

# Optimization Of The Compressive And Flexural Strength Of Sand - Latratic Concrete Using Scheffe's And Osadebe's Theories

Onyeike, Ogwumike Onyeike  
MOUAU/PG/M.ENG/CIE/23/122798  
Department Of Civil Engineering  
Michael Okpara University Of Agriculture, Umudike

---

## **Abstract**

*This thesis focuses on optimizing the compressive and flexural strengths of sand-lateritic concrete. It involves casting concrete cubes and prototype sand-lateritic concrete beams. In total, ninety (90) sand-lateritic concrete cubes measuring 150 × 150 × 150 mm and ninety (90) prototype sand-lateritic beams measuring 600 × 150 × 150 mm were cast according to the employed mix proportions. The test specimens were cured for 28 days and then tested. Portions of the comprehensive test results were used to formulate models based on Scheffé's and Osadebe's theories, while portions of the flexural test results were used to formulate models based on Scheffé's and Osadebe's theories for predicting the flexural strengths of sand-lateritic concrete. The adequacies of the models were evaluated and confirmed using Fisher and Student's T-tests. Computer programs based on these models were developed in the QBASIC language. The developed programs can predict concrete mix proportions given a desired compressive or flexural strength, and vice versa. The predicted compressive and flexural strength values from the models agreed with the corresponding experimental results, as well as with each other. The optimum compressive and flexural strengths correspond to the mix ratios 0.51:1:2.25:0.25:4 and 0.535:1:1.85:0.65:5, yielding 19.15 N/mm<sup>2</sup> and 2.37 N/mm<sup>2</sup>, respectively. Thus, incorporating some laterite into the mix proportioning is feasible since the optimum strengths fall within the acceptable range. Hence, the formulated models can be used to predict the compressive and flexural strength of sand-lateritic concrete with accuracy.*

---

Date of Submission: 16-03-2026

Date of Acceptance: 26-03-2026

---

## I. Introduction

### Background Of The Study

Experimentation is the vital part of scientific or engineering method. Researchers perform experiments virtually in all fields of inquiry in order to discover something about a particular process or system. Well-designed experiments can often lead to a model of system performance.

The development of supplementary fine aggregate materials is fundamental to advancing low cost materials to be used in the production of a self-sufficient means of shelter, especially in developing countries. Future interest in concrete will revolve around the merits and benefits associated with using supplementary fine aggregate materials. Apart from improving concrete properties, the main benefit comes from saving material resources (Elinwa *et al*, 2005).

Test is dynamic and the quest for contemporary building structures is therefore inevitable. This has made men to desire higher standards of living which brought about the development of new ways of shaping the environment hence the development of increasing sophisticated construction practices and building materials.

Concrete is one of the mostly used construction materials in the world. As the use of concrete becomes more widespread, the specification of concrete such as quality, durability, strength and optimization becomes more important in building and construction industries. Thus, the construction industries rely heavily on concrete for its operations in the development of shelter and other infrastructural facilities. And, because of the natural or man-made forces or loads that the erected structures are subjected to after construction, such as wind, accidental push, vibration caused by explosions or earth moving equipment etc, the need for concrete of high strength arose, it then becomes extremely difficult for majority of people to erect their own houses. There is also incessant collapse of structures in an attempt to reduce cost by compromising constituent components of concrete.

The use of concrete of low strength, in structures that require concrete of high compressive or flexural strength, results in failed or collapsed structures when they were subjected to the designed impact or weather conditions. Cement is an essential, but expensive constituent of concrete, which has the property of setting and hardening under water by virtue of chemical reaction (Neville, 1096). Its high cost is traceable to the industrial

processes undergone during manufacturing stage and the limited raw materials. Concrete is a man-made composite, whose major constituent is natural aggregate, such as gravel and sand. Alternatively, artificial aggregates like blast furnace slag, expanded clay, broken brick and steel shot may be used where appropriate.

Over the years, there has been astronomical increase in the cost of construction materials. The worldwide consumption of sand as fine aggregate in concrete production is very high and several developing countries have encountered some strain in the supply of sand in order to meet the increasing needs of infrastructural development in recent years. According to Raman et al (2007), the situation is responsible for the increase in the price of sand, and the cost of concrete.

The concrete is used structurally in building for foundation, bridges, sewage treatment works, railway, roads, dams etc. In its hardened state, concrete is a rock-like material with a high compressive strength. Normal concrete has a comparatively low tensile strength and for structural applications, it is normal practice to incorporate steel bar to resist any form of tensile forces. As a construction material, concrete offers many advantages including flexibility in design since it is economical, durable, fire resistant, able to fabricate on site and aesthetic in appearance.

Lateritic soil is a widely available and sustainable material that can be used as a partial replacement for sand in concrete production, the increasing demand for sustainable and eco-friendly construction materials has led to a growing interest in the use of lateritic soil in concrete production (Osinubi, 2006). Sand lateritic concrete (SLC) is a type of concrete that combines the benefits of sand and lateritic soil to produce a durable and sustainable building material (Afolayan, 2013). and this SLC has shown to exhibit good strength properties, making it a viable option for construction projects, However, the optimization of SLC's strength properties is crucial to ensure its safe and efficient use in construction project and its widespread adoption in the construction industry.. Scheffe's and Osadebe's theories are statistical techniques that can be used to optimize the mix proportions of SLC and predict its strength properties (Scheffe, 1958; Osadebe, 2011).

There remains a notable research gap in the study of sand-lateritic concrete (SLC), despite its considerable promise as a durable and economical construction material. While the material has attracted interest for its environmental and resource efficiency benefits, there is surprisingly little systematic exploration of how its key mechanical properties—specifically compressive and flexural strengths—can be optimized through rigorous statistical approaches. In particular, there is a paucity of work that applies advanced experimental-design methodologies and statistical theories to identify optimal mix configurations and processing parameters that maximize strength performance, while also accounting for variability in raw materials and curing conditions.

Two influential theoretical tools—Scheffé's theory (Scheffé, 1958) and Osadebe's theory (Osadebe, 2011)—offer robust frameworks for modeling and optimizing multi-factor systems. Yet, investigations that leverage these theories to dissect and enhance the strength properties of SLC are limited. Existing studies often rely on conventional empirical fitting, limited design spaces, or qualitative assessments that fail to quantify interaction effects, nonlinearity, and the uncertainty inherent in material behavior. As a result, the potential gains from a design-optimization perspective—gains that could translate into more efficient mixes, improved structural performance, and broader adoption in practice—remain largely untapped within the SLC domain.

Addressing this lacuna, the present study sets out to systematically probe the optimization of SLC strength characteristics through the dual lenses of Scheffé's and Osadebe's theoretical constructs. By constructing carefully designed experiments that vary relevant factors—such as sand-to-laterite ratios, cementitious content, aggregate properties, curing regimes, and processing variables—the research seeks to illuminate not only the main effects of each factor but also their interactions and higher-order relationships. The objective is to develop predictive, statistically grounded models that can guide practitioners in selecting mix designs and processing conditions that yield maximum compressive and flexural strengths, while also providing quantifiable measures of confidence and precision.

In essence, this work aims to fill a critical gap by integrating advanced statistical theory with material optimization for SLC. Through rigorous modeling, validation, and interpretation within Scheffé's and Osadebe's frameworks, the study aspires to produce actionable insights, contribute to a more robust understanding of how SLC strength responds to controlled design choices, and lay a foundation for more reliable, scalable applications of sand-lateritic concrete in real-world construction.

### **Statement Of The Problem**

Sand is a major constituent used in the production of concrete, blocks and mortar needed for construction purposes. The worldwide consumption of sand as fine aggregate is very high and many developing countries have encountered some strain in the supply of sand needed to meet increasing needs of infrastructural development. In order to overcome this problem, an alternative is being sought for use in place of sand.

To obtain good quality concrete, the constituent materials must be properly proportioned. Concrete mix design is the process of ascertaining the appropriate proportions of the ingredients of concrete required for

a specified grade of concrete. Conventional methods of designing concrete mixers are trial and error in nature and give approximate results. Besides, the methods are tedious, costly and time consuming. However, the use of SLC in construction projects is limited due to the lack of understanding of its strength properties and the optimal mix proportions. This study aims to address this problem by investigating the optimization of the compressive and flexural strength of SLC using Scheffé's and Osadebe's theories.

### **Objectives Of The Study**

The primary objective of this study is to optimize the compressive and flexural strength of SLC using Scheffé's and Osadebe's theories. The specific objectives are:

- (i) To determine workability changes with varying percentages of Sand Lateritic concrete and compare these mixes to sharp-sand mixes, and to assess the variation in the initial and final setting times of SLC at different curing times (7, 14, 21 and 28 days).
- (ii) To determine compressive and flexural strength at varying laterite and sand percentages (10%, 20%, 30%) of SLC and obtain optimal mix ratios and corresponding target strengths after incorporating laterite.
- (iii) To formulate models to predict the compressive strength and flexural strength of sand-lateritic concrete (SLC) using experimental data; confirm their validity using Fisher and Student's t-tests.
- (iv) To utilize Scheffé's theory and Osadebe's theory to optimize the flexural and compressive strength of sand-lateritic concrete (SLC).
- (v) To validate model adequacy and reliability to ensure fit and adequacy of the Scheffé and Osadebe-based theories.
- (vi) To compare and analyze the contributions of each theory to strength predictions to assess the agreement between model predictions, experimental results, and inter-model consistency.
- (vii) To develop computer programs (in QBASIC) based on the formulated models and assess predictive mix proportions for a given target strength (compressive or flexural).

### **Significance Of The Study**

The study makes a meaningful contribution to sustainable and durable concrete by leveraging locally available resources to advance sand-lateritic concrete (SLC). It aims to produce optimum concrete mixtures that reduce cost, while optimized blends enhance material properties and performance, offering an affordable alternative fine aggregate for housing development. This work also streamlines and speeds up the mix-design process for sand-laterite concrete.

More broadly, the research serves as a practical reference for civil engineers and building technologists by providing a partial substitute for conventional sand in concrete production. It presents validated models based on Scheffé's and Osadebe's theories to predict and optimize the compressive and flexural strengths of SLC, enabling more reliable and cost-effective mix designs in real-world projects.

The QBASIC-based programs empower practitioners to determine mix proportions for target strengths or to predict strengths from given proportions, accelerating material-selection decisions. Demonstrating optimum strength at specific mix ratios shows that incorporating laterite into concrete can be done without sacrificing performance, supporting sustainable material use.

