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(S, d)Magic Labeling of Non - Unicyclic Graphs -Paper II

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Abstract
Let G (p, q) be a connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → $\{ s, s+d, s+2d, \ldots s+(q+1)d \}$ and g be an injective function g: E(G) → $\{d, 2d, 3d \ldots 2(q-1)d\}$.Then the function f is said to be (s, d) magic labeling if $f(u) + g(uv) + f(v)$ is a constant, for all $u, v \in V(G)$ and $uv \in E(G)$. A graph G is called (s, d) magic graph if it admits (s, d) magic labeling.

Keywords: *Shell graph, Jelly fish graph, Butterfly graph, Jahangir graph and Petersen graph*

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I. Introduction

The graphs under consideration are simple and finite. Many branches of science and technology, including astronomy, circuit design, coding theory, and others, use graph labeling. Deb and Limaye have defined shell graph and proved that the graph is harmonious. Jeba Jesintha and Ezhilarasi Hilda have introduced shell graph ,shell-butterfly graph [2] and proved that they are graceful. We introduce (s, d) Magic labeling of graphs. If G admits (s, d) Magic labeling, then G is called as (s, d) Magic graph. In this paper, a new concept of (s, d) Magic labeling has been introduced for some graphs.

[5] Let $G\ (p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: V(G) \to \{s, s+d, s+2d, \ldots s+(q+1)d\}$ and

$g: E(G) \to \{d, 2d, 3d \ldots 2(q-1)d\}$ be an injective function. Then, for any $u, v \in V(G)$ and $uv \in E(G)$, $f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be (s, d) magic labeling. If a graph G admits (s, d) magic labeling, then it is referred to as a (s, d) magic graph.

II. Definitions

Definition 2.1 [2] A shell graph is a cycle $C_n$with $(n-3)$ chords sharing a common end point called the apex.
Definition 2.2: [2] A Butterfly graph is defined as a double shell graph with exactly two pendant edges at the apex.
Definition 2.3: [3] The jelly fish graph, $JF_{m,n}$ is obtained from a 4-cycle with vertices x, y, u, v, by joining x and y with a prime edge and appending m pendent edges to u and n pendent edges to v. The prime edge in jelly fish graph is defined to be the edge joining the vertices x and y.
Definition 2.4 [6] Jahangir graph $J_{m,n}$ for $m \geq 2, n \geq 3$ is a graph with $mn+1$ vertices comprising a certain cycle C_{mn} possessing single vertex that is additional and is beside n vertices of C_{mn} placed at a distance m between the C_{mn}.
Definition 2.5 **[7]** The Generalized Petersen graph $P(n, k)$ for all $n \geq 3, 1 \leq k \leq \left[\frac{n}{2}\right]$ is a 3-regular graph with 2n vertices $u_0\ , u_1 \ldots \ldots u_{n-1,} v_0, v_1 \ldots v_{n-1}$ and edges $(u_i v_i)$, $(u_i u_{i+1})$, $(v_i v_{i+m})$ for all $i \in \{0,1,2, \ldots . n-1\}$, where the subscripts are taken modulo n.

III. Main Result

Theorem 3.1 The Shell graph C (η, η-3) is (s, d) magic labeling
Proof: Let G be a shell graph and $v_1, v_2, \ldots . v_n$ be successive vertices of G
Let $|v| = \eta$, $|E| = 2\eta - 3$,
Define the function f from the vertex set to $\{\{s, s+d, s+2d \ldots . s+(q+1)\}$,
$g: E(G) \to \{d, 2d, 3d \ldots . .2(q-1)d\ \ \}$ to label the edges.
Case (a) when η is odd

Labeling of vertices of shell graph C(η,η-3)
$f(v_\eta) = s + (\frac{3\eta-5}{2})$ d

Value of i	$f(v_{i+1})$	$f(v_{2i})$	$f(v_{2i-1})$
$i = 0$	s	$-$	$-$
$1 \leq i \leq \frac{\eta-1}{2}$	$-$	$s+\{(\frac{\eta-1}{2})+(i-1)\}d$	$-$
$2 \leq i \leq \frac{\eta-1}{2}$	$-$	-	$s+(i-1)d$

Labeling of Edges of Shell graph C(η,η-3)			
Value of i	$g(v_i v_{i+1})$	$g(v_1 v_i)$	$g(v_\eta v_{i+1})$
$1 \leq i \leq \eta-1$	$2s + 2(q-1)d - (f(v_i) + f(v_{i+1}))$	$-$	$-$
$1 \leq i \leq \eta$-3	$-$	$-$	$2s + 2(q-1)d - (f(v_\eta) + f(v_{i+1}))$
$i = \eta$	$-$	$2s + 2(q-1)d - (f(v_1) + f(v_i))$	$-$

