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Hidden Variables versus Bell's Inequality and Conflicts of Superposition, Complementarity and Entanglement in Quantum Mechanics

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[Abstract]: *The mistake of using Bell's Inequality in defeating Hidden Variables as a reasonable solution of EPR Paradox is revealed and discussed. Both entangled photon and electron pairs can change their quantum energy states (hidden variables) through a transformation process (polarization or spin measurement) to a new corresponding entangled quantum state no matter of Bell's Inequality. Because of the existence of predetermined quantum energy states, Schrödinger's Cat and Superposition Theory cannot be true. In addition, the phase angle of a particle wave can be changed by a detector which results in the destruction of interference patterns in Double Slit Interference Experiment, therefore, Complementarity Principle is not true neither.*

[Keywords]: *Yangton and Yington, Wu's Pairs, Photon, Polarization, Photon Spin, Electron Spin, Double Slit Interference, Quantum Entanglement, Superposition, Schrödinger's Cat, Complementarity, Quantum Mechanics, Hidden Variables, EPR Paradox, Bell's Inequality.*

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I. Schrödinger's Cat and Quantum Superposition

Schrödinger's cat is a thought experiment, devised by Austrian physicist Erwin Schrödinger in 1935, though the idea originated from Albert Einstein. It illustrates what he saw as the problem of the Copenhagen interpretation of quantum mechanics applied to everyday objects. The scenario presents a hypothetical cat that may be simultaneously both alive and dead, a state known as a quantum superposition. There were many discussions on this topic in the past century. However, it is my belief that superposition cannot be true, simply because that alive and dead as well as up spin and down spin cannot coexist. Otherwise, it will be against of common sense and fundamental principle of logic.

II. Complementarity and Double Slit Interference

In modern physics, the double-slit experiment is a demonstration that light and matter can display characteristics of both classically defined waves and particles. Moreover, it displays the fundamentally probabilistic nature of quantum mechanical phenomena. This type of experiment was first performed, using light, by Thomas Young in 1801, as a demonstration of the wave behavior of light.

In the basic version of this experiment, a coherent light source, such as a laser beam, illuminates a plate pierced by two parallel slits, and the light passing through the slits is observed on a screen behind the plate. The wave nature of light produces interference (Fig. 1) that would not be expected if light consisted of classical particles.

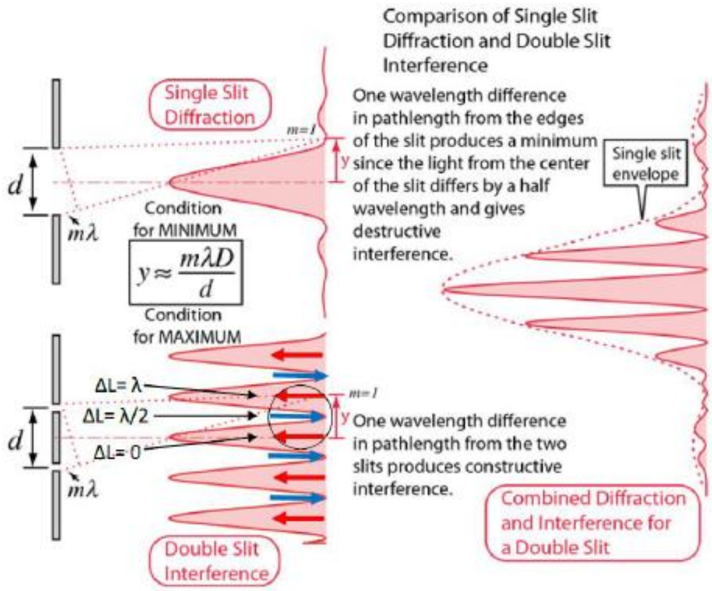

Fig. 1 Single Slit Diffraction and Double Slit Interference.

Other versions of the experiment that include detectors at the slits find that each detected photon passes through one slit (as would a classical particle), and not through both slits (as would a wave) [1]. As a result, two single slit diffraction patterns can be found without interference.

In 1961, Claus Jönsson of the University of Tübingen performed the experiment with electron beams [2]. In 1974, the Italian physicists Pier Giorgio Merli, Gian Franco Missiroli, and Giulio Pozzi repeated the experiment using single electrons and biprism (instead of slits). Sending particles such as electrons through a double-slit apparatus one at a time results in single particles appearing on the screen, however, an interference pattern emerges when these particles are allowed to build up one by one (Fig. 2). This demonstrates the wave particle duality, which states that all matter exhibits both wave and particle properties: the particle is measured as a single pulse at a single position, while the wave describes the probability of absorbing the particle at a specific place on the screen [3].