### **Scope Of The Study**

This study focuses on optimizing the compressive and flexural strengths of sand-laterite concrete (SLC) using Scheffé's and Osadebe's theories. It investigates the effect of mix proportions on the strength properties of SLC and develops mathematical models to predict compressive and flexural strength. Given the demand-supply imbalance for traditional construction materials, the study also develops models for designing concrete mixes that incorporate laterite as a fine aggregate. The models enable prediction of the compressive and flexural strengths of sand-laterite concrete for a specified mix, and vice versa. Accordingly, the scope is limited to developing models and computer programs for designing concrete mixes produced from sand-laterite concrete, with optimization focused on mix proportions and concrete grades using Scheffé's and Osadebe's statistical theories. Test results for compressive and flexural strengths will be collected at 28 days. Additionally, Student's t-test and Fisher's F-test will be employed to verify the adequacy of the formulated models.

## **II. Literature Review**

### **Preamble**

A number of investigations have explored the strength characteristics of sand-lateritic concrete (SLC), focusing on its compressive and flexural performance. For instance, Anya et al. (2021) developed a statistical model to predict the flexural strength of concrete containing lateritic quarry dust. Ogunley (2023) reported that concrete properties improved up to about 9% replacement with waste granite powder (WGP) before subsequently diminishing.

Adeyemi et al. (2020) observed that the compressive strength of laterized concrete increases with age. SLC is a form of concrete produced by combining sand with lateritic soil to yield a durable and sustainable building material (Afolayan, 2013). The strength and durability of SLC are affected by the mix proportions, curing regimes, and the characteristics of its constituent materials (Osinubi, 2006).

## **Concrete**

Concrete is a stone – like material obtained by designing a careful proportioned mixture of cement, sand, gravel or other aggregates and water to harden the mixture in form of the shape and dimension of the desired structure (Sharmin *et al*, 2006), As a matter of fact, it is a construction materials composed cement (commonly Portland cement), aggregates (fine or coarse) of different sizes (sand, gravel, crushed stone/chippings or other inert materials such as slag/vermiculite), and water.

Self compacting concrete is defined as a category of high performance concrete that has excellent deformability in the fresh state and high resistance to segregation, and can be placed and compacted under itself weight without applying vibration (Seshadric and Srinivasa, 2008). Its widespread use in many Roman Structures ( a key event in the history of architecture termed the Roman Architectural Revolution), freed Roman construction from the restrictions of stone and brick material, and allowed for revolutionary new designs in terms of both structural complexity and dimension.(Lancaster, 2005). Concrete, as the Romans knew it, was a new and revolutionary material, laid in the shape of arches, vaults and domes, it quickly hardened into a rigid mass, free from many of the internal thrusts and strains that troubled the builders of similar structures in stone or brick.

In order to obtain the best out of a concrete, it is necessary to have an understanding, not only of the properties of concrete but also of the factors that influence their performance, durability and strength. Concretes used in structures have to meet a number of different functional requirements while they are exposed continuously to a wide variety of destructive agencies. Factors associated with these requirements include strength, heat, water, fire, sound and production. Concrete must be the strength and workable, a careful balance of the cement to water is required when making concrete (Chamberlain, 1995), Concrete can be unreinforced or reinforced. Unreinforced concrete is the earliest form of concrete, but due to new development, it has been replaced by reinforced concrete. Reinforced concrete is a type of concrete strengthened by the addition or inclusion of metal bars that help to increase the tensile strength of concrete.

Other types of concrete include; Pre -stressed Concrete, Post – tensioned concrete, precast concrete and cast – in – concrete.

Concrete is unarguably the commonest construction material used in structural and civil works. It is a versatile construction material adaptable for different uses. Concrete is a mixture of cement, aggregates (fine and coarse), and water. The slurry of cement and water serves as a binder for the fine and coarse aggregates, by the chemical reaction of the cement and water known as hydration. Fresh cement paste is a network of particles of cement in water, it is plastic and normally remains thus for an hour or more, after which this plastic mass sets. In the first few minutes of mixing, the reaction rate is rapid and calcium silicate hydrate-cement gel (C-S-H) forms a coating around the cement grains. As hydration progresses, hydration products gradually fill the capillary pores formed (Attah *et al.*, 2020). Early hydration is confined to the narrow pores producing tobermorite crystallite which is a stronger material and acts as a solid link between the coated cement particles, so producing a continuous solid matrix within the cement paste. Hardened cement paste can thus be described as a multi-phase material composed of unhydrated cement particles embedded into a continuous matrix of cement gel which, in turn, is a matrix of semi-amorphous intertwined fibrous or needle shaped particles, and thin crumpled sheets and foils which form a continuous matrix having a continuous system of minute water-filled voids called gel pores. Cement gel is interpenetrated by capillary pores or cavities filled with water to a degree dependent upon the amount of drying. Concrete, however, is a very complex multi-phase material composed of at least seven components, i.e. coarse aggregate, sand, unhydrated cement particles, cement gel, capillary and gel pores, and pore water and accidentally or deliberately entrapped air voids. Concrete can also be considered as two-phase material consisting of a uniform distribution of coarse particles in a homogeneous matrix of different composition (Alaneme and Mbadike, 2021).

## **Properties of Concrete**

Concrete properties refer to its characteristics or basic qualities. The special property desired of concrete is a function of the particular purpose for which the concrete is intended. It is therefore necessary to select the constituent materials and combined them in such a manner as to develop the special properties needed as economically as possible in order to for a concrete to be suitable for a specific purpose. Concrete properties can also be addressed in terms of its state/condition i.e. in its fresh or hardened state. Visually, thought of as two major components: paste and essentially inert materials. The paste consists of Portland cement, water, and some air, either in the form of naturally entrapped air voids or minute initially entrained air bubbles. The inert

materials are usually composed of sand and gravel, crushed stone and slag (Alaneme *et al*, 2021).

### **Fresh Concrete**

According to Shetty (2002), fresh concrete or plastic concrete is a freshly mixed material which can be moulded into any shape. Concrete is termed 'fresh' when the constituents are first mixed together and is in the plastic state. The relative quantities of cement, aggregates and water mixed together, control the properties of concrete in the wet state as well as in the hardened state. Fresh concrete is a transient material with continuously changing properties. However, it is essential that concrete can be handled, transported, placed, compacted and finished to form a homogenous, usually void-free, solid mass that realizes the full potential hardened properties (Domone, 2003). Among these qualities, two properties cover all that is required of the freshly mixed concrete; they are (a) workability and (b) Stability. Both are essentially practical properties and are therefore intuitive to everyone dealing with concrete production. However, each is highly complex and not easily, precisely defined.

### **Workability**

Domone (2003) observed that a satisfactory definition of workability is by no means straight forward. Workability, according to Indian Standard, IS: 6461 Part VII (1973), is that property of a mortar or freshly mixed concrete that determines the ease and homogeneity with which it can be mixed, placed, compacted and finished while Road Research Laboratory, U.K. based on extensive study of the field of compaction and workability, defined workability as —the property of concrete which determines the amount of useful internal work required to produce full compaction. Another definition which envelopes a wider meaning is that, it is the —ease with which concrete can be compacted hundred percent having regard to mode of compaction and place of deposition (Shetty, 2002). ASTM (1993) considered workability as the amount of work needed to produce full compaction; thereby relating it to the placing rather than the handling process. ACI definition encompassed other operations; it was defined as that property of mortar that determines the ease and homogeneity with which it can be mixed, placed, consolidated and finished (ACI, 1990). However, this made no attempt to define in what way workability can be measured or specified. ASTM definition of workability as that property determining the effort necessary to handle a freshly mixed quantity of concrete with minimum loss of homogeneity has been similarly criticized (ASTM, 1993). Workability is largely dependent on aggregate (shape and size distribution), water content, cement content and level of hydration (age), and can be modified by adding chemical admixtures. Higher water content or adding chemical admixtures increases concrete workability. Excess water will lead to increased bleeding (surface water) and/or aggregate segregation (separation of cement and aggregates), which results to the concrete having decreased quality. The use of an aggregate with an undesirable gradation can result in a very harsh mix design with a very low slump, which cannot be readily made more workable by addition of reasonable amounts of water. Numerous tests have been devised for this purpose. Domone (2003) identified four tests that have a current British Standard: slump, compacting factor, Vebe and flow table (or more simply, flow). Shetty (2002) included Kelly Ball test as being among the commonly employed methods of measuring workability.

### **Concrete in its Hardened State**

After water is added, the outer layers dissolve and, as the cement cures, it becomes solid again. A chemical reaction has occurred. If there are no moistures, there will be no chemical reaction. When Portland cement is mixed with water, the compounds of the cement react to form a cementing substance. As the hydration reactions proceed, not only do the reaction product take up what was originally free water, but in the gel and other reaction product begin to occupy more space, and the mobility of the space is decreased. Finally, increasing number of particles of gel and product make sufficiently close contact and develop bonds of increasing strength, and if the mass is left undisturbed, it begins to develop rigidity. In normally and correctly mixed cement, each particle of sand and coarse aggregate is completely surrounded and coated by this paste, and all spaces between the particles are filled with it. As the cement set and hardens, it binds the aggregate into a solid mass—the hardened cement paste, and at some point, the mass can sustain more or less arbitrary load without flowing, and the paste is said to have set (Bogue and Lerch, 1984).

### **Mechanical Strength**

Mehta and Monteiro (2006) opined that strength of concrete is commonly considered its most valuable property especially by designers and quality control engineers, although, in many practical cases, other characteristics, such as durability and permeability, maybe more important. Nevertheless, strength usually indicates the quality of concrete because strength is directly related to the structure of the hydrated cement paste. Moreover, concrete strength is specified for compliance purposes and is invariably a vital element of structural design. Concrete strength in compression and tension (both direct tension and flexural tension) are closely related, however the interrelationship is not of the type of direct proportionality. The ratio of the two

strengths depends on general level of strength of concrete. Some factors affect tensile and compressive strength differently e.g. the tensile is less sensitive to variations in the water/cement (W/C) ratio. Consequently, the ratio of tensile to compressive strength is not constant and decreases with increasing concrete strength. In most cases, it varies from 0.01 to 0.20 for strong and weak concretes respectively, when the tensile strength is determined in flexure (Soroka, 1993). Of the various strengths of concrete the determination of compressive strength is of most important because concrete is primarily meant to withstand compressive stresses. In situations where the shear or tension strength is of importance, the compressive strength is usually used as a measure of these properties (Gupta and Gupta, 2004).

