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Simplex-Based Concrete Mix Design

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Abstract: *Normal concrete is a mixture of cement, water, fine and coarse aggregates. Concrete mix design involves selecting the correct proportions of these constituent materials to produce concrete having the specified properties. Various mix design methods have some limitations. Time, energy and money are sometimes being wasted in order to get the appropriate mix proportions. In this paper, a mathematical model based on simplex method is formulated for the optimization of concrete cube strength. The model can provide all the possible mix ratios that can yield the desired concrete cube strength. It can also produce the concrete cube strength if mix proportions are given as well as the optimum value. Statistical tests were used to verify the adequacy of the model. They all agreed to the acceptance of the model.*
Keywords: *model; optimisation; concrete cube strength; simplex method.*

I. Introduction

Concrete which is the most widely used construction material is composed of cement, water, fine and coarse aggregates.

Two main objectives of hardened concrete tests are control of quality and compliance with specifications [1]. Concrete cube strength test is one of the major tests carried out on concrete before it can be used effectively. Also concrete grades are usually specified in standard construction work.

Basically, the problem of designing a concrete mix consists of selecting the correct proportions of cement, fine and coarse aggregates and water to produce concrete having the specified properties [2]. Various methods have been developed in order to achieve the desired properties of concrete cube strength. These methods are time, money and energy consuming.

To minimize some of these limitations an optimization procedure has been proposed. It is a process that seeks for the maximum or minimum value of a function of several variables while at the same time, satisfying a number of other requirements [3]. In this paper, a mathematical model using simplex method is formulated for the optimisation of concrete cube strength.

II. Methodology

The main materials used in the work are cement, fine and coarse aggregates and water.

Eagle cement, a brand of Ordinary Portland Cement, conforming to British Standard [4] was used in the test.

The fine aggregate used in the work was river sand free from deleterious matters such as dirts, clay and organic matters. The fine aggregate falls into zone 3 of the grading curve.

The coarse aggregate was normal weight, irregular shaped coarse aggregate with a maximum size of 20mm. Both the fine and coarse aggregate were hard and durable, and conform to the specifications of British Standard [5].

Portable drinking water was used for the production of the concrete specimen tested.
Scheffe's simplex method was used in the optimisation of concrete mix design.

Formulation of optimisation model based on Scheffe's simplex theory

A simplex lattice is described as a structural representation of lines joining the atoms of a mixture .The atoms are constituent components of the mixture. For a normal concrete mixture, the constituent elements are water, cement, fine and coarse aggregates. And so it gives a simplex of a mixture of four components. Hence the simplex lattice of this four- component mixture is a three- dimensional solid equilateral tetrahedron. Mixture components are subject to the constraint that the sum of all the components must be equal to one [6].
In order words:

$$X_1 + X_2 + X_3 + \ldots\ldots\ldots.. + X_q = 1 \tag{1}$$

$$\sum_{i=1}^{q} X_i = 1 \tag{2}$$

Where q is the number of components of a mixture
X_i is the proportion of the ith component in the mixture.
The (q,n) simplex lattice design introduced by Scheffe in 1958 [6], are characterized by the symmetric arrangements of points within the experimental region and a well chosen polynomial equation to represent the response surface over the entire simplex region. The response represents the property studied, namely, the concrete cube strength. The polynomial is obtained by using the restriction given by 'equation (1)' or 'equation (2)'.

A polynomial function of degree n in the q variables $X_1, X_2, X_3, \ldots, X_q$ is given in form of

$$y = b_o + \sum b_i X_i + \sum b_{ij} X_i X_j + \sum b_{ijk} X_i X_j X_k + \sum b i_1 i_2 \ldots i_n X i_1 X i_2 \ldots X i_n \quad (3)$$

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq q$, $1 \leq i \leq j \leq k \leq q$, $i \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively)
and b =constant coefficients
In general, the reduced form of 'equation (3)' is in the form of 'equation (4)' for a polynomial function with n = 4.

$$\begin{aligned} Y = b_o + b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_{12} X_1 X_2 + b_{13} X_1 X_3 \\ + b_{14} X_1 X_4 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{34} X_3 X_4 + b_{11} X_1^2 \\ + b_{22} X_2^2 + b_{33} X_3^2 + b_{44} X_4^2 \end{aligned} \quad (4)$$