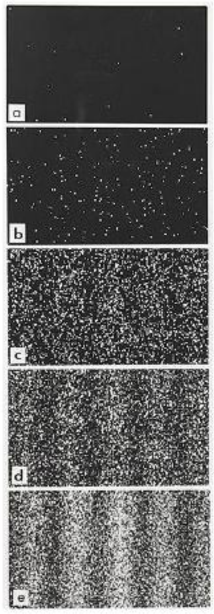

Fig. 2 Step by step build up pattern of electron Double Slit Interference.

This phenomenon has been shown to occur with photons, electrons, atoms and even some molecules, including buckyballs. So experiments with electrons add confirmatory evidence to the view that electrons, protons, neutrons, and even larger entities that are ordinarily called particles nevertheless have their own wave nature and even a wavelength (related to their momentum).

The double-slit experiment (and its variations) has become a classic thought experiment, for its clarity in expressing the central puzzles of quantum mechanics. Because it demonstrates the fundamental limitation of the ability of the observer to predict experimental results, Richard Feynman called it "A phenomenon which is impossible to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the only mystery of quantum mechanics" [4].

A well-known thought experiment predicts that if particle detectors are positioned at the slits, showing through which slit a photon goes, the interference pattern will disappear.[4] This which-way experiment illustrates the complementarity principle that photons can behave as either particles or waves, but cannot be observed as both at the same time.

Despite all arguments, I believe that the interaction between particle and detector is the reason to cause the changes of the interference patterns [5].

III. Quantum Entanglement and EPR Paradox

Quantum entanglement is the physical phenomenon that occurs when a pair or group of particles is generated at the same time, they interact or share spatial proximity in a way such that the quantum state of each particle of the pair or group cannot be described independently of the state of the others, even when the particles are separated by a large distance.

Measurements of physical properties such as position, momentum, spin and polarization performed on entangled particles are found to be perfectly correlated. For example, if a pair of entangled particles is generated such that their total spin is known to be zero, and one particle is found to have clockwise spin on a first axis, then the spin of the other particle, measured on the same axis, even instantly will be found to be counterclockwise. However, this behavior gives rise to paradoxical effects: any measurement of a property of an entangled particle results in an irreversible wave function collapse of that particle which can cause interruption of the entanglement and subsequently a random state of the other particle can be measured.

In 1935, Albert Einstein, Boris Podolsky, and Nathan Rosen [6] brought up EPR paradox, in which Einstein and others considered such behavior to be impossible unless instant communication can be fulfilled for an infinite distance. It violates the local realism view of causality (Einstein referring to it as "spooky action at a distance") and argued that the accepted formulation of quantum mechanics must therefore be incomplete. The weak point in EPR's argument was not discovered until 1964, when John Stewart Bell proved by his inequality that the Hidden Variables interpretation hoped for by EPR, was mathematically inconsistent with the predictions of quantum theory.

In fact, there are four possible arrangements for entangled particles. Table 1 shows the results of instant measurement and consistent measurement of each arrangement:

Table 1. Instant measurement and consistent measurement for entangled particles of possible arrangements.

	Instant measurement	Consistent measurement
Predetermined (Hidden Variables)	Anti State	Anti State
Prescheduled	Anti State	Random
Superposition (One each time)	Upon communication speed and frequency	Random
Superposition (Co-exist)	Upon communication speed and frequency	Random

Obviously, Hidden Variables (predetermined quantum states) is the only arrangement that can ensure to find the counter entangled particle always in anti quantum state.

IV. Hidden Variables and Bell's Inequality

When a light beam passing through three polarizers with polarization angles A = 0^o, B = 22.5^o and C = 45^o, the intensity of the transmitted light can be displayed in Table 2. Where "Real Transmission" is the measurement results and "Bell Transmission" is the theoretical results based on Bell's Inequality. Since the real results are different from Bell's Inequality, therefore, Hidden Variables is excluded from the solution of EPR Paradox.

Table 2. The Real Transmission and Bell Transmission of three polarizers with polarization angels A = 0^o, B = 22.5^o, C = 45^o.