#### **Effects of Aggregate/Cement Ratio on Concrete Strength**

The richness of a concrete mix affects the strength of a concrete. For a constant water/cement ratio, a leaner mix leads to a higher strength. The reasons for this are not clear, in certain cases; some water may be absorbed by the aggregate: a large amount of aggregate absorbs a greater quantity of water, the effective water/cement ratio being thus reduced. In other cases, a high aggregate content could lead to a lower shrinkage and lower bleeding, and therefore to less damage to the bond between the aggregate and the cement paste; likewise, the thermal changes caused by the heat of hydration of cement would be smaller (Neville, 1999). The most likely explanation lies in the fact that the total water content per cubic meter of concrete is lower in a leaner mix than in a rich one. As a result, in a leaner mix, the voids form a smaller fraction of the total volume of concrete and it is these voids that have an adverse effect on strength. Studies on the influence of aggregate content on the strength of concrete with a given quality of cement paste indicate that, when the volume of aggregate (as a percentage of the total volume) is increased from zero to 20, there is a gradual decrease in compressive strength, but between 40 to 80 percent there is an increase. The influence of the volume of aggregate on tensile strength is broadly similar (Neville, 1999).

#### **Durability of concrete**

For a long time, concrete required little or no maintenance as it was considered to be a very durable material (Bogue and Lerch, 1984). This assumption is predominantly true, except when the concrete is subjected to highly aggressive environments. Concrete structures are erected in very polluted urban and industrial areas, harmful sub-soil water in coastal area, aggressive marine environments, and many other unfavorable conditions where other materials of construction are found to be non-durable. Concrete usage in recent years having spread to highly harsh and hostile conditions, threatens the initial assumption that concrete is a very durable material, particularly on account of premature failures of number of structures in the recent past. In the past, concrete mix design procedures only considered strength, assuming concrete strength is an all-encompassing factor for all other required properties of concrete including durability. Although compressive strength can be used to determine durability, to a large extent, it is not entirely true that the strong concrete is a determinant for a durable concrete. It has been proved that the degree exposure of concrete throughout its life span to harsh environmental condition is equally important. Therefore, durability and strength are to be explicitly put into consideration at the design stage. ACI Committee 201(2002) defines durability of cement concrete as the ability to resist chemical attack, weathering action, abrasion, or/and any other deterioration process. Concrete that is durable, sustains its original form, quality, and serviceability when exposed to its environment. A concrete is said to be durable when it performs satisfactorily under anticipated exposure (working) condition during its life span (Mehta and Monteiro, 2006). Therefore, the mix proportions and materials used should be such that maintains the integrity of the concrete and possibly protect the embedded metal from corrosion. One of the major features that influences the durability of a concrete is its permeability to the ingress of water, carbon dioxide, oxygen, chloride, sulphate, and other potentially harmful substances, thus resulting in micro and macro-cracks, and voids developed during production and service of concrete structures. Most of the durability problems in concrete can be attributed to the volume change in the concrete. Volume change in concrete is caused by many factors. The entire process of hydration is but an internal change in volume, the impact of heat of hydration, the pozzolanic action, the carbonation, the sulphate attack, the moisture movement, the effect of chlorides, all types of shrinkages, corrosion of steel reinforcement and a host of other aspects come under the preview of volume change in concrete (Neville and Brooks, 1993). The internal or external restraints to changes in volume in concretes bring about cracks. It is the cracks that increases permeability and afterwards becomes a part of cyclic action, until the deterioration, degradation, disruption and eventual failure of the concrete. (Shetty, 2002).

#### **Permeability of Concrete**

Theoretically, the introduction of low-permeability aggregate particles into a high permeability cement paste (especially with high water-cement ratio; pastes at early ages when the capillary porosity is high) is expected to minimize the permeability of the system because the aggregate particles should intercept the

channels of flow within the cement paste matrix. In comparison to a neat cement paste, therefore, a mortar or a concrete with the same water-cement ratio and degree of maturity should bring about a low coefficient of permeability. Test data by Neville and Brooks (1990) indicated that, in practice, this does not happen. The two sets of data clearly show that the addition of aggregate to a cement paste or a mortar increased the permeability considerably; in fact, the larger the aggregate size, the greater the coefficient of permeability. Typically, the permeability coefficients for moderate-strength concrete (containing 38 mm aggregate, 356 kg/m<sup>3</sup> cement, and an 0.5 water-cement ratio), and low-strength concrete used in dams (75 to 150 mm aggregate, 148 kg/m<sup>3</sup> cement, and an 0.75 water-cement ratio) are of the order of  $1 \times 10^{-10}$  and  $30 \times 10^{-10}$  cm/s, respectively. The explanation as to why the permeability of mortar or concrete is higher than the permeability of the corresponding cement paste lies in the micro-cracks normally present in the interfacial transition zone between aggregate and the cement paste. Studies have shown that, the aggregate size and grading affect the bleeding characteristic of a concrete mixture that, in turn, influences the interfacial transition zone (Neville and Brooks, 1993).

### **Concrete Materials**

The word concrete comes from the Latin word 'concrete' ( meaning compact or condensed), the perfect passive participle of the 'concrecere' come from two words, named (together) and 'crescere' ( to grow). Concrete was used for construction in many ancient structures (Stalla, 1996). During the Roman Empire, Roman concrete was made from quick lime, pozzolana and an aggregate of pumice.

Concrete has been in use as a major building material ever since its inception world over, the last three – four decades have seen construction of innumerable reinforced concrete structures with compressive strength of concrete in the range of 60 – 100 Mpa (Krishnan, 2001).

### **Cement**

Cement is a powdery material which when mixed with water produces a binding substance that has both cohesive and adhesive properties that bind mineral fragments into a compacted whole, used in construction works. Cement used in construction is characterized as hydraulic or non – hydraulic. Hydraulic cements harden because of hydration and chemical reactions that occur independently of the mixture's water content; they can harden even underwater or when constantly exposed to wet weather. The chemical reaction that results when the anhydrous cement powder is mixed with water produces hydrates that are not water – soluble. Non – hydraulic cement must be kept dry in order to retain their strength (Aitcin, 2000).

Cement consists of a mixture of oxides of calcium, silica, alumina and iron. It is the major component of concrete that provide and improves the strength of concrete, (Finn *et al*, 2009).

Cement as one of the concrete materials are of different types and when each type is used, an entirely different strength of concrete is produced as compared when another type is used. (ASTM standard, 1976). For the purpose of this research, it will be limited to ordinary Portland (OPC), though there are so many types of cement but ordinary Portland cement is the most common and available in Nigeria.

The type of cement to be used for the concrete depends on the type of structure to be construction. Like high early strength cement may be required for precast concrete products/element or in high rise building frames to permit rapid removal of frameworks and early load carrying capacity while cements of low heat of hydration are required for massive structures. American Concrete Institute. (ACI committee, 1970).

### **Aggregates**

Aggregates are important constituents of concrete. They give body to concrete, reduce shrinkage and affect economy (Shetty, 2003). Aggregate occupy 60% to 80% of concrete volume making its selection highly important (Neville and Brookes, 1987). Aggregate is one of the important constituents in concrete which has effect in strength development in the theory that the gaps of coarse aggregate is filled by the fine aggregate and the gaps of fine aggregate is filled by the binding materials (Aziz, 1995). Fine and Coarse aggregates makes up the bulk of concrete mixture. Redistribution of aggregates after compaction often creates in homogeneity due to the influence of vibration. This can lead strength gradients (Veretennykov *et al*, 2008). Marine aggregate should not use unless washed completely to remove the magnesium and sodium chloride salts, which are deliquescent and attract moisture (Hendry, *et al*, 1987).

Aggregates are generally classified into two groups, fine and coarse aggregates.

Fine Aggregate.

Fine aggregate is one of the main constituents of concrete making, being about 35% of volume of concrete used in the construction industry (Saeed and Shadid, 2008). Fine aggregate consist of natural or manufactured sand with particle sizes up to 5mm. It consists of inert natural sand conforming to BS 882, 1992.

Coarse Aggregate.

The size distribution above 5mm diameter is classified as coarse aggregates. The most suitable aggregate would appear to be one that is graded well with a balance between rounded and angular particle and a surface texture that is not too smooth. In practice, it has been found that a natural river aggregate with grading complying with BS 822, 1992 is the most suitable.

### Laterite

Lateritic Soil is one of the soils rich with vital nutrients and is widely used. It is a soil and rock type rich in iron and aluminum and is commonly considered to have formed in hot and wet tropical areas, nearly all laterites are of high iron oxide content and is developed by intensive and long-lasting weathering of the underlying parent rock.

Although, lateritic materials have been widely studied, no general accepted definition of laterite exists. Many definitions have been proposed since the first description of laterites by Buchanan (1807). Excellent reviews of the old definitions are given by Sivarajasingham *et al*(1962). Lateritic soils are mixtures of phyllosilicate and iron minerals, general  $Fe_2O_3(FeO)$  in  $Al_2O_3 - SiO_2-H_2O$  matrices, with the crystalline habitus typical of Kaolinite in which a high proportion of  $Al^{3+}$  is replaced by  $Fe^{2+}$  or  $Fe^{3+}$  (Kamseu *et al.*, 2013). These materials are available in tropical and sub-tropical areas all around the globe (Mbumbia *et al.*; 2000). Lateritic soils, also known as laterites, are a group of highly weathered soils prevalent in tropical and subtropical regions where high temperatures and heavy rainfall are common. These soils are characterized by their rich iron and aluminum oxide contents, which give them their distinctive reddish color. Lateritic soils are reddish-brown residual soils formed by the chemical weathering of pre-existing rocks such as granite, shale, limestone, schist, sandstone, and gneiss (Wahab, Roshid, & Rizal, 2021). They are commonly formed in tropical climatic regions of the world; over 85% of their major oxide constituents are made up of  $SiO_2$  and  $Al_2O_3$ ,  $Fe_2O_3$  (Adeyemi, 2023). The reddish-brown coloration of these soils is due to their  $Fe_2O_3$  contents. They are often found in areas with high-altitude plateaus and gentle slopes, particularly in parts of Africa, South America, India, and Southeast Asia. In Nigeria, lateritic soils are abundant in different parts of the country.

The unique engineering properties of lateritic soils, such as their variable consistency, permeability, and strength, make them a subject of interest in civil engineering and construction projects. These soils can exhibit a wide range of behaviors under different moisture conditions, ranging from hard and brittle when dry to soft and pliable when wet. Their physical and mechanical properties are heavily influenced by their mineralogical composition, degree of weathering, and the environmental conditions to which they have been exposed.

In terms of stability, lateritic soils pose significant challenges. The safety and stability concerns of using lateritic soils are critical for infrastructure projects such as roadways and building constructions (Emeh, Igwe, & Onwo, 2019). Failures in lateritic terrains can lead to significant economic losses, environmental damage, and even loss of life.