Multiplying 'equation (1)' by bo gives 'equation (5)'

$$b_o X_1 + b_o X_2 + b_o X_3 + b_o X_4 = b_o \quad (5)$$

Multiplying 'equation (1)' successively by X_1, X_2, X_3, and X_4 and rearranging gives 'equation (6)'

$$\begin{aligned} X_1^2 = X_1 - X_1 X_2 - X_1 X_3 - X_1 X_4 \\ X_2^2 = X_2 - X_1 X_2 - X_2 X_3 - X_2 X_4 \\ X_3^2 = X_3 - X_1 X_3 - X_2 X_3 - X_3 X_4 \\ X_4^2 = X_4 - X_1 X_4 - X_2 X_4 - X_3 X_4 \end{aligned} \quad (6)$$

Substituting 'equations (5) and (6)' into 'equation (4)' and simplifying yields 'equation (7)'

$$\begin{aligned} Y = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 + \alpha_{12} X_1 X_2 + \alpha_{13} X_1 X_3 + \alpha_{14} X_1 X_4 \\ + \alpha_{23} X_2 X_3 + \alpha_{24} X_2 X_4 + \alpha_{34} X_3 X_4 \end{aligned} \quad (7)$$

Where the coefficients, α_1 and α_{34} are defined in general as follows:

$$\begin{aligned} \alpha_i = b_o + b_i + b_{ii} \\ \alpha_{ij} = b_{ij} - b_{ii} - b_{jj} \end{aligned} \quad (8)$$

'Equation (7)' can be reduced further as follows:

$$Y = \sum_{1 \leq i \leq q} \alpha_i X_i + \sum_{1 \leq i \leq j \leq q} \alpha_{ij} X_i X_j \quad (9)$$

'Equation (9)' is the response to the pure component, i and the binary mixture of components i and j.

Determination of the coefficients of the (4,2) polynomial
Assuming the response function for the pure component, i and that for the binary mixture of components i and j are y_i and y_{ii} respectively, then

$$y_i = \sum_{i=1}^{4} \alpha_i X_i \quad (10)$$

and

$$y_{ij} = \sum_{1 \leq i \leq 4} \alpha_i X_i + \sum_{1 \leq i \leq j \leq 4} \alpha_{ij} X_i X_j \quad (11)$$

Substituting the values of X_1, X_2, X_3, and X_4 at the ith point (i.e. any of the vertices of the lattice) into 'equation (10)' gives the following general equation.

$$y_i = \alpha_i \quad (12a)$$

For example, at point one, the value of X_1 =1 while the values of X_2, X_3 and X_4 are equal to zero because $\sum X$ = 0. Substituting the values of X_1, X_2, X_3, and X_4 into 'equation (10)' gives

$$y_1 = \alpha_1 \quad (12b)$$

Substituting the values of X_1, X_2, X_3, and X_4 at the point ij (that is at the mid point of the borderline connecting points i and j) of the lattice, into 'equation (11)' yields:

$$y_{ij} = \tfrac{1}{2} \alpha_i + \tfrac{1}{2} \alpha_j + \tfrac{1}{4} \alpha_{ij} \quad (13a)$$

For point 12, that is at the midpoint of the borderlines connecting points 1 and 2 of the lattice, the values of $X_1 = X_2 = \tfrac{1}{2}$ while the values of X_3, and X_4 are equal to zero because $\sum X_i = 1$. Substituting the values of X_1, X_2, X_3, and X_4 into 'equation (11)', gives 'equation (13b)'

$$y_{12} = \tfrac{1}{2} \alpha_1 + \tfrac{1}{2} \alpha_2 + \tfrac{1}{4} \alpha_{12} \quad (13b)$$