Polarizer	Real Transmission	Bell Transmission
A	100%	100%
A and B	85%	75%
A but B	15%	25%
(A and B) and C	72.25%	50%
(A and B) but C	12.75%	25%
(A but B) and ((A and B) but C)	27.75%	50%
A and C	50%	50%
A but C	50%	50%
(A and C) and B	42.5%	50%
Bell's Inequality (A but C) ≤ (A but B) and ((A and B) but C)	50% ≤ 27.75%?	50% ≤ 50%
Bell's Inequality (A and B) and C = (A and C) and B	72.25% = 42.5%?	50% = 50%
Remarks	Doesn't meet Bell's Inequality	Meets Bell's Inequality

Bell's Inequality is based on Set Theory (Fig. 3). However, I wonder Bell's Inequality can be used as an effective mathematical tool to analyze and prove that whether if predetermined quantum states (Hidden Variables) is the solution of EPR Paradox. More detailed discussion will be included in the latter section of this paper.

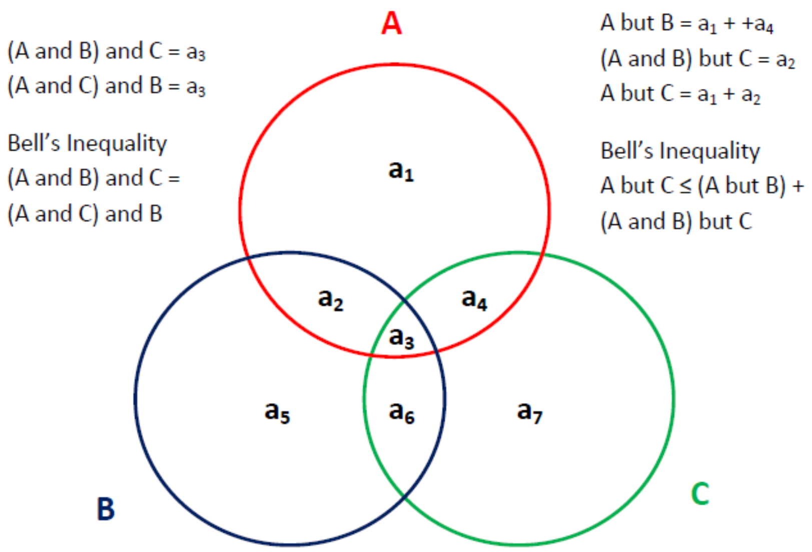

Fig. 3 Bell's Inequality Diagram – The distribution of elements in three domains (sets). All elements must stay unchanged no matter of distribution.

V. Electron Entanglement

1. Dual Spins

According to Yangton and Yington Theory [7], electron has a ball structure (Fig. 4) which is composed of an outer shell (a group of circulating Yingtons) and an inner core (a cluster of rotating Yangtons).

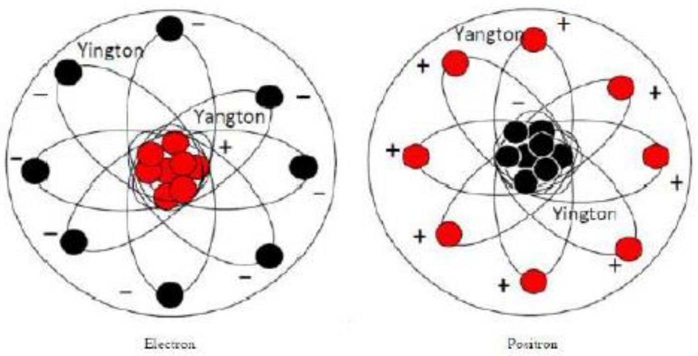

Fig. 4 Hypothetical structures of electrons and positrons.

It is proposed when electron spins, they can move either in the same directions or the opposite directions. This phenomenon is named "Dual Spins" [8]. In Dual Spin System, there are two major categories: "Up Spin" and "Down Spin" which are defined by the circulation direction of Yington Shell. In addition, there are two minor categories: "Parallel Spin" and "Anti Parallel Spin" which are defined by both directions of Yington Shell and Yangton core. Together, there are a total of four spin modes: Up-Up (U_u) and Up-Down (U_d) modes for Up Spin; and Down-Down (D_d) and Down-Up (D_u) modes for Down Spin (Fig. 5).