The determination of the right proportion of lateritic soil in concrete mix is crucial for ensuring the safety of both the construction process and the long-term stability of the structure. The critical ratio is defined as the maximum proportion of laterite that can be maintained without failure, given the soil's properties and the prevailing environmental conditions (Moayed, Osouli, Nguyen, & Rashid, 2019).

Lateritic soils are a common type found in tropical regions and are formed due to the weathering of basaltic or other igneous rocks. They are characterized by their high iron and aluminium content, giving them a distinct red or brown color. In engineering, lateritic soils present several challenges due to their unique characteristics, such as low strength, high compressibility, and susceptibility to erosion.

In engineering, lateritic soil present several challenges due to their unique characteristics. Some of these challenges include their low strength and high compressibility, which can make them unsuitable for certain types of construction projects. However, with proper understanding and management, lateritic soils can be used effectively in many engineering applications. Lateritic soils can also be used in the construction of dams and embankments. However, their susceptibility to erosion and settlement over time requires careful planning and management. A study of Bhutto *et al.* (2019) found that lateritic soils can be used as embankment material for low-height dams, but soil stabilization method must be used to increase its shear strength and prevent erosion.

Lateritic soils are widely used as a base material in engineering construction due to their abundance and low cost. However, the low strength of lateritic soils can make them unsuitable for many construction works. Therefore, proper design and construction methods must be used to ensure their stability and durability.

Lateritic soils typically exhibit low strength and tend to deform under load. Their strength can vary considerably with their mineral makeup and the level of compaction. These soils are relatively impermeable, meaning they are not very porous and water drains slowly through them (Rizal, Hezmi, Razali, & Walab, 2022). While this low permeability can be advantageous for structures needing water resistance, it can also create

drainage and flooding challenges. Regarding shrinkage and swelling, lateritic soils tend to shrink or swell with moisture changes, potentially affecting foundation stability and overall structural integrity. Designers must account for these volume changes when working with lateritic soils. Additionally, their high clay content can make compaction difficult, hindering the achievement of the desired density for buildings and infrastructure. Employing appropriate compaction methods and equipment is essential to ensure proper soil compaction.

Lateritic soils form through the weathering of rocks in tropical and subtropical climates. They are usually rich in iron and aluminum oxides, which gives them a reddish-brown hue. The mineral makeup of lateritic soils is complex and depends on the parent rock and how far weathering has progressed (Tamassoki, Norsyahariati, Jakarmi, & Kusin, 2022). Predominant minerals include kaolinite, gibbsite, and goethite. Kaolinite is a clay mineral produced by the weathering of feldspar-rich rocks, while gibbsite and goethite are aluminum and iron oxide minerals formed from rocks rich in these elements. Additional minerals such as quartz, feldspar, mica, and pyroxene may also be present. The mineralogy of lateritic soils significantly influences their engineering properties, including strength, permeability, and durability. For example, kaolinite can reduce strength and increase plasticity, whereas gibbsite and goethite contribute to high iron and aluminum oxide content, which can lead to corrosion of metal structures.

In geotechnical engineering, specific gravity is used to derive various soil properties—unit weight, porosity, void ratio, and degree of saturation. It also helps classify soils (e.g., via the Unified Soil Classification System, USCS), which is essential for designing foundations, retaining structures, and other geotechnical works. Several methods exist to determine soil specific gravity, such as the water pycnometer, gas jar, and heavy liquid methods. The choice depends on soil type, test purpose, and available equipment.

The specific gravity of lateritic soils typically lies between 2.0 and 2.8, influenced by mineral composition and weathering extent. It is affected by the quantity and type of minerals (notably iron and aluminum oxides), as well as compaction and moisture content. This parameter is crucial in geotechnical design, affecting density, porosity, and void ratio, and it can also be used to evaluate the effectiveness of stabilization approaches (e.g., cement or lime stabilization) in enhancing strength and stability. Ayodele, Pamukcu, & Agbede (2020) found that moisture content and compaction energy reduce specific gravity.

Unit weight, or weight density, is the soil's weight per unit volume (commonly expressed as  $\text{kN/m}^3$  or  $\text{t/m}^3$ ). It is important for geotechnical design and earthworks estimation (Raja & Thyagaraj, 2020).

Typical unit weight for lateritic soils ranges from about 15 to 20  $\text{kN/m}^3$ , depending on compaction and moisture content (Raja & Thyagaraj, 2020). Local conditions and soil characteristics can cause substantial variation in this value. Given their distinctive properties and variable unit weight, a thorough understanding is essential for effective use in engineering and construction. Accurate unit weight measurements are critical for designing stable structures and ensuring civil engineering project safety.

Shear strength is a key geotechnical parameter used to assess stability and bearing capacity of foundations, slopes, retaining walls, and other earth structures. It is measured using direct shear tests, triaxial shear tests, or unconfined compression tests. These tests apply controlled stress or strain to a soil sample and record deformation or failure. Shear strength can be expressed as shear stress, through cohesion and friction angle, or via shear strength envelopes.

## **Water**

Water is the most needed ingredient for the production of concrete. For best strength, durability and other desirable properties, the concrete should be placed with the minimum quantity of mixing water constituent with proper handling.

Water used for mixing concrete should be clean and free from both organic and inorganic materials. Impurities in mixing water can affect the setting time of a concrete and cause surface efflorescence. Portable water is usually suitable for concrete mix because it is free from any form of contamination. Water facilitates mixing, placing and compacting of the fresh concrete.

## **Admixtures**

Admixtures are addition to concrete mix that can help control the set time and other aspects of fresh concrete. With the use of admixes, control can be exercised over concrete. It can improve the performance of problem concrete by modifying its characteristics and enhancing workability. In normal use, admixture dosages is less than 5% by mass of cement and are added to the concrete at the time of batching/mixing (U.S. FHA, 2007); Admixture is of two types; Mineral and Chemical admixtures.

### **Mineral Admixtures**

These are inorganic materials that have pozzolanic or latent hydraulic properties. These very fine – grained materials are added to the concrete mix to improve the properties of concrete (U.S.FHA, 2007) or as a replacement for Portland cement (Kosmatha and Panarese, 1988). Mineral admixtures include (US.FHA, 2007).

- (i) Fly Ash; This is a by – product of coal – fired electric generating plants; it is used to partially replace Portland cement. The properties of fly ash depend on the type of coal burnt. In general, siliceous fly ash is pozzolanic, while calcareous fly ash has latent hydraulic properties.
- (ii) Silica Fume; This is a by – product of silicon and ferrosilicon alloys. Silica fume is similar to fly ash but smaller. This is used to increase strength and durability of concrete, but generally requires the use of super plasticizers for workability.
- (iii) Ground granulated blast furnace slag; this is a by – product of steel production used to partially replace Portland cement. It has latent hydraulic properties.

#### Chemical Admixtures

Chemical admixtures are materials in the form of powder or fluids that is added to the concrete to give it certain characteristics not obtainable with plain concrete mixes. Chemical admixture is used to improve the quality of concrete during mixing, transporting and curing. The common types of admixture (Cement admixture Association, 2010) are as follows;

- (i) Accelerators speed up the hydration of the concrete. Setting and hardening accelerators increase the rate of both setting and early strength development. The most common admixture for this purpose is calcium chloride. Since its use may result in several adverse effects such as increased drying, shrinkage, reduced resistance to sulphate attack and increased risk of corrosion of steel reinforcement, it should only be used with extreme caution and in accordance with any relevant specifications.
- (ii) Retarders slow the hydration and are use in large or difficult pour where partial setting before the pour is complete and undesirable.
- (iii) Air entrainments add and entrain tiny air bubbles in the concrete , which will reduce damage during freeze than cycles, thereby increasing the concrete’s durability.
- (iv) Plasticizers increase workability of plastic or “fresh’ concrete , allowing it to be placed more easily, with less consolidating effort.
- (v) Pigments can be used to change the colour of concrete for aesthetics. Pigments do not normally affect concrete properties although those base on carbon may cause some loss of strength at early ages and can also reduce the effectiveness of air – entraining admixtures.

#### Concrete Mix Design

Concrete mix design is the process in selecting ingredient which is cement, aggregates, water and determining their relative proportions to give the required strength, workability and durability. The proportioning of ingredients of concrete is governed by the required performance of concrete in two state, namely plastic and hardened states.

The two main objective of concrete mix design are:

- (i) To determine the proportion of concrete mix constituents of cement, fine aggregate, coarse aggregate and water.
- (ii) To produce concrete of the specific proportion.

#### Strengths Of Concretes

When concrete is being used, it is important to consider a number of factors which will determine the strength of the concrete structure. The important strength of the concrete include:

#### Compressive Strength

The compressive strength is normally obtained expectantly by means of a compressive test. The apparatus used for this experiment is the same as the one used in tensile strength (Callister, 2003).

This is the most critical property of concrete in relation to its long-term structural behavior. It forms the basis of compliance with most specification.

The compressive strength of concrete is taken as the maximum compressive load it can carry per unit area. Concrete strengths of up to 80Nmm<sup>2</sup> can be achieved by selective use of the type of cement, mix proportions, method of compaction and curing conditions. The strength of the concrete in compression is much greater than either its tensile or shear strength. The strength increases with age, but at a gradually decreasing rate in a relationship which varies between different types of concrete. Hence any measurement of compressive strength most specify the age of the concrete. Although test is carried out at different ages for different purposes, the most important test is done at an age when the strengths has seized to increase significantly. Hence, the majority of crushing strength has test are carried out at age of 28 days. The strength at any age is increased by increasing cement content of concrete within certain limit or by decreasing the water- cement is provided that sufficient water is present to hydrate all the cement. The 28days cube strength cube strength is

usually in the range of 10 – 80 Nmm<sup>2</sup>.

### **Flexural Strength**

Flexural strength is also known as modulus of rupture (MR), bending strength or fracture strength (Hodgkinson, 2000). It is one measure of the tensile strength of concrete. It is a measure of ability of unreinforced concrete beam or slab to resist failure in bending. It is also a measurement that indicates a materials <sup>resistance</sup> to deforming, when it is placed under a load. It is measured by loading 6 \* 6 inch (150mm \* 150mm) concrete beam with a span length of at least three times the depth. Flexural strength is expressed as modulus of ruptures (MR) in MPa and is determined by standard test methods namely Third point loading ASTM C78 or centre-point loading special by ASTM C292. Flexural strength (M.R) is about 10% to 20% of compressive strength depending on the type, size and volume of coarse aggregate used (Kosmatha, 1985). However, the best correlation for specific materials is obtained by laboratory tests for given materials and mix design. The mr determined by third – point is lower than the MR determined by centre – point loading, sometimes being as much as 15%.