From 'equation (12a)',

$$\alpha_i = y_i \quad (14)$$

Similarly,

$$\alpha_j = y_j \quad (15)$$

Rearranging 'equation (13a)' yields:

$$\alpha_{ij} = 4y_{ij} - 2\alpha_i - 2\alpha_j \quad (16a)$$

Substituting 'equations (14) and (15)' into 'equation (16)' gives:

$$\alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (16b)$$

When 'equations (14), (15) and (16b)' are substituted, 'equation (7)' becomes:

$$\begin{aligned} y = {} & y_1X_1 + y_2X_2 + y_3X_3 + y_4X_4 + (4y_{12} - 2y_1 - 2y_2)X_1X_2 \\ & + (4y_{13} - 2y_1 - 2y_3)X_2X_3 + (4y_{14} - 2y_1 - 2y_4)X_1X_4 \\ & + (4y_{23} - 2y_2 - 2y_3)X_2X_3 + (4y_{24} - 2y_2 - 2y_4)X_2X_4 \\ & + (4y_{34} - 2y_3 - 2y_4)X_3X_4 \end{aligned} \quad (17)$$

Let the coefficient of $y_1 = X_1 --2X_1(X_2+X_3+X_4)$ (18)

From 'equation (1)',

$$X_2 + X_3 + X_4 = 1 - X_1 \quad (19)$$

Substituting 'equation (19)' into equation (18)' gives the coefficient of y_1 as follows:

$$\begin{aligned} y_1 &= X_1 - 2X_1(1 - X_1) \\ &= X_1 (2X_1 - 1) \end{aligned} \quad (20)$$

Rearranging 'equation (17)' and transferring all the coefficients of y_1 in like manner, gives the following mixture design model for optimization of a 4-component concrete.

$$\begin{aligned} y = {} & X_1(2X_1 - 1)y_1 + X_2(2X_2 - 1)y_2 + X_3(2X_3 - 1)y_3 \\ & + X_4(2X_4 - 1)y_4 + 4X_1X_2y_{12} + 4X_1X_3y_{13} + 4X_1X_4y_{14} \\ & + 4X_2X_3y_{23} + 4X_2X_4y_{24} + 4X_3X_4y_{14} \end{aligned} \quad (21)$$

The terms y_i and y_{ii} are responses (representing concrete cube strength) at the points i and ij. They are determined by carrying out laboratory test.

III. Components transformation

It is impossible to use the normal mix ratios such as 1:2:4 or 1:3:6 at given water /cement ratio because of the requirement of the simplex that sum of all the components must be one. Hence it is necessary to carry out a transformation from actual to pseudo components. The actual components represent the proportion of the ingredients while the pseudo components represent the proportion of the components of the ith component in the mixture i.e. X_1, X_2, X_3, X_4. Considering the four-component mixture tetrahedron simplex lattice, let the vertices of this tetrahedron (principal coordinates) be described by A_1, A_2, A_3, A_4.

The arbitrary mix proportions prescribed for the vertices of the tetrahedron shown in "Figure 1",

A_1 (0.55: 1: 2: 4)
A_2 (0.50: 1: 2.5: 6)
A_3 (0.45: 1: 3: 5.5)
A_4 (0.6: 1: 1.5: 3.5)
are based on past experiences and literature.

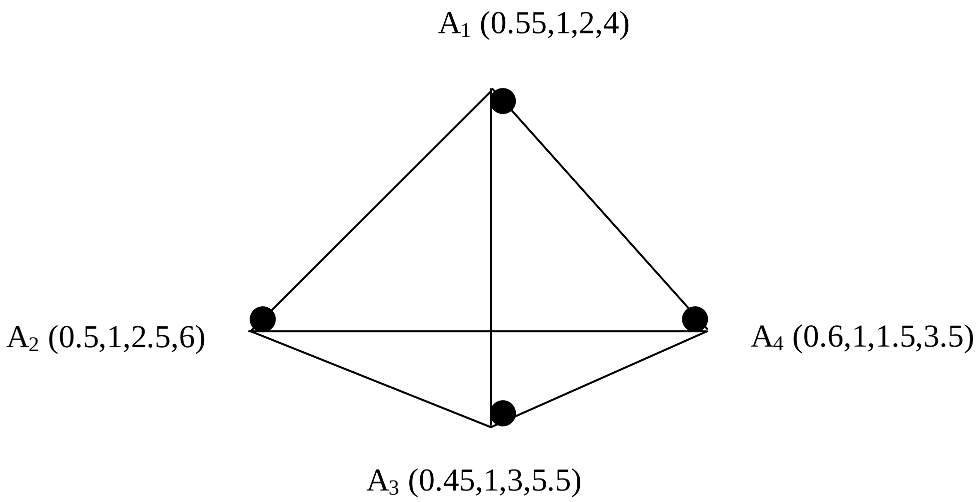

Fig 1: Vertices of a (4,2) lattice (actual)