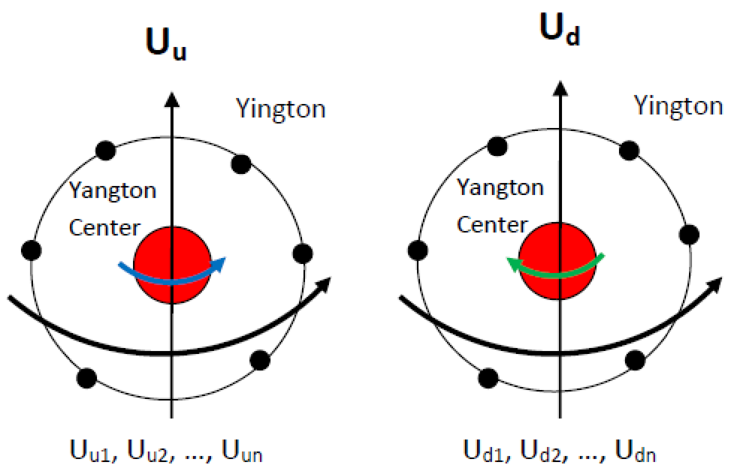

Fig. 5 Electron Spin contains Four Modes: (1) Up Spin: UP-UP (U_u) and UP-Down (U_d) modes, (2) Down Spin: Down-Down (D_d) and Dow-Up (D_u) modes. Each mode contains equal amount of energy states.

2. Quantum States

Subject to the difference of the angular momentums between Yington Shell and Yangton Core, there are a number of quantum states in each of the spin modes. Each quantum state can be represented by a composite code, for example U_u5 means the 5th energy level of Up-Up (U_u) Mode. According to Pauli Exclusion Principle [9], an electron can only occupy one quantum state at a time, therefore a pair of entangled electrons should have quantum states of the same energy but opposite spin modes such as U_u5 and D_d5. In addition, all spin modes have equal amounts of quantum states. Furthermore, it is proposed that backward spin U_d has higher energy than that of forward spin U_u (same for D_u and D_d). Also, all electrons prefer to stay in the low energy quantum states rather than the high energy quantum states.

3. Transformation

To measure the electron spin, a magnetic field is applied to the electron in a specific direction and the electron is detected in either spin up or spin down directions. Fig 6 shows an electron spin measurement, where B_1 is the internal magnetic field of the electron, B_2 is the external magnetic field applied by the measurement device and Θ is the angel between B_1 and B_2.

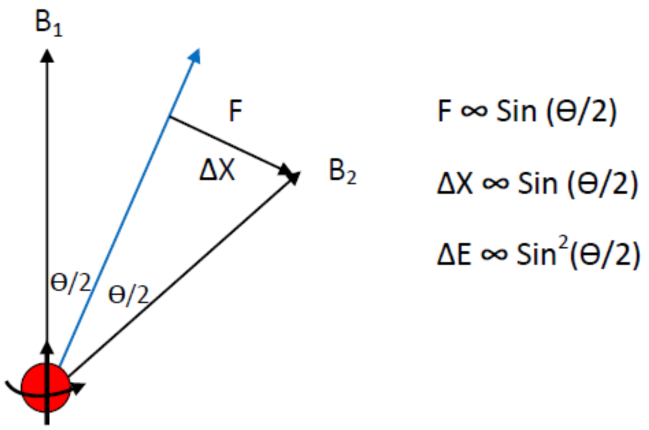

Fig. 6 Force and energy applied on an electron to change its spin direction in the measurement process (transformation).

Because
$F \propto \text{Sin}\,(\Theta/2)$
$\Delta X \propto \text{Sin}\,(\Theta/2)$

$$\Delta E \propto \text{Sin}^2(\Theta/2)$$

In Up-Down mode, the highest energy quantum state is EU_dn (Fig. 7). Any quantum state has higher energy than EU_dn will be transformed to Down-Up mode in the new direction, therefore,

$$E_m(\Theta) + \Delta E(\Theta) = E_{Ud}n$$

$E_m(\Theta) + K\ \text{Sin}^2\,(\Theta/2) = E_{Ud}n$

Where $E_m(\Theta)$ is the minimum energy quantum state to be transformed and $\Delta E(\Theta)$ is the transformation energy at angle Θ.

Because at $\Theta = 90^o$, all quantum states in Up-Down mode will be transformed to the Down-Up Mode in the new direction (Fig. T), therefore,

$E_m(90^o) = ½\ E_{Ud}n$

$½\ E_{Ud}n + K\ \text{Sin}^2\,(45^o) = E_{Ud}n$

$K\ \text{Sin}^2\,(45^o) = ½\ E_{Ud}n$

$$K = E_{Ud}n$$

Where $E_{Ud}n$ is the highest quantum energy state in U_d mode.

