IOSR Journal of Polymer and Textile Engineering (IOSR-JPTE)
e-ISSN: 2348-019X, p-ISSN: 2348-0181, Volume 5, Issue 5 (Sep. - Oct. 2018), PP 15-20
www.iosrjournals.org

Dual prime numbers.

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Abstract: *After defining, dual prime numbers will be presented: Bölcsföldi prime numbers from 23 to 33333333223, Birkás prime numbers from 227 to 77772727777, Bölcsföldi-Balogh prime numbers from 353 to 53333335333, Bölcsföldi- Madar prime numbers from 557 to 57775777757 . How many dual prime numbers are there in the interval (10^{p-1}, 10^p) (where p is a prime number)? On the one hand, it has been counted by computer at least to 13 digits. On the other hand, the function (1) gives the approximate number of dual prime numbers in the interval (10^{p-1},10^p). Near-proof reasonig has emerged from the conformity of Mills' prime numbers with dual prime numbers. The sets of dual prime numbers are probably infinite.*

Date of Submission: 15-09-2018 Date of acceptance: 30-09-2018

I. Introduction

The sets of special prime numbers within the set of prime numbers are well-known. For instance, the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes (F_0=0, F_1=1, $F_n=F_{n-1}+F_{n-2}$), Gauss-primes (in the form 4n+3), Leyland-primes (in the form x^y+y^x, where $1\le x\le y$), Pell-primes (P_0=0, P_1=1, $P_n=2P_{n-1}+P_{n-2}$), Bölcsföldi-Birkás primes (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found 4 further sets of special prime numbers within the set of prime numbers. It are the sets of Bölcsföldi-, of Birkás-, of Bölcsföldi-Balogh- and of Bölcsföldi-Madar primes.

II. Bölcsföldi prime numbers [3], [9], [10], [11], [12].

Definition:a positive integer number is a Bölcsföldi prime number, if
a/ the positive integer number is prime, b/ all digits are 2 or 3, c/ the number of digits is prime,
d/ the sum of digits is prime.
The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi prime numbers .
Bölcsföldi prime number p has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{2, 3\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \quad \text{and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The Bölcsföldi prime numbers are as follows (the last digit can only be 3):
23,
223,
32233, 32323,33223,
2222333, 2223233, 2232323, 2233223, 2332333, 2333323, 3223223, 3223333, 3233323, 3332233,
22222222223, 22223333333, 22233332333, 22233333323, 22332233333, 22332333323, 22333233233,
22333332233, 23222333333, 23223332333, 23232233333, 23233223333, 23233223333, 23233232333,
23233333223, 23322323333, 23323322333, 23332223333, 23332332323, 23333233333, 32223332333,
32232333233, 32233323233, 32323223333, 32323333223, 32332232333, 32332333333, 32333223233,
32333322323, 32333332333, 33222323333, 33223332323, 33223333333, 33233323223, 33322233323,
33332333233, 33333222233, 33333332323, 33333333223, etc.

T(p) is the factual frequency of Bölcsföldi prime numbers in the interval (10^{p-1}, 10^p).
T(2)=1, T(3)=1, T(5)=3, T(7)=10, T(11)=39, T(13)=104, T(17)=2526, T(19)=4915, etc.
S(p) function gives the number of Bölcsföldi prime numbers in the interval (10^{p-1},10^p). We think that

$S(p)=1,0404x1,6^{p-1}$, where p is prime. (1)

The factual number of Bölcsföldi primes and the number of Bölcsföldi primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Bölcsföldi primes in the interval $(10^{p-1},10^{p})$ T(p)	The number of Bölcsföldi primes according to function $S(p)=1,0404x1,6^{p-1}$	T(p)/S(p)
2	1	1,6	0,63
3	1	2,66	0,38
5	3	6,82	0,44
7	10	16,78	0,60
11	39	114,39	0,34
13	104	292,85	0,36
17	2526	1919,20	1,32
19	4915	4913,15	1,00

III. Birkás prime numbers [3], [9], [10], [11], [12].

Definition:a positive integer number is a Birkás prime number, if
a/ the positive integer number is prime, b/ all digits are 2 or 7, c/ the number of digits is prime,
d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Birkás prime numbers (Fig.1, Fig.2).