Case (b) when η is even

Labeling of vertices of the Shell graph C(η,η-3)			
$f(v_\eta) = s + (\frac{3\eta-4}{2})d$			
Value of i	$f(v_{i+1})$	$f(v_{2i})$	$f(v_{2i-1})$
$i = 0$	s	$-$	$-$
$1 \leq i \leq \frac{\eta}{2}-1$	$-$	$s+\left[\frac{\eta}{2}+(i-1)\right]d$	$-$
$2 \leq i \leq \frac{\eta}{2}$	$-$	$-$	$s+(i-1)d$

Labeling of Edges of the Shell graph C(η,η-3)			
Value of i	$g(v_i v_{i+1})$	$g(v_1 v_i)$	$g(v_\eta v_{i+1})$
$1 \leq i \leq \eta-1$	$2s + 2(q-1)d - (f(v_i) + f(v_{i+1}))$	$-$	$-$
$1 \leq i \leq \eta$-3	$-$	$-$	$2s + 2(q-1)d - (f(v_\eta) + f(v_{i+1}))$
$i = \eta$	$-$	$2s + 2(q-1)d - (f(v_1) + f(v_i))$	$-$

From above table we find that f and g are injective therefore $f(v_i) + f(v_{i+1}) + g(v_i v_{i+1})$, $f(v_1) + f(v_i) + g(v_1 v_i)$, $f(v_\eta) + f(v_{i+1}) + g(v_\eta v_{i+1})$, are constant equals to $2(s + (q-1)d)$.Hence shell graph admits (s, d) magic labeling.

Example 3.1 (s, d) magic labeling of Shell graph C (7,4) is shown below.

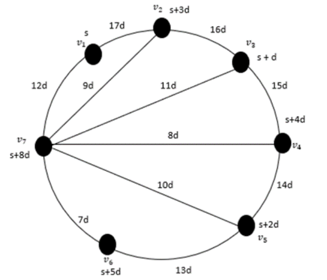

Figure 3.1: Shell graph C (7,4)

Theorem 3.2 The Jelly fish graph $JF_{m,\eta}$ is (s,d) magic labeling.

Proof: Let $G = JF_{m,\eta}$ be a Jelly fish graph. The vertex set $V(JF_{m,\eta})$ = $\{u, v, x, y, u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq \eta\}$, $E(JF_{m,\eta})$ =$\{ux, uy, vx, vy, uu_i, vv_j : 1 \leq i \leq m, 1 \leq j \leq \eta\}$.let $|V(JF_{m,\eta})| = m + \eta + 4$, $|E(JF_{m,\eta})| = m + \eta + 5$. Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$, $g: E(G) \rightarrow \{d, 2d, 3d \ldots . .2(q - 1)d\ \}$ to label the edges.

Labeling of vertices of the graph $JF_{m,\eta}$		
$f(x) = s$		
$f(v) = s + d$		
$f(y) = s + (m + \eta + 3)d$		
$f(u) = s + (m + \eta + 2)d$		
Value i &j	$f(u_i)$	$f(u_j)$
$1 \leq i \leq m$	$s + (\eta + i) + 1)d$	$-$
$1 \leq j \leq \eta$	$-$	$s + (j + 1)d$

Labeling of edges of the graph $JF_{m,\eta}$		
$g(ux) = 2s + 2(q - 1)d - (f(u) + f(x))$		
$g(uy) = 2s + 2(q - 1)d - (f(u) + f(y))$		
$g(vy) = 2s + 2(q - 1)d - (f(v) + f(y))$		
$g(vx) = 2s + 2(q - 1)d - (f(v) + f(x))$		
Value i	$g(uu_i)$	$g(vv_j)$
$1 \leq i \leq \eta$	$2s + 2(q - 1)d$ $-(f(u) + f(u_i))$	$-$
$1 \leq \mathrm{j} \leq \mathrm{m}$	-	$2s + 2(\mathrm{q} - 1)d - (f(v) + f(v_j))$

From above table we find that f and g are injective ,therefore $f(u) + f(u_i) + g(uu_i)$, $f(v) + f(v_j) + g(vv_j)$, $f(u) + f(x) + g(ux)$, $f(u) + f(y) + g(uy)$, $f(v) + f(y) + g(vy)$, $f(v) + f(x) + g(vx)$ are constant equals to $2(s + (q - 1)d)$.Hence the Jelly fish graph $JF_{m,\eta}$ admits (s,d) magic labeling.