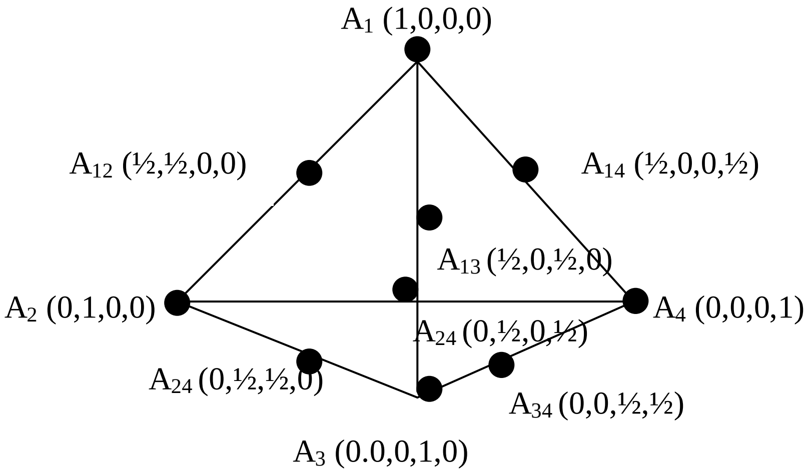

Fig 2: Vertices of a (4,2) lattice (pseudo)

Let X represent pseudo components and Z, actual components. For component transformation we use the following equations:

$$X = BZ \tag{22}$$

$$Z = AX \tag{23}$$

where A = matrix whose elements are from the arbitrary mix proportions chosen when 'equation (23)' is opened and solved mathematically.
B = the inverse of matrix A
Z = matrix of actual components
X = matrix of pseudo components obtained from "Figure 2".
Expanding and using 'equations (22) and (23)' the actual components Z were determined and presented in Table 1.

Table 1. Actual Components Z

N	X_1	X_2	X_3	X_4	Response	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	0.55	1	2	4
2	0	1	0	0	Y_2	0.50	1	2.5	6
3	0	0	1	0	Y_3	0.45	1	3	5.5
4	0	0	0	1	Y_4	0.6	1	1.5	3.5
5	0.5	0.5	0	0	Y_{12}	0.525	1	2.25	5
6	0.5	0	0.5	0	Y_{13}	0.5	1	2.5	4.75
7	0.5	0	0	0.5	Y_{14}	0.575	1	1.75	3.75
8	0	0.5	0.5	0	Y_{23}	0.475	1	2.75	5.75
9	0	0.5	0	0.5	Y_{24}	0.55	1	2	4.75
10	0	0	0.5	0.5	Y_{34}	0.525	1	2.25	4.5

Control points

11	0.5	0.25	0.25	0	C_1	0.5125	1	2.375	4.875
12	0.25	0.25	0.25	0.25	C_2	0.525	1	2.25	4.75
13	0	0.25	0.25	0.5	C_3	0.5375	1	2.125	4.625
14	0	0.25	0	0.75	C_4	0.575	1	1.75	4.125
15	0.75	0	0.25	0	C_5	0.525	1	2.25	4.375
16	0	0.5	0.25	0.25	C_6	0.5125	1	2.375	5.25
17	0.25	0	0.5	0.25	C_7	0.5125	1	2.375	4.625
18	0.75	0.25	0	0	C_8	0.5375	1	2.125	4.5
19	0	0.75	0.25	0	C_9	0.4875	1	2.625	5.875
20	0	0.4	0.4	0.2	C_{10}	0.5	1	2.5	5.3

If there is need to use bulk volume in the mix design, one has to carry out components transformation as stated above using equations '(22) and (23)'. It is worthy of note here that the equation derived has no need of the value of the specific gravity.