Birkás prime number p has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) € \{2, 7\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) € \{ 7\} \quad \text{and} \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The Birkás prime numbers are as follows (the last digit can only be 7):
227,
2227727, 2272727, 2772227, 7272227,
22222272727, 22222777277, 22227222727, 22227227777, 22227277727, 22227722227, 22227772727, 22272272227, 22272277277, 22272727727, 22277227727, 22277772227, 22722222277, 22722222727, 22722277727, 22727222227, 22727277227, 22772227727, 22772722277, 27222222277, 27222227777, 27222722227, 27222727277, 27222777227, 27227222777, 27272222227, 27272227727, 27272722727, 27277777777, 27722227277, 27727272227, 27727722227, 27772227227, 27772777777, 27777777277, 72222722227, 72222722777, 72222772277, 72227222777, 72227722727, 72227727227, 72272222227, 72272227277, 72272277227, 72277227227, 72722222777, 72722727227, 72727222727, 72777222227, 72777777727, 77222227727, 77272272227, 77277772777, 77772727777, etc.

3

H(p) is the factual frequency of Birkás prime numbers in the interval $(10^{p-1}, 10^{p})$. H(2)=0, H(3)=1, H(5)=0, H(7)=4 , H(11)=54 , H(13)=207, H(17)=2485, H(19)=7950, H(23)=59043,. etc. S(p) function gives the number of Birkás prime numbers in the interval $(10^{p-1},10^{p})$. We think that **$S(p)=1,9078x1,6^{p-1}$**, where p is prime. (1)

The factual number of Birkás primes and the number of Birkás primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Birkás primes in the interval $(10^{p-1},10^{p})$ H(p)	The number of Birkás primes according to function $S(p)=1,9078x1,6^{p-1}$	H(p)/S(p)
2	0	3,0525	0
3	1	4,8840	0,20
5	0	12,5030	0
7	4	32,0076	0,12

11	54	209,7648	0,26	
13	207	536,9980		0,39
17	2485	3519,2698	0,71	
19	7950	9009,3308	0,88	
23	59043	59043,5501	1,00	

Fig.1

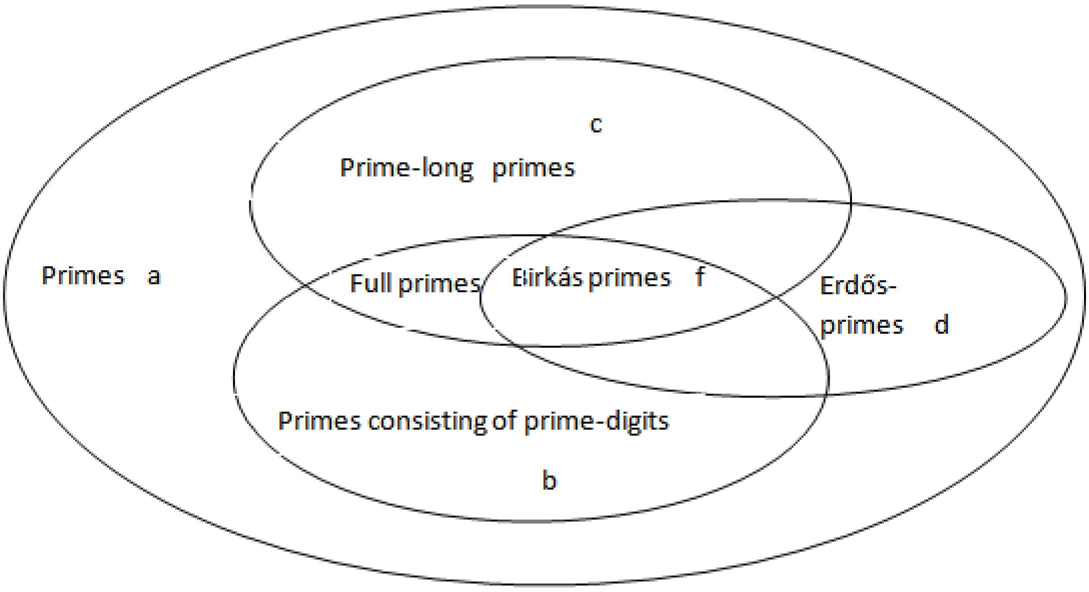
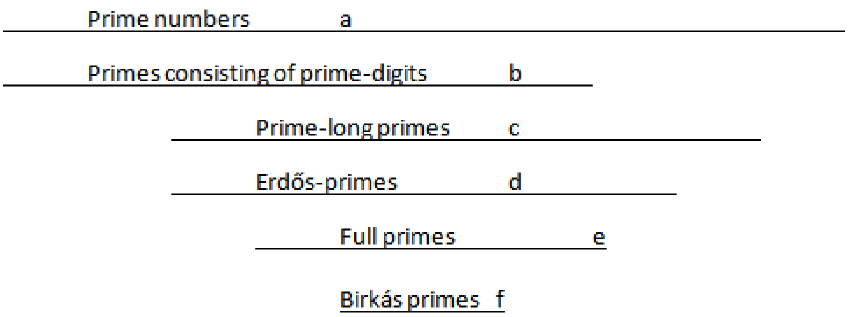