Example 3.2 (s, d) magic labeling of Jelly fish graph $JF_{5,3}$is shown below.

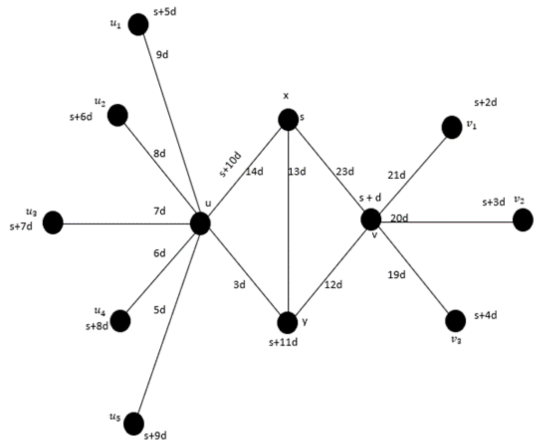

Figure 3.2: Jelly fish graph $JF_{5,3}$

Theorem 3.3 The Butterfly graph $BF_{m,\eta}$ is (s,d) magic labeling.

Proof: Let $G = BF_{m,\eta}$ be a butterfly graph.

Let the vertex set be $V(BF_{m,\eta}) = \{u, v, w, u_i, v_j\ : 1 \leq i \leq m, : 1 \leq j \leq \eta\}$,
$E(BF_{m,\eta}) = \{uw, vw, wu_i, wv_j : 1 \leq i \leq m, : 1 \leq j \leq \eta\} \cup \{u_i u_{i+1};\ 1 \leq i \leq m - 1\} \cup \{v_j v_{j+1}; 1 \leq i \leq \eta - 1\}$

Define the function f from. the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$, $g: E(G) \rightarrow \{d, 2d, 3d \ldots . .2(q - 1)d\ \}$ to label the edges

Labeling of vertices of the graph $BF_{m,\eta}$		
f(u) = s + (m + η + 1)d		
$f(v) = s + (\eta + m)d$		
Value i &j	$f(u_{i+1})$	$f(v_{j+1})$
$i = 0$	s	$s + d$
$1 \le i \le \eta - 1$	$s + 2id$	-
$1 \le j \le m - 1$	-	$s + (2j + 1)d$

Labeling of edges of the graph $BF_{m,\eta}$				
$g(uw) = 2s + 2(q-1)d - (f(u) + f(w))$				
$g(vw) = 2s + 2(q-1)d - (f(v) + f(w))$				
Value i	$g(wu_i)$	$g(wv_j)$	$g(u_iu_{i+1})$	$g(v_jv_{j+1})$
$1 \le i \le$ m	$2s + 2(q-1)d$ $-(f(w) + f(u_i))$	-	-	-
$1 \le$ j $\le \eta$	-	$2s + 2(\text{q} - 1)d$ $-\left(f(w) + f(v_j)\right)$	-	-
$1 \le i \le$ m − 1	-	-	$2s + 2(q-1)d$ $-(f(u_i) + f(u_{i+1}))$	-
$1 \le$ j $\le \eta$-1	-	-	-	$2s + 2(q-1)d$ $-(f(v_j) + f(v_{j+1}))$

From above table we find that f and g are injective, therefore $f(w) + f(u_i) + g(wu_i), f(w) + f(v_j) + g(wv_j)$, $f(u) + f(w) + g(uw)$,$f(v) + f(w) + g(vw)$,$f(u_i) + f(u_{i+1}) + g(u_iu_{i+1}), f(v_j) + f(v_{j+1}) + g(v_jv_{j+1})$ are constant equals to $2(s + (q-1)d)$.Hence the Butterfly graph $BF_{m,\eta}$ admits (s,d) magic labeling.

Example 3.3 (s, d) magic labeling of Butterfly graph $BF_{4,4}$ is shown below.

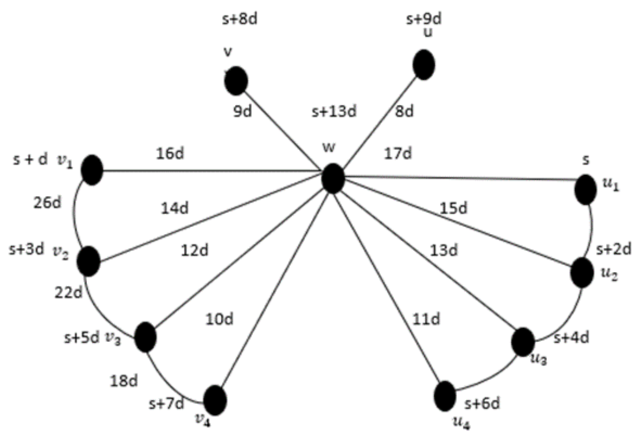

Figure 3.3: Butterfly graph $BF_{4,4}$

Theorem 3.4: The Jahangir graph $J_{m,\eta}$ for m is even, $\eta \ge 2$ is a (s,d) magic labeling.