Experimental method

The actual components as transformed from 'equation (5)' and (Table 1) were used to measure out the quantities water (Z_1), cement (Z_2), sand (Z_3), and coarse aggregates (Z_4) in their respective ratios for the concrete cube strength test. For instance, the actual ratio for the test number 20 means that the concrete mix ratio is 1: 2.5: 5.3 at 0.5 free water/cement ratio. A total of 20 mix ratios were used to produce 40 prototype concrete cubes measuring 150mm x 150mm x 150mm that were cured and tested on the 28th day. Ten out of 20 mix ratios were used as control mix ratios to produce 20 cubes for the confirmation of the adequacy of the mixture design model given by 'equation (21)'. The cubes were then tested for concrete cube strength using the universal testing machine. The load under which the cube specimen failed was recorded and used to compute the strength of the concrete cubes.

IV. Results and analysis

The test result of the concrete cube strength (Y_i) based on day 28-day strength, is presented as part of (Table 2).

The concrete cube strength was obtained from the following equation:

$$f_{cu} = P/A \tag{24}$$

where f_{cu} is the concrete cube strength in Mega Pascals (MPa) or Newtons per millimeters squared (Nmm^{-2}).

P = failure load in Newtons (N).

A = nominal cross-sectional area in millimetres squared (Nmm^{-2}).

Table 2. Test Results and Replication Variance

Exp No.	Replicates	Response Y_i (N/mm$^{2)}$	Response Symbol	Y	$\sum Y_i$	$\sum Y_i^2$	S_i^2
1	1A IB	27.10 25.34	Y_1	26.22	54.44	1376.53	1.55
2	2B 2B	31.12 29.32	Y_2	30.22	60.44	1828.12	1.62
3	3A 3B	25.20 22.80	Y_3	24	48.00	1154.88	2.88
4	4A 4B	27.90 27.20	Y_4	27.55	55.10	1518.25	0.25
5	5A 5B	27.58 30.22	Y_{12}	28.89	57.78	1672.70	3.44
6	6A 6B	23.31 25.57	Y_{13}	24.44	48.88	1197.18	2.55
7	7A 7B	20.13 23.41	Y_{14}	21.77	43.54	953.25	5.38
8	8A 8B	33.01 29.21	Y_{23}	31.11	62.22	1942.88	7.22
9	9A 9B	23.22 21.66	Y_{24}	22.44	44.88	1008.32	1.21
10	10A 10B	26.88 25.12	Y_{34}	26.00	52.00	1008.32	1.55
11	11A 11B	22.22 29.40	C_1	25.81	51.62	1358.09	25.77
12	12A 12B	22.22 30.56	C_2	26.39	52.78	1427.64	34.78
13	13A 13B	26.67 27.29	C_3	26.98	53.96	1456.03	0.19
14	14A 14B	23.78 22.86	C_4	23.32	46.64	1088.07	0.43
15	15A 15B	28.01 28.47	C_5	28.24	56.48	1595.10	0.10
16	16A 16B	29.33 26.17	C_6	27.75	55.50	1545.12	4.99
17	17A 17B	20.00 27.38	C_7	23.69	47.38	1149.66	27.23

18	18A 18B	26.44 21.60	C_8	24.02	48.04	1165.63	11.71
19	19A 19B	22.22 29.70	C_9	25.96	51.92	1375.82	27.98
20	20A 20B	24.66 28.48	C_{10}	26.57	53.14	1419.23	7.30
						∑	168.13

The values of the mean of responses, Y and the variances of replicates S_i^2 presented in columns 5 and 8 of (Table 2) were gotten from the following 'equations (25) and (26)':

$$Y = \sum_{i=1}^{n} Y_i/n \quad (25)$$

$$S^2_i = [1/(n-1)]\{ \sum Y_i^2 - [1/n(\sum Y_i)^2]\} \quad (26)$$

Where $1 \leq i \leq n$ and this equation is an expanded form of 'equation(27)'

$$S^2_i = [1/(n-1)][\sum_{i=1}^{n} (Y_i - Y)^2] \quad (27)$$

Where Y_i = responses
Y = mean of the responses for each control point
n = number of parallel observations at every point
n−1 = degree of freedom
S^2_i = variance at each design point
Considering all the design points, number of degrees of freedom,

$$V_e = \sum N - 1 = 20 - 1 = 19 \quad (28)$$

Where N is the number of points

Replication variance, $$S^2_y = (1/Ve) \sum_{i=1}^{N} S_i^2 \quad (29)$$

$$=168.13/19 = 8.848$$

Where S_i^2 is the variance at each point
Using 'equations (28) and (29)', the replication error, S_y can be determined as follows

$$S_y = \sqrt{S^2_y} = 2.97 \quad (30)$$

This replication error value was used below to determine the t-statistics values for Scheffe's simplex model.