### **Shear Strength**

This is used to describe the strength of a material or component against the type of structural failure or yield when the material or component fails in shear. The shear strength of a component is important in structural engineering for designing the dimensions and materials to be used for manufacture/construction of the component (Neville, 1996). In a reinforced concrete beam, the main purpose of stirrups are to increase the shear strength.

### **Optimization History**

The desire for optimality (perfection) is inherent for humans. The search for extremes inspires mountaineers, scientist, mathematicians, and the rest of human race. A beautiful and practical mathematical theory of optimization (i.e search –for-optimum strategies) is developed since the sixties when computers became available.

This mathematical theory is the creation of reliable methods to capture the extreme values of a function through an intelligent arrangement of its evaluations or measurements (Cherkaev, 1997). Optimization, in its simplest sense, means solving problems in which one seeks to minimize or maximize a real function by systematically choosing values of real or integer variables from within an allowed set. It also entails finding the best available values of some objective function, given a defined domain, including a variety of different types of domains. The optimization process is required to select the very best solution from among those available. What is meant by “best” depends on the problem at hand; it might mean the solution that provides the most profit, or the one that consumes the least of some limited resources. According to Chinneck and Ramadan (2000), practical optimization is the art and science of using scarce resources to the best possible effect. Optimization techniques are called into play every day in questions of modern engineering, industrial planning, allocation, decision making, etc.

### **Methods Of Optimization**

Different methods of optimization exist which can be applied to the crushing and flexural strengths of concrete. These methods include Scheffe’s Method, Regression method, neural network method, Genetic algorithm method etc.

#### **Scheffe’s Method**

This is a statistical method of optimization developed by Henry Scheffe in (1959). Scheffe’s method is a single step multiple comparison procedure which applies to the set of estimates of all possible contrasts among the factor level means, not just the pair wise differences considered by the Turkey-Kramer method (Scott and Harold, 2004).

Scheffe’s principle is based on the adjustment of significance level in a linear regression analysis to account for multiple comparisons. It is particularly useful in the analysis of variance, and in constructing simultaneous confidence bands for regress involving basis functions. The simultaneous confidence coefficients is exactly 1 -  $\alpha$ , whether the factor level sample sizes are equal or unequal.. it has been widely used in the optimization of concrete mixtures, including those containing supplementary cementitious materials (Simon, 2003, Yusuf, 2015).

#### **Regression Method**

This is one of the methods used in optimization. It is a statistical technique that allows predictions about an individual’s performance on one variable based on his performance on another variable. Models are

formulated to test for “goodness of fit.” This Osadebe theory has been used in the optimization of concrete mixtures, including those containing industrial by-products (Osadebe, 2013; Anwasi, 2017) integration. According to Osadebe (2003), the response function  $F_s$  is continuous and differentiable with respect to its predictors  $Z_i$ .

The simplest form of regression involves fitting a line to data (Statman, 1997). According to him, regression methods go beyond fitting lines to data; they explore fitting curves, use more than one variable to make predictions, and help determine whether the chosen model fits the data well.

### **Neural Network Method**

These are data processing systems consisting of a large number of simple, highly, interconnected processing element (artificial neurons) in an architecture inspired by the structure of the central cortex of the brain. They have the ability to learn from experience in order to improve their performance and to adapt themselves to changes in the environment (Holla and Schabowicz, 2005). A typical neural network has three layers: The input layer, the hidden layer and the output layer. The input layer represents the value of one independent variable. Each of the output neuron computes one dependent variable. For a good neural network, the training input features must be properly selected, and the type of input data identified. Signals are received at the input layer, pass through the hidden layer, and reach the output layer (Hakim *et al.*, 2011). An important application of neural network is pattern recognition. Pattern recognition can be implemented by using a feed forward neural network that has been trained accordingly.

### **Genetic Algorithm Method**

In using genetic algorithm, you must represent a solution to your problems as a genome (chromosome), which then creates a population of solutions and applies genetic operators such as mutation and crossover to evolve the solutions in order to find the one(s). Genetic algorithm is based on the supposed functioning of the living. The method is very different from classical optimization algorithm (Goldberg and Holland, 1988).

A genetic algorithm is a search heuristic that mimics the process of natural evolution. The heuristic is routinely used to generate useful solution to optimization and search problems. This belongs to the class of evolutionary algorithm, which generate useful solution to optimization problems using techniques inspired by natural evolution, such as cross over (Goldberg, 1989).

A typical algorithm requires the following:

- a) A genetic representation of the solution domain
- b) A fitness function to evaluate the solution domain

Although crossover and mutation are known as the main genetic operation. It is possible to use other operators such as regrouping, colonization, extinction, or migration in genetic algorithms.

Genetic algorithm do not scale well with complexity. That is, where the number of elements which are exposed to material is large there is often an exponential increase in search of space size. This makes it extremely difficult to use the technique on problems to evolutionary search, they must be broken down into the simplest representation possible. Hence, we typically see evolutionary algorithm encoding designs for fan blades instead of engines, building shapes instead of detailed construction plans, aerofoils instead of whole aircraft designs (Wolpert and Macready, 1995).

## **III. Materials And Methods**

### **Materials**

The materials that are used in this research work include the following: Ordinary Portland cement (Ibeto), fine aggregate (sand and laterite), coarse aggregate (granite) and water.

### **Cement**

The cement used in this work is Ibeto cement, a brand of ordinary Portland cement conforming to the requirements of BS12 (1978). It was source locally and use in producing the sand-lateritic concrete specimen tested. The properties of the cement were tested and the results obtained are shown in Tables 4.1 and 4.2. These tests of the properties were performed at Project Development Institute PROD A) Emene in Enugu State.

### **Fine Aggregate**

Laterite soil obtained from a borrow-pit in Alayi, Bende L.G.A, Abia State was used as the fine aggregate. The laterite used was oven dry. Pistol and hammer was used to crush and grind it to powdered form. It is free from impurities, sieved and graded using British Standard Sieves and grading curve and used to mix the sharp sand used to produce the concrete.

The results are presented in Table 4.3 and Fig 4.1. The fine aggregates, have specific gravity of 2.61, while that of bulk density is 1550kg/m.

### **Coarse Aggregate**

The coarse aggregate used in this research work, is crushed granite rock sourced locally from Ishiagu in Ebonyi State, Nigeria. The sizes of coarse aggregates used, varies between 5mm and 20mm in diameter. The crushed rock was sieved through a 20mm British test sieve and graded in accordance to specification and those aggregate that passed through were used together with sharp sand and laterite to produce the concrete. The coarse aggregate used, have a specific gravity of 2.55, while the bulk density is 1490kg/m<sup>3</sup>.

### **Water**

Portable water obtained from bore-hole tap in Ntalakwu Community in Bende local Government was used in producing the sand- lateritic concrete. The density of water used is 994kg/m<sup>3</sup>.

### **Methods**

Two methods were used in this work, namely: experimentation method and optimization methods.

### **Method Of Experimentation**

In this project tests were carried out. They include the following: (i). Sieve analysis of fine aggregate (ii). Compressive strength test (iii). Flexural strength test (iv). Slump test (v). Setting times test of cement.

### **Sieve Analysis Of Fine Aggregate**

There are different methods of carrying out sieve analysis, depending on the material to be measured. However, in this work, the dry sieve method was used. This was done by drying samples of fine aggregates (sharp river sand) in the laboratory for seven day before they were sieved with set of sieves so arranged, that the largest aperture size was placed on top and the smallest aperture was placed at the bottom of the sieves. This experiment was carried out in accordance with the BS 812 part 103 of 1985. The grain size distribution obtained from sharp river sand were determined and the result presented in Table 4.3.

### **Compressive Strength Test**

In the compressive strength, the mixing of material was done manually. First step was cleaning the surface for the mixing, followed by depositing the fine aggregate (sand-laterite) in the required proportions. Thirdly, was the adding of cement to the sand-laterite before it was mixed together. After mixing, granite was added and mixed together with cement and sand-clay before water was added finally. The concrete cubes are cast in steel mould measuring 150mm x 150mm . 150mm. Concrete beams that are cast in steel mould measuring 600mm x 50mm x 150mm. Three cubes of concrete were produced in each mix proportions for models. A total of ninety (90) concrete cubes were produced for - actual and control mix ratios for compressive.

Each mould was filled with concrete in two equal layers as specified by BS 1881, part 108 of 1983. Each layer of concrete was given thirty blows with tamping rod. The top surface of the concrete cube was smoothed with the aid of a trowel, and the cubes were stored uninterrupted for twenty four hours. After setting for 24hours, the specimen were de-moulded and transferred to where they are kept immersed in water for the remaining 28days.

The compressive strength test was carried out on the 28th day. The concrete cubes were taken out of the curing tank and allowed to dry, weighed and thereafter placed in contact with the platens of the compression testing machine. The testing machine was loaded at a constant rate until failure occurs on the concrete cubes. The load causing failure was recorded. Three concrete cubes for each mix proportion were loaded to failure. A total of ninety concretes cubes were produced for actual and control mix ratios for compressive. The crushing load strength were noted and the average crushing strength were determined. Compressive strength = Failure Load (N) / Cross Sectional Area in mm<sup>2</sup>. The values of the respective concrete cube strength results are shown on Table 4.6.

### **Flexural Strength Test Of Concrete Beams**

Like in concrete cube tests, the mass for each component for every mix ratio in flexural was measured out and used to form the concrete that was poured in 600mm \* 150mm \* 150mm steel mould. The sides of the mould were tapped thoroughly to release air pockets on all the side. After setting for 24 hours, the specimen were de-moulded and transferred to where they are kept immersed in water for the remaining 27 days.

The crushing was done after 28 days on the concrete beams to determine their respective flexural strength using the flexural testing machine.

$$\text{Flexural Strength} = (PL/bd^2) * 10^3 \quad (3.1)$$

where:

P = Maximum Load (N)

L = Distance between supporting rollers (mm)

b = Width of the beam (mm)

d = Depth /thickness of the beam (mm)

Three concrete beams were loaded for failure and average flexural strength determined. A total of ninety concrete beams were produced for actual and - control mix ratios for flexural.

**Slump Test**

Concrete constituency is most frequently measured by slump test. Slump test can be used to find out the workability of concrete.

For every concrete mix for each mix ratio for the crushing strength test, a slump test was carried out. The side of the cone was rubbed with oil to facilitate easy lifting of the cone. The cone was placed on the platform and filled with concrete to one- third of its height and compacted twenty five times with the steel rod. The standard cone was later filled with concrete to two-third of its height and compacted twenty five times. Finally the cone was filled up completely and twenty five times of compaction was equally carried out. After compaction, the surface of the concrete in the cone was smoothened. At this stage, the cone was lifted and upside down with respect to its original position. The straight edge will then be placed on the reserved cone and the difference between the cone and the concrete was measured using the measuring rule. The difference in height between the cone and the concrete gives the workability of the concrete.