Fig.2



IV. Bölcsföldi-Balogh prime numbers [3], [9], [10], [11], [12].

Definition:a positive integer number is a Bölcsföldi-Balogh prime number, if
a/ the positive integer number is prime, b/ all digits are 3 or 5, c/ the number of digits is prime,
d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi-Balogh prime numbers (Fig.3).

Bölcsföldi-Balogh prime number p has the following sum form:

$$p=\sum_{j=0}^{k(p)} e_j(p).10^j \quad \text{where } e_j(p) \in \{3,5\} \text{ and } k(p)+1 \text{ is prime and } e_0(p) \in \{3\} \quad \text{and } \sum_{j=o}^{k(p)} e_j(p) \text{ is prime.}$$

The Bölcsföldi-Balogh prime numbers are as follows (the last digit can only be 3):

{353}, {33353, 33533, 35353, 35533, 53353, 55333},
{3353333, 3355553, 3553553, 3555353, 5353553, 5533553},
{33333353533, 33333355553, 33333555553, 33335353333, 33335553353, 33353335553,
33353353333, 33353355353, 33353553533, 33355333553, 33355535533,
33355553333, 33355553353, 33533355533, 33533555353, 33535333553,
33535355333, 33535533533, 33553335533, 33553353553, 33553553353,

33553553533, 33555333353, 33555353353, 33555533533, 33555535553,
35333335553, 35333533553, 35333553533, 35335355353, 35335533533,
35335533553, 35335555553, 35353333333, 35353335553, 35353353353,
35355335333, 35355533333, 35533333553, 35533555333, 35533555553,
35535335533, 35535355553, 35535533353, 35535533533, 35553333353,
35553335353, 35553353533, 35553533333, 35553535553, 53333335333, etc.

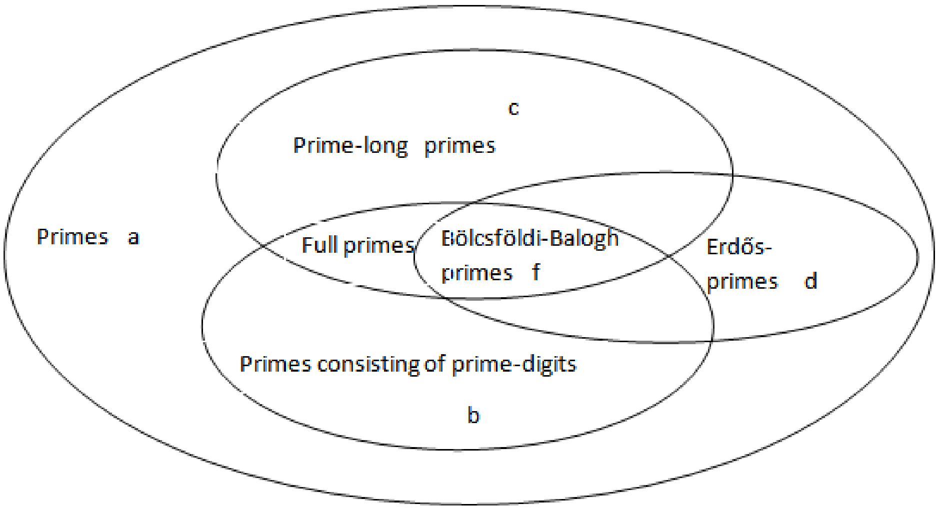
R(p) is the factual frequency of Bölcsföldi-Balogh prime numbers in the interval $(10^{p-1}, 10^{p})$.
R(2)=0, R(3)=1 , R(5)=6 , R(7)=6 , R(11)=95 , R(13)=187 , R(17)=3184 , R(19)=11477 , R(23)=94435, etc.
S(p) function gives the number of Bölcsföldi-Balogh prime numbers in the interval $(10^{p-1},10^{p})$. We think that