Determination of the optimisation model based on Scheffe's theory

Using 'equation (31)' and (Table 2), the coefficients of the second degree polynomial were determined as follows:

$$\alpha_1 = y_1 \text{ and } \alpha_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (31)$$

$\alpha_1 = 26.22$, $\alpha_2 = 30.22$, $\alpha_3 = 24$, and $\alpha_4 = 27.55$,
$\alpha_{12} = 4(28.89) - 2(26.22) - 2(30.22) = 2.68$
$\alpha_{13} = 4(24.44) - 2(26.22) - 2(24) = -2.68$
$\alpha_{14} = 4(21.77) - 2(26.22) - 2(27.55) = -20.46$
$\alpha_{23} = 4(31.11) - 2(30.22) - 2(24) = 16$
$\alpha_{24} = 4(22.44) - 2(30.22) - 2(27.55) = 25.78$
$\alpha_{34} = 4(26) - 2(24) - 2(27.55) = 0.9$

Substituting the values of these coefficients into 'equation (21)' yields:

$$Y = 26.22X_1 + 30.22X_2 + 24X_3 + 27.55X_4 + 2.68X_1X_2 - 2.68X_1X_3 - 20.46X_1X_4 + 16 X_2X_3 - 25.78X_2X_4 + 0.9X_3X_4 \quad (32)$$

'Equation (32)' is the Scheffe's mathematical model for concrete cube strength based on the 28-day strength.

Test of the adequacy of the model

The model equation was tested for adequacy against the controlled experimental results. The statistical hypothesis for this mathematical model is as follows:

Null Hypothesis (H_0): There is no significant difference between the experimental and the theoretically expected results at an α-level of 0.5.
Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected results at an α-level of 0.05.
The student's t-test and fisher test statistics were used for this test. The expected values ($Y_{predicted}$) for the test control points were obtained by substituting the values of X_1 from (Table 1) into the model equation ie 'equation (32)'. These values were compared with the experimental result ($Y_{observed}$) given in (Table 2).

Student's t-test

For this test, the parameters Δ_y, Є and t are evaluated using the following equations respectively

$$\Delta_Y = Y_{(observed)} - Y_{(predicted)} \quad (33)$$

$$Є = (\sum a_i^2 + \sum a_{ij}^2) \quad (34)$$

$$t = \Delta_y\sqrt{n} / [Sy\sqrt{(1+ Є)}] \quad (35)$$

where Є is the estimated standard deviation or error,
t is the t-statistics,
n is the number of parallel observations at every point
S_y is the replication error
a_i and a_{ij} are coefficients while i and j are pure components
$a_i = X_i(2X_i-1)$
$a_{ij} = 4X_iX_j$
$Y_{obs} = Y_{(observed)}$ = Experimental results
$Y_{pre} = Y_{(predicted)}$ = Predicted results

Table 3. T –Statistics for test control points

N	CN	i	j	a_i	a_{ij}	a^2_i	a^2_{ij}	Є	Y_{obs}	Y_{pre}	Δ_Y	t
		1	2	0	0.5	0	0.25					
		1	3	0	0.5	0	0.25					
		1	4	0	0	0	0					
1	C_1	2	3	-0.125	0.25	0.0156	0.0625		25.81	26.21	-0.4	-0.15
		2	4	-0.125	0	0.0156	0					
		3	4	-0.125	0	0.0156	0					
		4	-	0	-	0	0					
					∑	0.0468	0.5625	0.6093				
		1	2	-0.125	0.25	0.0156	0.0625					
		1	3	-0.125	0.25	0.0156	0.0625					
		1	4	-0.125	0.25	0.0156	0.0625					
2	C_2	2	3	-0.125	0.25	0.0156	0.0625		26.39	25.85	0.54	0.21
		2	4	-0.125	0.25	0.0156	0.0625					
		3	4	-0.125	0.25	0.0156	0.0625					
		4	-	-0.125	-	0.0156	-					
					∑	0.1092	0.375	0.4842				
		1	2	0	0	0	0					
		1	3	0	0	0	0					
		1	4	0	0	0	0					
3	C_3	2	3	-0.125	0.25	0.0156	0.0625		26.98	25.22	1.76	0.66
		2	4	-0.125	0.5	0.0156	0.25					
		3	4	-0.125	0.5	0.0156	0.25					
		4	-	0	-	0	-					
					∑	0.0468	0.5625	0.6093				
		1	2	0	0	0	0					
		1	3	0	0	0	0					
		1	4	0	0	0	0					
4	C_4	2	3	-0.125	0	0.0156	0		23.32	23.38	-0.06	-0.02
		2	4	-0.125	0.75	0.0156	0.5625					
		3	4	0	0	0	0					