**Setting Times Test Of Cement**

The course of this work, the initial setting time of cement pastes was determined with the help of a needle, which was attached to the plunger of vicat apparatus. Initial setting time due occur when the needle penetrates to the point 5mm from the bottom of the mould, while the final setting time was determined when the needle with a metal attachment and a circular cutting edge of 5 mm in diameter, is set at 0.5mm behind the top of the needle. The final set is said to have occurred when the needle makes an impression on the paste surface, but the cutting edge fail to do so. This was done on each of the batches of the sand-lateritic concrete cubes. The actual mix ratio for the component of concrete mixture are given in Table 3.1

**Table 3.1: Actual Mix Ratio for the component of concrete mixture:**

S/N	w/c	Cement	Fine Aggregate (Sand & laterite)	% of Sand: Laterite		Granite
1	0.51	1	2.5	90 :	10	4
2	0.53	1	2.0	80	20	4
3	0.54	1	3.0	70	: 30	6
4	0.56	1	1.5	60	40	3
5	0.57	1	2.5	50	50	3
6	0.52	1	2.25	86	: 14	4
7	0.525	1	2.75	79 :	21	5
8	0.535	1	2.0	78.75	: 21.25	3.5
9	0.54	1	2.5	70 :	30	3.5
10	0.535		2.5	74 :	26	5
11	0.545	1	1.75	71 :	29	3.5
12	0.55	1	2.25	63 :	37	3.5
13	0.55	1	2.25	67 :	33	4.5
14	0.535	1	2.75	61 :	39	4.5
15	0.565	1	2.0	53.75	: 46.25	3

**Determination of the Quantities of the Constituents of Concrete**

**(a). For Compressive Strength Test:**

In this section, various quantities of constituents of concrete are calculated based on 150mm cubic mould.

Volume of a 150mm cubic mould = 0.15x0.15x0.15

=0.003375m<sup>3</sup>

But the mass of 1m<sup>3</sup> of concrete=2400kg/m<sup>3</sup>

Therefore, the mass of 150mm concrete cubic given by

M = 2400 x 0.003375

= 8.1kg  
 Mass of three 150mm concrete cubes = 3x 8.1kg  
 = 24.3kg  
 Allowing 10 percent for wastages gives a total mass, M, of concrete:  
 $M = 24.3 + 0.1 \times 24.3$   
 = 26.73kg  
 = 27kg

**(b) For Flexural Strength Test:**

A 600mm x 150mm x 150mm prototype beam were used in the flexural tests.  
 Volume of one prototype beam =  $0.6 \times 0.15 \times 0.15$   
 =  $0.013\text{m}^3$   
 Mass of one prototype beam =  $2400 \times 0.0135$   
 = 32.4kg  
 For three prototype beam, Mass =  $32.4 \times 3 = 97.2\text{kg}$   
 Allowing 10 percent wastage, gives a total mass of concrete, M as:  
 $M = 97.2 + 0.1 \times 97.2$   
 = 106.92kg  
 = 107kg

**Determination of the Masses of each mix proportion used for actual and control of both compressive and flexural strength test.**

**(i) For Compressive Strength Test:**

For ratio of 0.51: 1 : 2.25 : 0.25 : 4  
 Total Ratio -  $0.51 + 1 + 2.25 + 0.25 + 4 = 8.01$   
 Total mass of each concrete cube = 27kg  
 Mass of cement =  $(1/8.01) 27 = 3.37\text{kg}$ ,  
 Mass of water =  $0.51 \times 3.37 = 1.72\text{kg}$   
 Mass of sand =  $2.25 \times 3.37 = 7.58\text{kg}$   
 Mass of laterite =  $0.25 \times 3.37 = 0.84\text{kg}$   
 Mass of granite =  $4 \times 3.37 = 13.48\text{kg}$

**(ii) For Flexural Strength Test:**

For ratio of 0.51: 1 : 2.25 : 0.25 : 4  
 Total Ratio =  $0.51 + 1 + 2.25 + 0.25 + 4 = 8.01$   
 Total mass of each concrete beam = 107kg  
 Mass of cement =  $(1/8.01) 107 = 13.36\text{kg}$ ,  
 Mass of water =  $0.51 \times 13.36 = 6.81\text{kg}$   
 Mass of sand =  $2.25 \times 13.36 = 30.06\text{kg}$   
 Mass of laterite =  $0.25 \times 13.36 = 3.34\text{kg}$   
 Mass of granite =  $4 \times 13.36 = 53.44\text{kg}$ .

This was also done for N2 - N15 for the mix proportion of both compressive and flexural strength casting of cubes and beam.

The values of all the masses of concrete constituents are calculated using actual mix ratios on

Table 3.1 and presented in Tables 3.2-3.3 for compressive strength tests, Tables 3.4-3.5 for flexural strength tests.

**Table 3.2: Mix Proportions used for actual compressive strength tests.**

Mix	Water (kg)	Cement (kg)	Sand (kg)	laterite (kg)	Granite (kg)
N1	1.72	3.37	7.58	0.84	13.48
N2	1.90	3.59	5.74	1.43	14.34
N3	1.38	2.56	5.38	2.31	15.37
N4	2.50	4.46	4.01	2.67	13.37
N5	2.18	3.82	4.77	4.77	11.46
N6	1.81	3.49	6.62	1.13	13.94
N7	1.53	2.92	6.28	1.68	14.59
N8	2.06	3.85	5.97	1.64	13.48
N9	1.34	3.59	6.20	2.69	12.57
N10	1.60	2.99	5.33	1.94	14.94
Nil	2.17	3.97	4.97	1.99	13.91
N12	2.03	3.70	5.27	3.05	12.95
N13	1.79	3.25	4.88	2.44	14.64
N14	1.64	3.07	5.15	3.30	13.83

N15	2.32	4.11	4.42	3.80	12.34
-----	------	------	------	------	-------

**Table 3.3: Mix Proportion used for controlled comprehensive strength tests**

Mix	Water(kg)	Cement(kg)	Sand(kg)	laterite(kg)	Gramitel(kg)
C1	1.87	3.45	5.55	2.34	13.79
C2	1.67	3.10	5.92	1.78	14.56
C3	1.85	3.45	5.85	2.02	13.82
C4	2.06	3.72	5.17	3.39	12.66
C5	1.78	3.35	5.94	1.87	14.06
C6	1.86	3.42	5.24	2.11	14.38
C7	2.00	3.60	5.37	3.42	12.61
C8	1.71	3.14	5.32	2.69	14.13
C9	1.72	3.20	5.72	2.09	14.26
C10	2.09	3.84	5.29	2.36	13.42
C11	1.75	3.31	6.27	1.75	13.91
C12	1.82	3.38	5.68	2.23	13.07
C13	2.24	4.07	5.00	2.68	13.01
C14	2.36	4.18	4.34	3.59	12.53
C15	1.74	3.24	5.72	2.03	14.27

**Table 3.4: Mix Proportion used for actual flexural strength tests.**

Mix	Water(kg)	Cement(kg)	Sand(kg)	Laterite(kg)	Granite(kg)
N1	6.81	13.36	30.06	3.34	54.43
N2	7.53	14.21	22.74	5.68	56.84
N3	5.48	10.15	21.32	9.14	60.91
N4	9.89	17.66	15.89	10.59	52.97
N5	8.63	15.13	18.92	18.92	45.40
N6	7.18	13.82	26.25	4.49	55.26
N7	6.07	11.57	24.87	6.65	57.84
N8	8.17	15.26	23.66	6.49	53.42
N9	7.69	14.24	24.56	10.68	49.83
N10	6.34	11.84	21.91	7.70	59.21
N11	8.58	15.75	19.68	7.87	55.11
N12	8.06	14.66	20.89	12.09	51.30
N13	7.09	12.89	19.34	9.67	58.01
N14	6.52	12.18	20.40	13.09	54.01
N15	9.21	16.30	17.52	15.08	48.90

**Table 3.5: Mix Proportion used for controlled flexural strength tests**

Mix	Water(kg)	Cement(kg)	Sand(kg)	laterite(kg)	Granite(kg)
C1	6.63	12.48	24.97	12.23	50.68
C2	8.45	15.58	18.93	8.10	55.94
C3	6.58	11.88	19.73	13.07	55.73
C4	7.90	14.96	17.95	8.90	57.29
C5	5.84	10.06	22.12	14.28	54.70
C6	6.94	12.35	22.84	9.69	55.19
C7	6.14	11.14	25.08	16.72	47.92
C8	7.79	13.50	29.02	8.91	47.78
C9	7.08	12.08	24.15	9.60	54.10
C10	6.82	12.20	18.72	16.10	53.17
C11	6.43	11.19	21.93	8.05	59.40
C12	7.39	13.84	18.68	7.89	59.21
C13	6.93	12.34	23.07	11.10	53.55
C14	8.33	15.15	17.95	7.88	57.70
C15	7.44	13.59	25.01	11.35	49.61

**Methods Of Optimization**

For the purpose of this research, Scheffe’s simplex optimization and Osadebe’s regression optimization method was used.

**Scheffe’s Simplex Optimization Method**

A factor space is a line (one dimension), a plane (two dimensions), tetrahedron (three dimension), a pentahedron (four dimensions) etc. or any imaginary dimensional space where components of a mixture can interact. This space defines the boundary with which the components can exist. Though there are many types of factor space, but Scheffe’s simplex lattice factor space was considered here. Scheffe’s stated that the number of dimensional factor space to be used must be one less than the number of components i.e.  $q - 1$  (Scheffe, 1958). Thus, if there are five components in a mixture, the factor space (simplex) to be used is four dimensional factor space.

**Scheffes Factor Space**

The factor space creates an environment for interaction of mixture components. Scheffe propounded that if  $q$  components interacts to form a mixture, then  $aq-1$  space can be used to analyze the mixture. Thus, if there are five components in a mixture, the factor space to be used is a four-dimensional factor space. In this space, the total amount (mass or Volume) of the mixture is restricted to one. This implies that no individual component in the mixture should be greater than one.

$$X_i \leq 1 \tag{3.2a}$$

$X_i$  is the  $i$ th component in the Scheffe factor space interacting with other components to form a mixture. It is assumed here that the component of the mixture is negative.

$$X_i \geq 0 \tag{3.2b}$$

$$\text{Combining Equation (3.1) and Equation (3.2), thus } 0 \leq X_i \leq 1 \tag{3.3}$$

Since the total amount of the mixture is one, then

$$x_1 + x_2 + x_3 + x_4 + x_5 + \dots + x_{q-1} + x_q = 1 \tag{3.4}$$

$$\text{Therefore, } \sum_{i=1}^q x_i = 1 \tag{3.5}$$

**Relationship Between Pseudo And Actual Components**

The actual or normal components are those proportions of the ingredients that make up the concrete mixture, while the pseudo component represents proportions of the pseudo components in the concrete mixture that satisfy the condition  $\sum x = 1$

The Pseudo components can be transformed into actual components by multiplying with certain coefficient and vice-versa.