Proof: Let for a Generalized Jahangir graph $J_{m,\eta}$, $v_0$be an apex vertex and $v_1, v_2, \ldots\ldots v_{m\eta}$be the rim vertices. Set of edges E $(J_{m,\eta})$ = $\{v_iv_{i+1,}\ ; i = 1,2,3 \ldots. m\eta - 1\} \cup \{v_{m\eta}v_1\} \cup \{v_0v_{1+(m(i-1)}\ ; i = 1,2,3 \ldots \eta\}$. So, for a Generalized Jahangir graph $J_{m,\eta}$ $|V| = m\eta + 1$ and |E|= $(m + 1)\eta$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots. s + (q+1)\}$, $g: E(G) \to \{d, 2d, 3d \ldots..2(q-1)d\}$ to label the edges

Case (i) When m=2

Labeling of vertices of the graph $J_{m,\eta}$		
Value i	$f(v_{2i+1})$	$f(v_{2i})$
$i = 0$	-	$s + [(m+1)\eta - 1)]d$
$i = \eta$	-	s
$0 \le i \le \eta - 1$	$s + (i+1)d$	−
$1 \le i \le \eta - 1$	−	$s + (\eta + i)d$

Labeling of edges of the graph $J_{m,\eta}$		
$g(v_{2\eta}v_1)=2s+2(q-1)-(f(v_{2\eta})+f(v_1))$		
Value i	$g(v_iv_{i+1})$	$g(v_0v_{2i+1})$
$1 \le i \le m\eta - 1$	$2s+2(q-1)d-(f(v_i)+f(v_{i+1}))$	$-$
$0 \le i \le \eta$-1	$-$	$2s+2(q-1)d-(f(v_i)+f(v_{2i+1}))$

From above table we find that f and g are injective ,therefore $f(v_i)+f(v_{i+1})+g(v_iv_{i+1}), f(v_{2\eta})+f(v_1)+g(v_{2\eta}v_1)$ and $f(v_0)+f(v_{2i+1})+g(v_0v_{2i+1})$ are constant equals to $2(s+(q-1)d)$.Hence the Jahangir graph $J_{m,\eta}$ for m = 2, $\eta \ge 2$ admits (s,d) magic labeling.

Example 3.4 (a) (s, d) magic labeling of Jahangir graph $J_{2,8}$, is shown below.

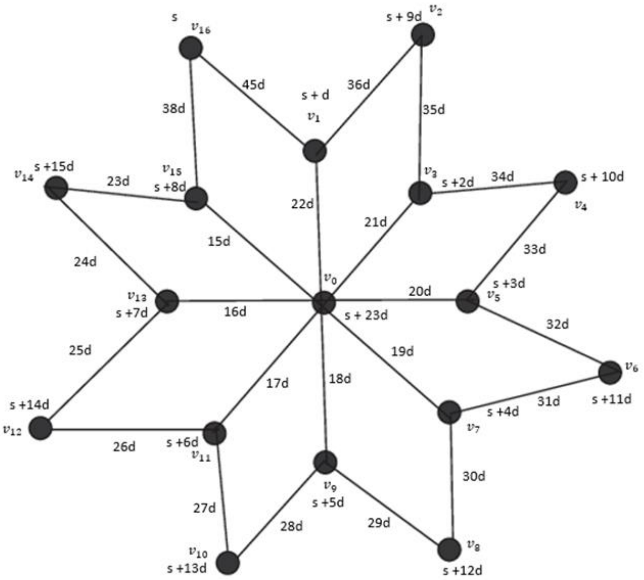

Figure 3.4: a Jahangir graph $J_{2,8}$

Case (ii) When m ≥ 4

Labeling of vertices of the graph $J_{m,\eta}$	
Value i	$f(v_i)$
$i=1$	s
$i=0$	$s+d$
$2 \le i \le m\,\eta$	$s+id$

Labeling of edges of the graph $J_{m,\eta}$		
$g(v_{m\eta}v_1)=2s+2(q-1)-(f(v_{m\eta})+f(v_1))$		
Value i	$g(v_iv_{i+1})$	$g(v_0v_{mi+1})$
$1 \le i \le m\eta - 1$	$2s+2(q-1)d-(f(v_i)+f(v_{i+1}))$	-
$0 \le i \le \eta$-1	-	$2s+2(q-1)d-(f(v_0)+f(v_{mi+1}))$