		4	-	0.375	-	0.1406	-					
					∑	0.1718	0.5625	0.7343				
		1	2	0.375	0	0.1406	0					
		1	3	0.375	0.75	0.1406	0.5625					
		1	4	0.375	0	0.1406	0					
5	C_5	2	3	0	0	0	0		28.24	26.01	2.23	0.75
		2	4	0	0	0	0					
		3	4	-0.125	0	0.0156	0					
		4	-	0	-	0	-					
					∑	0.4374	0.5625	0.9999				
		1	2	0	0	0	0					
		1	3	0	0	0	0					
		1	4	0	0	0	0					
6	C_6	2	3	0	0	0	0.25		27.75	26.83	0.92	0.35
		2	4	0	0.5	0	0.25					
		3	4	-0.125	0.25	0.0156	0.0625					
		4	-	-0.125	-	0.0156	-					
					∑	0.0312	0.5625	0.5937				
		1	2	-0.125	0	0.0156	0					
		1	3	-0.125	0.5	0.0156	0.25					
		1	4	-0.125	0.25	0.0156	0.0625					
7	C_7	2	3	0	0	0	0		23.69	25.58	-1.89	-0.71
		2	4	0	0	0	0					
		3	4	0	0.5	0	0.25					
		4	-	-0.125	-	0.0156	-					
					∑	0.0624	0.5625	0.6249				
		1	2	0.375	0.75	0.1406	0.5625					
		1	3	0.375	0	0.1406	0					
		1	4	0.375	0	0.1406	0					
8	C_8	2	3	-0.125	0	0.0156	0		24.02	25.71	-1.69	0.80
		2	4	-0.125	0	0.0156	0					
		3	4	0	0	0	0					
		4	-	0	0	0	0					
					∑	0.453	0.5625	1.0155				
		1	2	0	0	0	0					
		1	3	0	0	0	0					
		1	4	0	0	0	0					
9	C_9	2	3	0.375	0.75	0.1406	0.5625		25.96	31.67	-5.71	-1.99
		2	4	0.375	0	0.1406	0					
		3	4	-0.125	0	0.0156	0					
		4	-	0	-	0	-					
					∑	0.2968	0.5625	0.8593				
		1	2	0	0	0	0					
		1	3	0	0	0	0					
		1	4	0	0	0	0					
10	C_{10}	2	3	-0.08	0.64	0.0064	0.4096		26.57	27.77	-1.2	-0.45
		2	4	-0.08	0.32	0.0064	0.1024					
		3	4	-0.08	0.32	0.0064	0.1024					
		4	-	-0.12	-	0.0144	-					
					∑	0.0336	0.6144	0.648				

At significant level, $\alpha = 0.05$, $t_{\alpha/1}(Ve) = t_{0.05/10} = t_{0.005(9)} = 3.250$. The t – value is obtained from standard t – statistics table.

Since this is greater than any of the t- values calculated in (Table 3), we accept the Null hypothesis. Hence the model is adequate.

Fisher Test

For this test, the parameter y, is evaluated using the following equation:

$$y = \sum Y/n \quad (36)$$

Where Y is the response and n the number of responses.