Because

$E_m(\Theta) + K\ \text{Sin}^2\,(\Theta/2) = E_{Ud}n$

Therefore,

$$E_m(\Theta)/E_{Ud}n = \mathrm{Cos}^2(\Theta/2)$$

$$\Delta E(\Theta)/E_{Ud}n = \mathrm{Sin}^2(\Theta/2)$$

Because all quantum states below $E_m(\Theta)$ will remain in the same modes after transformation, therefore, the overall possibility to find the spin up mode in the new direction can be represented as:

$$P(\Theta) = \mathrm{Cos}^2(\Theta/2)$$

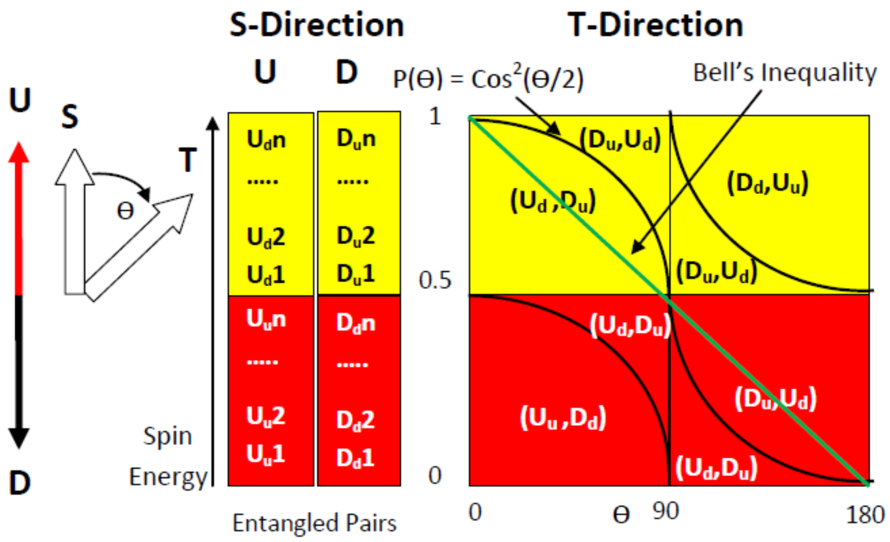

Fig. 7 Quantum Entanglement Phase Diagram: The transformation Diagram of Entangled Electron pairs from S direction to T direction.

Fig. 7 shows a detailed diagram of the transformation, in which entangled electron pairs (U_nx, D_dx) and (U_dy, D_uy) in S direction are transformed to T direction in different entangled modes at an angel from 0^o to 180^o. The probability of the transformation of electron entanglement from up spin to down spin is equal to $P(\Theta) = \mathrm{COS}^2(\Theta/2)$ which is different from the Bell's Inequality. This diagram is named "Quantum Entanglement Phase Diagram" [8].

A revised mathematical derivation of this phase diagram from my previous publication [8] is shown as follows:

A. $\Theta < 90^o$

a. $E_{Ud} + \Delta E(\Theta) < E_{Ud}n$

$E_{Ud} + (1 - \mathrm{COS}^2(\Theta/2))\, E_{Ud}n < E_{Ud}n$

$E_{Ud} < \mathrm{COS}^2(\Theta/2)\, E_{Ud}n$

Therefore,

$$U_d \rightarrow U_d$$

b. $E_{Ud} + \Delta E(\Theta) \geq E_{Ud}n$

$E_{Ud} + (1 - \mathrm{COS}^2(\Theta/2))\, E_{Ud}n \geq E_{Ud}n$

$E_{Ud} \geq \mathrm{COS}^2(\Theta/2)\, E_{Ud}n$

Therefore,

$$U_d \rightarrow D_u$$

c. $E_{Uu} + \Delta E(\Theta) < ½\, E_{Ud}n$

$E_{Uu} + (1 - \mathrm{COS}^2(\Theta/2))\, E_{Ud}n < ½\, E_{Ud}n$

$E_{Uu} < (\mathrm{COS}^2(\Theta/2) - ½)\, E_{Ud}n$

Therefore,

$$U_u \rightarrow U_u$$

d. $E_{Uu} + \Delta E(\Theta) \geq ½\, E_{Ud}n$

$E_{Uu} + (1 - \mathrm{COS}^2(\Theta/2))\, E_{Ud}n \geq ½\, E_{Ud}n$

$E_{Uu} \geq (\mathrm{COS}^2(\Theta/2) - ½)\, E_{Ud}n$

Therefore,

$$U_u \rightarrow U_d$$

B. $\Theta \geq 90^o$

a. $E_{Uu} + \Delta E(\Theta)\, E_{Ud}n \geq E_{Ud}n$

$E_{Uu} + (1 - COS^2 (\Theta/2))\, E_{Ud}n \geq E_{Ud}n$

$E_{Uu} \geq COS^2 (\Theta/2)\, E_{Ud}n$

Therefore,

$$U_u \rightarrow D_u$$

b. $E_{Uu} + \Delta E(\Theta)\, E_{Ud}n < E_{Ud}n$

$E_{Uu} + (1 - COS^2 (\Theta/2))\, E_{Ud}n < E_{Ud}n$

$E_{Uu} < COS^2 (\Theta/2)\, E_{Ud}n$

Therefore,

$$U_u \rightarrow U_d$$

c. $E_{Ud} + \Delta E(\Theta) < (1 + ½)\, E_{Ud}n$

$E_{Ud} + (1 - COS^2 (\Theta/2))\, E_{Ud}n < (1 + ½)\, E_{Ud}n$

$E_{Ud} < (COS^2 (\Theta/2) + ½)\, E_{Ud}n$

Therefore,

$$U_d \rightarrow D_u$$

d. $E_{Ud} + \Delta E(\Theta) \geq (1 + ½)\, E_{Ud}n$

$E_{Ud} + (1 - COS^2 (\Theta/2))\, E_{Ud}n \geq (1 + ½)\, E_{Ud}n$

$E_{Ud} \geq (COS^2 (\Theta/2) + ½)\, E_{Ud}n$

Therefore,

$$U_d \rightarrow D_d$$

A corresponding identical result can also be derived for Down-Up Mode (D_u) and Down-Down Mode (D_d) (Fig. 7).

All entangled electrons (Fig. 7) with predetermined quantum states in S direction can be transformed to T direction either remain in the same modes or change to a counter entangled modes (UP → Down and Down → Up). For example: a pair of entangled electrons (U_dx, D_ux) can be transferred to (D_uy, U_dy) or (D_dz, U_uz) at different angles. In other words, "Hidden Variables" (predetermined quantum states) can be modified by adding more energy and transformed to the new corresponding entangled quantum states under the influence of measurement. Because all entangled electron pairs gain additional energy through measurement process (in other words, they are no longer the same elements prior to the measurement), therefore the probability of distribution of the entangled electrons observed via measurement (transformation) doesn't have to follow Bell's Inequality and Einstein's Hidden Variables remains a reasonable solution of EPR paradox [8].

VI. Photon Polarization and Entanglement

1. Antimatter Revolution and Rotation Spins (ARRS)

According to Yangton and Yington Theory, photon has a disc structure which is composed of two anti particles, Yangton and Yington circulating on the same orbit [7]. It is proposed while Yangton and Yington circulating the orbit – revolution spin (photon spin), they can also rotate by them self (Yangton spin and Yington spin). This phenomenon is named "Antimatter Revolution and Rotation Spins" (ARRS). In ARRS, there are two major spin categories: "Up Spin" – photon spins in up direction and "Down Spin" – photon spins in the down direction. In addition, there are two minor spin categories: "Parallel Spin" – Yangton and Yington spin in the same direction as photon and "Anti Parallel Spin" – Yangton and Yington spin in the opposite directions. Together, there are a total of four spin modes: Up-Parallel (U_p) and Up-Anti Parallel (U_a) modes for Up Spin; and Down-Parallel (D_p) and Down-Anti Parallel (D_a) modes for Down Spin (Fig. 8).

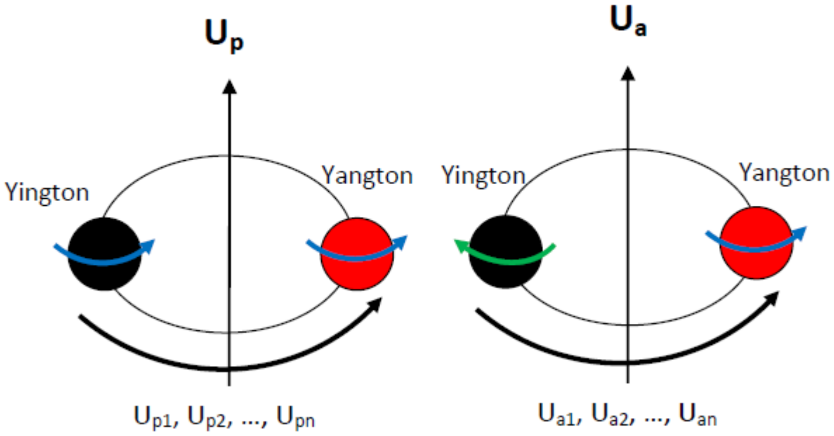

Fig. 8 Photon Spin contains Four Modes: (1) Up Spin: Up-Parallel (U_p) and Up-Anti Parallel (U_a) modes, (2) Down Spin: Down-Parallel (D_p) and Dow-Anti Parallel (D_a) modes. Each mode contains equal amount of energy states.