$S(p)=0,472x1,7^{p}$, where p is prime. (1)

The factual number of Bölcsföldi-Balogh primes and the number of Bölcsföldi-Balogh primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Bölcsföldi-Balogh primes in the interval $(10^{p-1},10^{p})$ R(p)	The number of Bölcsföldi-Balogh primes according to function $S(p)=0,472x1,7^{p}$	R(p)/S(p)
2	0	1,36	0
3	1	2,32	0,43
5	6	6,70	0,90
7	6	19,37	0,31
11	95	161,76	0,59
13	187	467,50	0,40
17	3184	3904,57	0,82
19	11477	11284,22	1,02
23	94435	94246,93	1,00

Fig.3



V. Bölcsföldi-Madar prime numbers [3], [9], [10], [11], [12].

Definition:a positive integer number is a Bölcsföldi-Madar prime number, if
a/ the positive integer number is prime, b/ all digits are 5 or 7, c/ the number of digits is prime,
d/ the sum of digits is prime.

The set of prime numbers meeting the conditions a/ and c/ is also well-known: it is the set of prime-long prime numbers [3], [9]. Positive integer numbers meeting all the four conditions (a/, b/, c/, d/) at the same time are Bölcsföldi-Madar prime numbers (Fig.4).

Bölcsföld-Madar prime number p has the following sum form:
k(p) k(p)

$p=\sum_{j=0} e_j(p).10^j$ where $e_j(p)$ € {5, 7} and k(p)+1 is prime and $e_0(p)$ € {7} and $\sum_{j=o} e_j(p)$ is prime.

The Bölcsföldi-Madar prime numbers are as follows (the last digit can only be 7):
{557, 577, 757}, {57557, 75557, 75577, 77557}, {5555777, 5557757, 5575777, 5577577, 5755577, 5775557, 5777557, 7575577, 7577777}, {55555555777, 55555557557, 55555575557, 55575555577, 55575575557, 55575777577, 55575777757, 55577777557, 55755577777, 55755777577, 55757775757, 55775557777, 55775577577, 55775777557, 55775777777, 55777555777, 55777755757, 55777775777, 57555577777, 57557557777, 57557577577, 57575755777, 57577577557, 57577577777, 57577775777, 57577777577, 57755575777, 57755775577, 57755775757, 57757757557, 57757757777, 57757777757, 57775775777, 57775777757, etc.

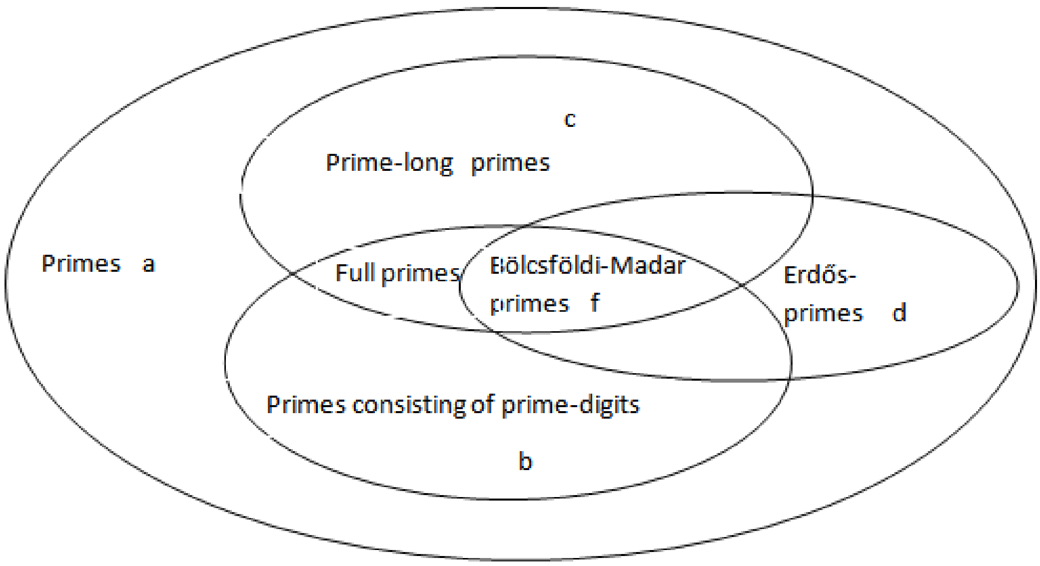
Q(p) is the factual frequency of Bölcsföldi-Madar prime numbers in the interval (10^{p-1}, 10^p).
Q(2)=0 , Q(3)=3 , Q(5)=4 , Q(7)=9 , Q(11)=67 , Q(13)=161 , Q(17)=3792 , Q(19)=5229, Q(23)=112849, etc. S(p) function gives the number of Bölcsföldi-Madar prime numbers in the interval (10^{p-1},10^p). We think that

