and direction of forces acting on the piece at each moment. In such cases, one may consider the Langevin equation, equation of motion with random forces, to study fluttery motion. However in this paper, we suppose that fluttery motion of falling pieces is described by random walk without taking equations of motion into account. In the first subsection, we give three models of random walk with different weight. Next, we discuss validity of the models from experimental point of view in the second subsection.

1.1 Theoretical Models

In order to describe the fluttery motion, we introduce following three theoretical models of random walk with different weight.

- Model A: pure random walk. Turning angle θ is chosen in completely random way in each step, and step a is constant.
- Model B: weighted random walk (1). Tuning angle θ and step a are chosen at each step yielding normal distribution. Its average and variance stay unchanged during the motion, given as "total" in Table 1.
- Model C: weighted random walk (2). Tuning angle θ and step a are chosen at each step yielding normal distribution. While its average and variance will continuously change as a function of time, we assume that they change from "1st" to "2nd" given in Table 1 at the halfway point of orbit.

By comparing theoretical predictions from these models and experiments, we will find the Model C is able to describe the motion. This also tells us that the fluttery motion is described by weighted random walk with normal distribution.

1.2 Verification

Here we verify the validity of our models given above in two ways. First we compare position of the piece at a definite time, $t = 3.0$ s as an example. Since the pieces go out of monitor in about 6 seconds (See Figure 2), pieces are still seen in the monitor and start to diffuse at $t = 3.0$ s. As an experimental data, we measure the position of the particle at $t = 3.0$ s for 100 times (24 + additional 76). As an theoretical data, we calculate those in three models A, B and C for 100 times each. The step a of Model A is assumed to be 0.95, which is the averaged value given in Table 1. The left panel of Figure 5 shows distance from the origin at $t = 3.0$ s of the experiment and Model A, B and C. One can see that Model A obviously conflicts with the experimental data, while Model B and C are consistent with those.

Next, we compare density of points for relatively small t for the experiments, Model B and C (Model A has been excluded). Drawing 2 cm × 2 cm square at the origin as shown in Fig. 2, and counting the number of points in the square, one can see, from the right panel of Figure 5, that Model C is consistent with the experimental data, while Model B is not.Therefore we can say that Model C can reproduce realistic random motion of plastic piece, which tells us such random motion is a kind of weighted random walk. While it is confirmed in our study that its weight of the first half and the second half of the motion is different, it will continuously change as a function of time. If one obtains precise form of the time-dependent function, it will be possible to describe random motion realistically. However to obtain such function is beyond the scope of this paper.

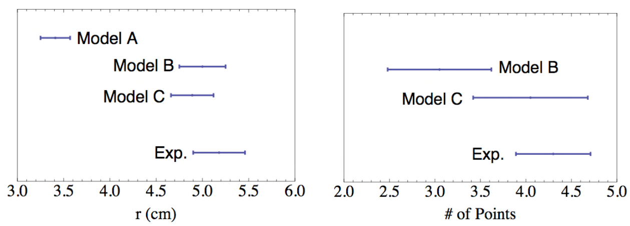

Figure 5: Distance from the origin at $t = 3.0$ sec. (left) and average number of points in the 2.0 cm × 2.0 cm square (right).

V. Conclusion

We have studied fluttery motion of falling plastic pieces as a students' activity in high school. The plastic piece is of radius 3.0 cm and mass 0.16 g. Motion of such relatively large pieces is quite random because of random forces of air resistance. In this work, we have statistically analyzed the random motion instead of solving Newtonian equations of motion with (unknown) random forces. While it occurs in three-dimensional space, we focus on the projective motion onto the horizontal plane.

We have recorded the horizontal plane motion of freely falling pieces by digital video camera inside the building of our school. From view point of horizontal plane motion, this random motion seems to be random walk with definite weights for turning angle and step at every step. Moreover, we found that a falling piece differently behaves in the first ("1st") and the second ("2nd") half of the motion. By taking 573 data of turning angle and step, we have obtained the average values and the variance of the random walk. In the three possible theoretical models, such as pure random walk (Model A), weighted random walk with time-independent weights (Model B) and that of time-dependent weights (Model C), we found that Model C can well describe the fluttery falling motion.

The average values and the variance of both turning angle and step will be determined by size and mass of the piece, condition of the air such as atmospheric pressure and humidity, etc. The time-dependent function form of those also depends on such experimental conditions. However, to find these precise knowledge on the random motion is beyond the scope of this paper.

As for the educational benefit of our work, it will be helpful to broaden students' horizons. In courses of basic physics, in order to understand physical thought, students learn how to solve Newtonian equations of motion with definite forces in simple situations. On the other hand, what if a student sees more complicated phenomena such as fluttery motion of falling objects? When he/she can not write down equations of motion, he/she might qualitatively consider what happens. Statistical analysis which we have done in this paper will be a way of quantitative consideration for random and complicated motions.

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