From above table we find that f and g are injective, therefore $f(v_i)+f(v_{i+1})+g(v_iv_{i+1}), f(v_{m\eta})+f(v_1)+g(v_{m\eta}v_1)$ and $f(v_0)+f(v_{mi+1})+g(v_0v_{mi+1})$ are constant equals to $2(s+(q-1)d)$.Hence the Jahangir graph $J_{m,\eta}$ for m ≥4, $\eta \ge 2$ admits (s,d) magic labeling

Example 3.4 b (s, d) magic labeling of Jahangir graph $J_{4,3}$ is shown below.

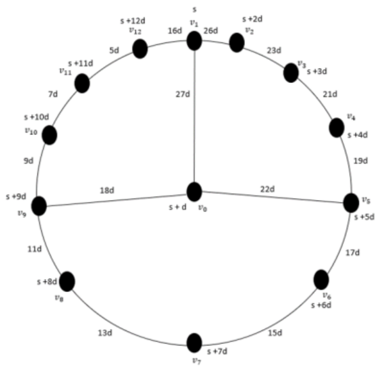

Figure 3.4: b Jahangir graph $J_{4,3}$

Theorem 3.5*:* The generalized Petersen graph $GP(\eta, k)$ is (S, d) magic labeling for all η is odd

Proof: Let G= $GP(\eta, k)$;$1 \leq k \leq \left[\frac{\eta}{2}\right]$ with $V(GP(\eta, k)) = \{x_i: 0 \leq i \leq \eta - 1 \cup y: 0 \leq i \leq \eta - 1\}$ and$\left(E(GP(\eta, k)) = \{x_i y_i: 0 \leq i \leq \eta - 1 \cup x_i x_{i+1}: 0 \leq i \leq \eta - 1\} \cup y_i y_{i+k}: 0 \leq i \leq \eta - 1\right.$ where subscripts are modulo here $|VGP((\eta, k)|$=2η and$|EGP((\eta, k)|$=3η

Define the function f from the vertex set to $\{s, s + d, s + 2d \ldots . s + (q + 1)\}$,

$g: E(G) \to \{d, 2d, 3d \ldots . .2(q - 1)d\}$

Labeling of edges of the graph $GP(\eta, k)$		
Value i	$f(x_i)$	$f(y_i)$
$0 \leq i \leq \eta - 1$	$s + id$	$s + (2\eta + i)d$

Labeling of edges of the graph $GP(\eta, k)$			
$g(x_{\eta-1} x_0)$=$2s + 2(q - 1) - (f(x_{\eta-1}) + f(x_0))$			
Value i	$g(x_i x_{i+1})$	$g(x_i y_i)$	$g(y_i y_{i+k})$
$0 \leq i \leq \eta - 2$	$2s + 2(q - 1)d - (f(x_i) + f(x_{i+1}))$	−	−
$0 \leq i \leq \eta - 1$	−	$2s + 2(q - 1)d - (f(x_i) + f(y_i))$	−
$0 \leq i \leq \eta - 1$, $1 \leq k \leq \left[\frac{\eta}{2}\right]$ Where subscripts are modulo n.	−	−	$2s + 2(q - 1)d - (f(y_i) + f(y_{i+k}))$

From the above table we find that f and g are injective, therefore $f(x_i) + f(x_{i+1}) + g(x_i x_{i+1}), f(x_i) + f(y_i) + g(x_i y_i)$ and $f(y_i) + f(y_{i+k}) + g(y_i y_{i+k})$ are constant equals to $2(s + (q - 1)d)$.Hence the generalized Petersen graph $GP(\eta, k)$ admits (S, d) magic labeling for all η is odd.

Example 3.5 (s, d) magic labeling of Petersen graph $GP(\eta, k)$ is shown below.

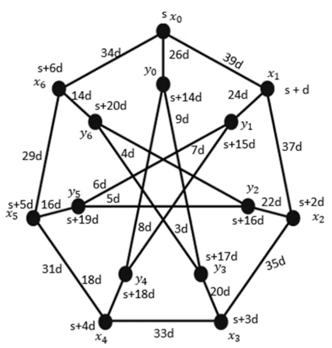

Figure 3.5: Petersen graph $GP(7, 3)$

IV. Conclusions

In this study, a (s,d) Magic Labeling has been discovered for a few graphs such as Shell graph, Jelly fish graph, butterfly graph, Jahangir graph and Petersen graph. Future research will examine the (s,d) Magic labeling of additional graphs and some graph families.

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