Using variance, $S^2 = [1/(n-1)][\sum (Y-y)^2]$ and $y = \sum Y/n$ for $1 \le i \le n$ (37)

Table 4. F-Statistics for the controlled points

Response Symbol	$Y_{(observed)}$	$Y_{(predicted)}$	$Y_{(obs)}-y_{(obs)}$	$Y_{(pre)}-y_{(pre)}$	$(Y_{(obs)}-y_{(obs)})^2$	$(Y_{(pre)}-y_{(pre)})^2$
C_1	25.81	26.21	-0.063	-0.213	0.003969	0.045369
C_2	26.39	25.85	0.517	-0.573	0.267289	0.328329
C_3	26.98	25.22	1.107	-1.203	1.225449	1.447209
C_4	23.32	23.38	-2.553	-3.043	6.517809	9.259849
C_5	28.24	26.01	2.367	-0.413	5.602689	0.170569
C_6	27.75	26.83	1.877	0.407	3.523129	0.165649
C_7	23.69	25.58	-2.183	-0.843	4.765489	0.710649
C_8	24.02	25.71	-1.853	-0.713	3.433609	0.508369
C_9	25.96	31.67	0.087	5.247	0.007569	27.53101
C_{10}	26.57	27.77	0.697	1.347	0.485809	1.814409
Sum	258.73	264.23			25.83281	41.98141
Mean	$y_{(obs)}=$ 25.873	$y_{(pre)}=$ 26.423				

Therefore from (Table 4), $S^2_{(obs)} = 25.83281/9 = 2.87$ and $S^2_{(pre)} = 41.98141/9 = 4.66$

But the fisher test statistics is given by:

$$F = S^2_1 / S^2_2 \quad (38)$$

where S^2_1 is the larger variance

Hence $S^2_1 = 4.66$ and $S^2_2 = 2.87$

Therefore, F = 4.66/2.87 = 1.62

From standard Fisher Table, $F_{0.95}(9,9) = 3.18$.Hence the regression equation is adequate.

Although the statistical tests were done for the same material (concrete), the grades are different because they were produced from different mixes obtained from the simplex analysis. However, if there were errors in the method used, the statistical tests will not agree to the acceptance of the model.

Comparison of results

The results obtained from the model were compared with those obtained from the experiment, as presented in Table 5

Table 5. Comparison of some Predicted Result with Experimental Results

S/N	Experimental Result ($N/mm^{2)}$)	Predicted Result ($N/mm^{2)}$)	Percentage Difference
1	25.81	26.21	1.55
2	26.39	25.85	2.05
3	26.98	25.22	6.52
4	23.32	23.38	0.26
5	27.75	26.83	3.32

A comparison of the predicted results with the experimental results shows that the percentage difference ranges from a minimum of 0.26% to a maximum of 6.52%, which is insignificant.

V. Conclusion / recommendation

(1) Scheffe's simplex method has been applied and used successfully to develop mathematical model for optimisation of concrete cube strength. However, the model applies to concrete of materials stated earlier. Interested researchers can learn the method and apply it to develop models for cube strength of concrete of different materials and sources.

(2) Concrete cube strength is a function of the proportions of the ingredients (cement, water, sand and coarse aggregate) of the concrete.

(3) The student's t-test and the fisher test used in the statistical hypothesis showed that the model developed is adequate. Although the statistical tests were done for the same material (concrete), the grades are different because they were produced from different mixes obtained from the simplex analysis. However, if there were errors in the method used, the statistical tests will not agree to the acceptance of the model.

(4) The maximum concrete cube strength with the model is $31.71Nmm^{-2}$

(5) Since the maximum percentage difference between the experimental result and the predicted result is insignificant (i.e. 6.52), the optimisation model will yield accurate values of concrete cube strength if given the mix proportions and vice versa.

References

[1] Neville, A.M., Properties of Concrete, third edition, Pitman, England, 1981.

[2] Teychenne, D.C., Franklin, R.E.and Erntroy, H.C., Design of Normal Concrete Mixes, A Publication of Building Research Establishment, Transport and Road Research Laboratory and Cement and Concrete Association, 1975.

[3] Majid, K.I., Optimum Design of Structures, Butterworth & Co. Ltd., London, 1974.

[4] British Standards Institution, BS 12: 1978, Specification for Portland Cement, 1978.

[5] British Standards Institution, BS 882: 1992, Specification for aggregates from natural sources for concrete, 1992.

[6] Scheffe, H., Experiments with Mixtures, Royal StatisticalSociety Journal, 20 (B), 1958, pp 344-360.