2. Quantum States

Subject to the difference of the angular momentums between Yington and Yangton, there are a number of quantum states in each of the spin modes. Each quantum state can be represented by a composite code, for example U_p5 means the 5th energy level of Up-Parallel (U_p) Mode. According to Pauli Exclusion Principle [9], a photon can only occupy one quantum state at a time, therefore a pair of entangled photons should have quantum states of the same energy but opposite spin modes such as U_p5 and D_p5. Also all spin modes have equal amounts of quantum states. Furthermore, it is proposed that Anti Parallel spin U_a has higher energy than that of Parallel spin U_p (as is D_a to D_p). In addition, all photons prefer to stay in the low energy quantum states rather than the high energy quantum states.

3. Polarization Transformation

When photons transform between two polarization directions, they need to overcome an energy barrier. Fig 9 shows a photon transformation between two polarization directions, where B_1 is the magnetic field of the photon in the original polarization direction, B_2 is the magnetic field of the new polarization direction and Θ is the angel between B_1 and B_2.

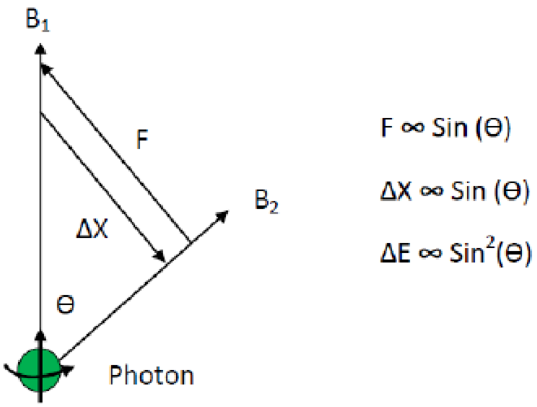

Fig. 9 Force and energy applied from a photon to change its original spin direction to a new polarization direction.

Because

$F \propto \mathrm{Sin}(\Theta)$

$\Delta X \propto \mathrm{Sin}(\Theta)$

$\Delta E \propto Sin^2(\Theta)$

Because only photons in Up molds (U_p and U_a modes) having higher energy than ΔE (energy barrier) can be transformed to the same Up modes (U_p and U_a modes) in the new polarization direction, therefore,

$$E_m(\Theta) = \Delta E(\Theta)$$

$$E_m(\Theta) = K\ Sin^2(\Theta)$$

Where $E_m(\Theta)$ is the minimum energy quantum state that can be transformed to the new polarization direction and $\Delta E(\Theta)$ is the energy barrier at angle Θ.

Because at $\Theta = 90^o$, all photons in the Up mode are blocked by the polarizer and no light can be transformed to the new polarization direction (pass through the polarizer), therefore,

$E_m(90^o) = E_{Ua}n$

$K\ Sin^2(90^o) = E_{Ua}n$

$$K = E_{Ua}n$$

Because

$E_m(\Theta) = K\ Sin^2(\Theta)$

Therefore,

$$E_m(\Theta)/ E_{Ua}n = Sin^2(\Theta)$$

Where $E_{Ua}n$ is the highest quantum energy state in U_a mode.

Because all photons with quantum states above $Sin^2(\Theta)\ E_{Ua}n$ can be transferred to the new polarization direction, therefore, the overall possibility to find the photons in the polarization direction (Θ) can be represented as:

$$P(\Theta) = Cos^2(\Theta)$$

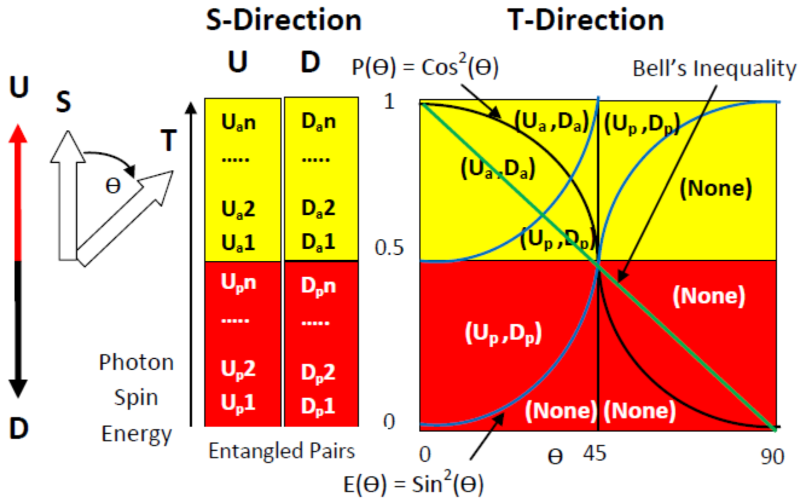

Fig. 10 Photon Polarization Transformation Diagram: The Transformation Diagram of Entangled Photon pairs from S direction to T direction.

Fig. 10 shows a detailed diagram of photon polarization transformation at a polarization angle from 0^o to 90^o. For those entangled photons (U_a, D_a) and (U_p, D_p), their quantum energy states are higher than the polarization transformation energy barrier $\Delta E(\Theta) = Sin^2(\Theta)\ E_{Ua}n$, they can overcome the energy barrier and transform to the lower quantum energy states (U_a, D_a) and (U_p, D_p) in the new polarization direction. Otherwise they will be blocked by the energy barrier if they are at lower quantum energy states. The probability of polarization transformation is equal to $P(\Theta\ ) = COS^2(\Theta)$ which is different from the Bell's Inequality. This diagram is named "Photon Polarization Transformation Diagram".

Similar to entangled electrons [8], however instead of gaining energy through measurement, the predetermined photon quantum states before polarization transformation, known as "Hidden Variables", can be changed by spending the internal energy to overcome the polarization energy barrier and then transformed to the new corresponding entangled quantum states through polarization process (Fig. 10). Because all entangled photon pairs loose energy through polarization transformation, they are no longer the same elements prior to the polarization transformation, therefore the probability of distribution of the entangled photons observed via polarization transformation process doesn't have to follow Bell's Inequality and Einstein's Hidden Variables remains a reasonable solution of EPR paradox.

VII. Confusion of Bell's Inequality

Bell's Inequality can be applied only in a space where all the elements in the space must stay unchanged no matter of distribution. However, in photon polarization and electron entanglement transformation processes, all photon and electron quantum states (Hidden Variables) have been changed either by adding or reducing energy to the particles due to the transformation. In other words, all elements in the domain space are not the same elements prior to transformation. Therefore, Bell's Inequality cannot be used to prove if Hidden Variables exist. In fact, Hidden Variables exist simply to obey Locality and Realism.

VIII. Conflicts in Quantum Mechanics

According to Yangton and Yington Theory, both photon and electron exist in a predetermined quantum states (Hidden Variables), therefore, it is believed that Schrödinger's Cat can only stay in either alive or dead status before the detection, but not in both at the same time. Therefore, "Superposition" that both alive and dead can coexist simultaneously cannot be true.

In addition, particle detector can change the interference patterns by influencing the phase angles of particle waves in Double Slit Interference experiment [5]. Therefore, "Complementarity" that both wave and particle properties cannot be observed or measured simultaneously is also not true.

Furthermore, Quantum Entanglement can be very well explained by Yangton and Yington Theory [8]. Both entangled photon and electron pairs can change their quantum energy states (hidden variables) through a transformation process (polarization or spin measurement) to new corresponding entangled quantum states no matter of Bell's Inequality.

As a result, all above facts have raised a serious challenge to Superposition Theory and Complementarity Principle – the heart of Quantum Mechanics.

IX. Conclusion

The mistake of using Bell's Inequality in defeating Hidden Variables as a reasonable solution of EPR Paradox is revealed and discussed. Both entangled photon and electron pairs can change their quantum energy states (hidden variables) through a transformation process (polarization or spin measurement) to a new corresponding entangled quantum state no matter of Bell's Inequality. Because of the existence of predetermined quantum energy states, Schrödinger's Cat and Superposition Theory cannot be true. In addition, the phase angle of a particle wave can be changed by a detector which results in the destruction of interference patterns in Double Slit Interference Experiment, therefore, Complementarity Principle is not true neither.

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