Assuming that the pseudo component is  $x$  and the actual component is  $z$  then

$$[z] = [A][x] \tag{3.6a}$$

$$[X] = [Z]^* [A]^{-1} \tag{3.6b}$$

where,  $[A]$  = matrix of coefficients of sand-clay concrete

$[z]$  = matrix of actual components of sand-clay concrete

$[x]$  = matrix of pseudo components of sand-clay concrete

Assuming  $[A]^{-1} = [B]$ , then Eqn (3.6b) becomes

$$[x] = [B]^* [z] \tag{3-7}$$

Where  $Z =$

Z	w/c	Cement	Sand	laterite	Granite
Z1	0.51	1	2.25	0.25	4
Z2	0.53	1	1.6	0.4	4
Z3	0.54	1	2.1	0.9	6
Z4	0.56	1	0.9	0.6	3
Z5	0.57	1	1.25	1.25	3
Z12	0.52	1	1.925	0.325	4
Z13	0.525	1	2.175	0.575	5
Z14	0.535	1	1.575	0.425	3.5
Z15	0.54	1	1.75	0.75	3.5
Z23	0.535	1	1.85	0.65	5
Z24	0.545	1	1.25	0.5	3.5
Z25	0.55	1	1.425	0.825	3.5
Z34	0.55	1	1.5	0.75	4.5
Z35	0.535	1	1.675	1.075	4.5
Z45	0.565	1	1.075	0.925	3

The mix ratios are arbitrary mix ratios or starting mix ratios from experience and past literatures.

And X represent

- |                                       |   |
|---------------------------------------|---|
| X <sub>1</sub> = (1, 0, 0, 0, 0)      | X <sub>2</sub> = (0, 1, 0, 0, 0)        |
| X <sub>3</sub> = (0, 0, 1, 0, 0)      | X <sub>4</sub> = (0, 0, 0, 1, 0)        |
| X <sub>5</sub> = (0, 0, 0, 0, 1)      | X <sub>12</sub> = (0, 5, 0, 5, 0, 0, 0) |
| X <sub>13</sub> = (0.5, 0, 0.5, 0, 0) | X <sub>14</sub> = (0.5, 0, 0, 0.5, 0)   |
| X <sub>15</sub> = (0.5, 0, 0, 0, 0.5) | X <sub>23</sub> = (0, 0.5, 0.5, 0, 0)   |
| X <sub>24</sub> = (0, 0.5, 0, 0.5, 0) | X <sub>25</sub> = (0, 0.5, 0, 0, 0.5)   |
| X <sub>34</sub> = (0, 0, 0.5, 0.5, 0) | X <sub>35</sub> = (0, 0, 0.5, 0, 0.5)   |
| X <sub>45</sub> = (0, 0, 0, 0.5, 0.5) |   |

Expanding Eqn(3.6a), gives:

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 \end{bmatrix}$$

Thus,

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad (3.9)$$

For point 1, N<sub>1</sub>, the values of the proportions of the actual components Z<sub>i</sub>, are as follows:

Z<sub>1</sub>= 0.51, Z<sub>2</sub> = 1.0, Z<sub>3</sub> = 2.25, Z<sub>4</sub> = 0.25, Z<sub>5</sub>=4

And, the corresponding values of the pseudo components X<sub>i</sub> are

$$x_1 = 1, x_2 = x_3 = x_4 = x_5 = 0$$

Substituting these values into Eqn (3.10) gives

$$\begin{bmatrix} 0.51 \\ 1.0 \\ 2.25 \\ 0.25 \\ 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{54} & a_{55} \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the resulting system of equations, gives

a<sub>11</sub> = 0.51, a<sub>21</sub> = 1, a<sub>31</sub> = 2.25, a<sub>41</sub> = 0.25, a<sub>51</sub> = 4

For point 2, N<sub>2</sub>, the values of the proportions of the actual components, Z<sub>i</sub>, are:

Z<sub>1</sub> = 0.53, Z<sub>2</sub> = 1.0, Z<sub>3</sub> = 1.60, Z<sub>4</sub> = 0.40, Z<sub>5</sub> = 4

And, the corresponding values of the proportions of the pseudo components are:

$$x_2 = 1, x_1 = x_3 = x_4 = x_5 = 0$$

$$\begin{bmatrix} 0.51 \\ 1.0 \\ 2.25 \\ 0.25 \\ 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{54} & a_{55} \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 40 \end{bmatrix}$$

a<sub>12</sub> = 0.53, a<sub>22</sub> = 1, a<sub>32</sub> = 1.6, a<sub>42</sub> = 0.4, a<sub>52</sub> = 4

Similarly,

For point 3, N<sub>3</sub>, the values of the proportions of the actual components, Z<sub>i</sub> are:

Z<sub>1</sub> = 0.54, Z<sub>2</sub> = 1.0, Z<sub>3</sub> = 2.10, Z<sub>4</sub> = 0.90, Z<sub>5</sub> = 6.0 and the corresponding values of the proportions of the pseudo components are x<sub>i</sub> are: X<sub>3</sub>=1. X<sub>1</sub>=X<sub>2</sub>=X<sub>4</sub>=X<sub>5</sub>=0

Solving the resulting system of equations, gives

a<sub>13</sub> = 0.56, a<sub>23</sub> = 1, a<sub>33</sub> = 2.1, a<sub>43</sub> = 0.9, a<sub>53</sub> = 6

For point 4, N<sub>4</sub>, the values of the proportions of the actual components, Z, are:

Z<sub>1</sub> = 0.56, Z<sub>2</sub> = 1.0, Z<sub>3</sub> = 0.90, Z<sub>4</sub> = 0.60, Z<sub>5</sub> = 3

And the corresponding values of the proportions of the pseudo components are X<sub>i</sub> are: X<sub>4</sub>=1. X<sub>1</sub>=x<sub>2</sub>=x<sub>3</sub>=X<sub>5</sub>=0

Solving the resulting system of equations, gives

a<sub>14</sub> = 0.56, a<sub>24</sub> = 1, a<sub>34</sub> = 0.9, a<sub>44</sub> = 0.6, a<sub>54</sub> = 3

For point 5, N<sub>5</sub>, the values of the proportions of the actual components, Z<sub>i</sub> are:

Z<sub>1</sub> = 0.57, Z<sub>2</sub> = 1.0, Z<sub>3</sub> = 1.25, Z<sub>4</sub> = 1.25, Z<sub>5</sub> = 3

And the corresponding values of the proportions of the pseudo components are:

X<sub>5</sub> = 1. X<sub>1</sub>=X<sub>2</sub>=X<sub>3</sub>=X<sub>4</sub>=0

Solving the resulting system of equations, gives

$$a_{15} = 0.57, a_{25} = 1, a_{35} = 1.25, a_{45} = 1.25, a_{55} = 3$$

Using Table 3.6 and eqn (3.9), the calculate values are shown in eqn (3.10):

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = \begin{bmatrix} 0.510.530.540.560.57 \\ 1 & 1 & 1 & 1 & 1 \\ 2.251.602.100.901.25 \\ 0.250.400.900.601.25 \\ 4.0 & 4.0 & 6.0 & 3 & 3 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad (3.10)$$

This implies that the matrix, A is given as follows:

$$[A] = \begin{bmatrix} 0.510.530.540.560.57 \\ 1 & 1 & 1 & 1 & 1 \\ 2.251.602.100.901.25 \\ 0.250.400.900.601.25 \\ 4.0 & 4.0 & 6.0 & 3 & 3 \end{bmatrix} \quad (3.11)$$

But,  $[A]^{-1} = [B]$

$$[B] = \begin{bmatrix} 83.72093 & -46.6047 & 3.395349 & -3.11628 & 0.48837 \\ -216.279 & 121.3953 & -6.60465 & 6.883721 & 0.511628 \\ 44.18605 & -25.9302 & 1.069767 & -1.25581 & 0.325581 \\ 130.2326 & -71.1628 & 2.837209 & -5.06977 & -0.09302 \\ -41.8605 & 23.30233 & -6.09767 & 2.55814 & -0.25581 \end{bmatrix} \quad (3.12)$$

### Five Component Factor Space

The study is based on a five component concrete mixture comprised of cement, coarse aggregate, sand, clay and water. Table 3.6 below shows the actual mix ratios for the five components of concrete mixture.

**Table 3.6: Actual mix ratio for the five components of concrete mixture.**

w/c	Cement	Sand	laterite	Granite	% of Sand: Clay	
0.51	1	2.25	0.25	4	90	: 10
0.53	1	1.60	0.40	4	80	: 20
0.54	1	2.10	0.90	6	70	: 30
0.56	1	0.90	0.60	3	60	: 40
0.57	1	1.25	1.25	3	50	: 50

### Number Of Coefficients

Henry Scheffe (1958) stipulated a formula for determining the number of coefficients, K, for a given number of components, q, at the presumed degree of polynomial, m. The formula is given as:

$$K = \frac{(q+m-1)!}{[(q-1)!-m!]} \quad (3.13)$$

In this study, the number of coefficients q = 5. Assuming the degree of polynomial, m = 2. Then,

$$K = \frac{(5 + 2 - 1)!}{[(5 - 1)! - 2!]} = \frac{6!}{[4! - 2!]} = \frac{6 * 5 * 4 * 3 * 2 * 1}{4 * 3 * 2 * 2 * 1} = \frac{720}{48} = 15$$

Therefore, the number of coefficients, K = 15

This is the basis of the fifteen mix proportions needed for the experimental work.

### Optimum Point Of Concrete

In imaginary space, different optimum points exists, that are dependent on different considerations. This means that for a particular consideration, an optimum point exists in the imaginary space. It is necessary to use a defined factor space to trap or enhance that optimum. Failure to do this means that the optimum point will fall outside the factor space with the consequence of not being able to achieve the optimum value at the end of the day. The factor space is four dimensional (i.e. q-1 =5-1 =4). The mix ratios listed below will be used as the actual mixes at the vertices of the pentahedron (shown in Fig. 3 .1) that encloses or traps the optimum point.

A<sub>1</sub> (0.51: 1: 2.25: 0.25: 4), A<sub>2</sub> (0.53: 1: 1.6: 0.4: 4),

A<sub>3</sub> (0.54: 1: 2.1: 0.9: 6), A<sub>4</sub> (0.56: 1:0.9: 0.6: 3),

A<sub>5</sub> (0.57: 1: 1.25: 1.25:3).