$S(p)=0{,}5651x1{,}7^p$, where p is prime. (1)

The factual number of Bölcsföldi-Madar primes and the number of Bölcsföldi-Madar primes calculated according to function (1) are as follows:

Number of digits p	The factual number of Bölcsföldi-Madar primes in the interval (10^{p-1},10^p) Q(p)	The number of Bölcsföldi-Madar primes according to function $S(p)=0{,}5651x1{,}7^p$	Q(p)/S(p)
2	0	1,63	0
3	3	2,78	1,08
5	4	8,02	0,49
7	9	23,19	0,39
11	67	193,67	0,35
13	161	559,70	0,29
17	3792	4674,73	0,81
19	5229	13510,00	0,39
23	112849	112836,73	1,00

Fig.4



VI. Number of the elements of the set for example of Birkás prime numbers [3], [9],[10], [11], [12].

Let's take the set of Mills' prime numbers! Definition: The number m=[M ad 3^n] is a prime number, where M=1,3063778838630806904686144926O2 is the Mills' constant, and n=1,2,3,… is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: m=2, 11, 1361, 2521008887,…

The connection n→m is the following:1→2, 2→11, 3→1361, 4→2521008887,… The Mills' prime number m=[M ad 3^n] corresponds with the interval (10^{m-1},10^m) and vice versa. For instance:2→(10, 10^2), 11→(10^{10},10^{11}), 1361→(10^{1360},10^{1361}), etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the intervals (10^{m-1}, 10^m) that contain at least one Mills' prime number is infinite. The number of Birkás primes in the interval (10^{m-1},10^m) is S(m)=1,9078x$1,6^{m-1}$. The number of Birkás prime numbers is probably infinite:

$\lim_{p\to\infty} H(p)=\infty$ is probably where p is prime.

VII. Conclusion

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

Acknowledgements

The authors would like to thank you for publishing this article.

References:

[1]. http://oeis.org/A019546
[2]. http://ac.inf.elte.hu → VOLUMES → VOLUME 44 (2015)→ VOLLPRIMZAHLENMENGE→FULL TEXT
[3]. http://mathworld.wolfram.com/SmarandacheSequences.html
[4]. http://listserv.nodak.edu/scripts/wa.exe?A2=ind0202&L=nmbrthry&P=1697
[5]. Harman, Glyn: Counting Primes whose Sum of Digits is Prime. Journal of Integer Sequences (2012. , Vol. 15, 12.2.2.)
[6]. ANNALES Universitatis Scientiarum Budapestiensis de Rolando Eötvös Nominate Sectio Computatorica, 2015, pp 221-226
[7]. International Journal of Mathematics and Statistics Invention, February 2017: http://www.ijmsi.org/Papers/Volume.5.Issue.2/B05020407.pdf
[8]. International Organisation of Scientific Research, April 2017Bölcsföldi Birkás prime numbers:http://www.iosrjournals.org/iosr-jm/pages/v13(2)Version-4.html or DIGITAL OBJECT IDENTIFIER NUMBER (DOI), May 2017Bölcsföldi-Birkás primenumbers: http://dx.doi.org or www.doi.org Article DOI is: 10.9790/5728-1302043841
[9]. International Refereed Journal of Engineering and Sciense 2018:Ács-Bölcsföldi-Birkás prime numbers:http://irjes.com/volume7issue6.html

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József Bölcsföldi1 "Dual prime numbers." IOSR Journal of Polymer and Textile Engineering (IOSR-JPTE) , vol. 5, no. 5, 2018, pp. 15-20.