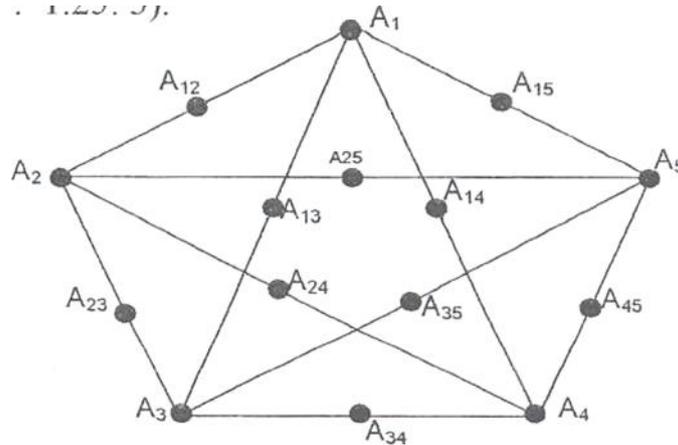


Fig 3.1: A four-dimensional factor space i.e. pentahedron showing actual components at the vertices.

The points of interest in the pentahedron space are  $A_1, A_2, A_3, A_4, A_5, A_{12}, A_{13}, A_{15}, A_{23}, A_{24}, A_{25}, A_{34}, A_{35},$  and  $A_{45}$ . These make up a total of fifteen pts corresponding to fifteen different experimental data required to : rimize the fifteen coefficients of the required polynomial function. The pentahedron space show fifteen points and the proportions of the: eracting components at each point.

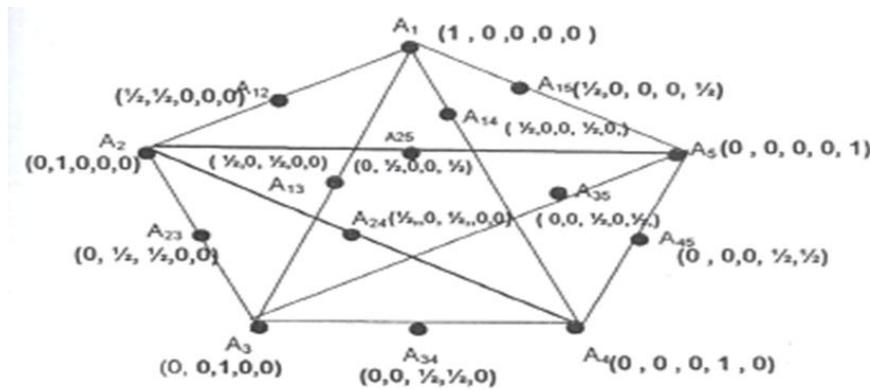


Fig. 3.2: A pentahedron space showing pseudo components at different points

**Coefficients Of Transformation**

The Figure 3.1 is used to obtain the values given in Table 3.7

**Table 3.7: Mix Ratio for 15 practical Tests for Actual and Pseudo Components.**

		Z <sub>1</sub>	Z <sub>2</sub>	Z <sub>3</sub>	Z <sub>4</sub>	Z <sub>5</sub>	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>
N	Response	Water	Cement	Sand	laterite	Granite					
1	Y <sub>1</sub>	0.51	1	2.25	0.25	4	1	0	0	0	0
2	Y <sub>2</sub>	0.53	1	1.6	0.4	4	0	1	0	0	0
3	Y <sub>3</sub>	0.54	1	2.1	0.9	6	0	0	1	0	0
4	Y <sub>4</sub>	0.56	1	0.9	0.6	3	0	0	0	1	0
5	Y <sub>5</sub>	0.57	1	1.25	1.25	3	0	0	0	0	1
6	Y <sub>12</sub>	0.52	1	1.925	0.325	4	0.5	0.5	0	0	0
7	Y <sub>13</sub>	0.525	1	2.175	0.575	5	0.5	0	0.5	0	0
8	Y <sub>14</sub>	0.535	1	1.575	0.425	3.5	0.5	0	0	0.5	0
9	Y <sub>15</sub>	0.54	1	1.75	0.75	3.5	0.5	0	0	0	0.5
10	Y <sub>23</sub>	0.535	1	1.85	0.65	5	0	0.5	0.5	0	0
11	Y <sub>24</sub>	0.545	1	1.25	0.5	3.5	0	0.5	0	0.5	0
12	Y <sub>25</sub>	0.55	1	1.425	0.825	3.5	0	0.5	0	0	0.5
13	Y <sub>34</sub>	0.55	1	1.5	0.75	4.5	0	0	0.5	0.5	0
14	Y <sub>35</sub>	0.535	1	1.675	1.075	4.5	0	0	0.5	0	0.5

15	Y <sub>45</sub>	0.565	1	1.075	0.925	3	0	0	0	0.5	0.5
----	-----------------	-------	---	-------	-------	---	---	---	---	-----	-----

where,

N stands for any point on the factor space

x and z are pseudo and actual components respectively

Y denotes the response.

**Mix Ratios For Practical**

Mix ratio when N=1, 2, 3, 4 and 5 can be seen in Table 3.6

The other mix ratios for points N=12, 13, 14, 15, 23, 24, 25, 34, 35 and 45 will be generated using the corresponding values of X<sub>i</sub> in Table 3.7 and Eqn (3.10)

At point 12, N<sub>12</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = \begin{bmatrix} 0.5 & 10.53 & 0.54 & 0.56 & 0.57 \\ 1 & 1 & 1 & 1 & 1 \\ 2.25 & 1.60 & 2.10 & 0.90 & 1.25 \\ 0.25 & 0.40 & 0.90 & 0.60 & 1.25 \\ 4.0 & 4.0 & 6.0 & 3 & 3 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(3.14)

- Z<sub>1</sub> = 0.52
- Z<sub>2</sub> = 1
- Z<sub>3</sub> = 1.925
- Z<sub>4</sub> = 0.325
- Z<sub>5</sub> = 4

At point 13, N<sub>13</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{bmatrix}$$

(3.15)

- Z<sub>1</sub> = 0.525
- Z<sub>2</sub> = 1
- Z<sub>3</sub> = 2.175
- Z<sub>4</sub> = 0.575
- Z<sub>5</sub> = 5

At point 14, N<sub>14</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

(3.16)

- Z<sub>1</sub> = 0.535
- Z<sub>2</sub> = 1
- Z<sub>3</sub> = 1.575
- Z<sub>4</sub> = 0.425
- Z<sub>5</sub> = 3.5

At point 15, N<sub>15</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}$$

(3.17)

- Z<sub>1</sub> = 0.54
- Z<sub>2</sub> = 1
- Z<sub>3</sub> = 1.75

$$Z_4 = 0.75$$

$$Z_5 = 3.5$$

At point 23, N<sub>23</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} \quad (3.18)$$

$$Z_1 = 0.535$$

$$Z_2 = 1$$

$$Z_3 = 1.85$$

$$Z_4 = 0.65$$

$$Z_5 = 5$$

At point 24, N<sub>24</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0.5 \\ 0 \end{bmatrix} \quad (3.19)$$

$$Z_1 = 0.545$$

$$Z_2 = 1$$

$$Z_3 = 1.25$$

$$Z_4 = 0.5$$

$$Z_5 = 3.5$$

At point 25, N<sub>25</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0.5 \end{bmatrix} \quad (3.20)$$

$$Z_1 = 0.55$$

$$Z_2 = 1$$

$$Z_3 = 1.425$$

$$Z_4 = 0.825$$

$$Z_5 = 3.5$$

At point 34, N<sub>34</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix} \quad (3.21)$$

$$Z_1 = 0.55$$

$$Z_2 = 1$$

$$Z_3 = 1.5$$

$$Z_4 = 0.75$$

$$Z_5 = 4.5$$

At point 35, N<sub>35</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \\ 0.5 \end{bmatrix} \quad (3.22)$$

$$Z_1 = 0.555$$

$$Z_2 = 1$$

$$Z_3 = 1.675$$

$$Z_4 = 0.075$$

$$Z_5 = 4.5$$

At point 45, N<sub>45</sub>

$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{bmatrix} = [A] \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \\ 0.5 \end{bmatrix} \quad (3.23)$$

$$\begin{aligned} Z_1 &= 0.565 \\ Z_2 &= 1 \\ Z_3 &= 1.075 \\ Z_4 &= 0.925 \\ Z_5 &= 3 \end{aligned}$$

**Responses**

The responses can be represented with the equation below

$$Y = b_0 + \sum b_i x_i + \sum b_{ij} x_i x_j + e \quad (3.24)$$

where b = constant coefficients, Xi = Pseudo components and e = random error term.

For five component mixture, the polynomial function is given by:

$$\begin{aligned} Y = & b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_{12} x_1x_2 + b_{13} x_1x_3 + b_{14} x_1 x_4 \\ & + b_{15} x_1x_5 + b_{23} x_2x_3 + b_{24} x_2x_4 + b_{25} x_2x_5 + b_{34} x_3x_4 \\ & + b_{45} x_4x_5 + b_{11} x_1^2 + b_{22} x_2^2 + b_{33} x_3^2 + b_{44} x_4^2 + b_{55} x_5^2 + e \end{aligned} \quad (3.25)$$

From Eqn (3.5), for a five component concrete mixture,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \quad (3.26)$$

Then, multiplying Eqn (3.26) by b<sub>0</sub> gives:

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = 1 \quad (3.27)$$

Multiplying Eqn. (3.26) by x<sub>1</sub> yields:

$$\begin{aligned} x_1^2 + x_1x_2 + x_1x_3 + x_1x_4 + x_1x_5 &= x_1 \\ x_1^2 = x_1 + x_1x_2 + x_1x_3 + x_1x_4 + x_1x_5 \end{aligned} \quad (3.28)$$

Similarly, multiplying Eqn (3.26) by x<sub>2, x<sub>3, x<sub>4</sub></sub> and x<sub>5</sub> will yields respectively Eqns (3.29), (3.30), (3.31) and (3.32)</sub>

$$x_2^2 = x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5 \quad (3.29)$$

$$x_3^2 = x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \quad (3.30)$$

$$x_4^2 = x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5 \quad (3.31)$$

$$x_5^2 = x_5 - x_1x_5 - x_2x_5 - x_3x_4 - x_4x_5 \quad (3.3.2)$$

Substituting Eqns. (3.27) - (3.32) into Eqn. (3.25) yields:

$$\begin{aligned} Y = & b_0 x_1 + b_0 x_2 + b_0 x_3 + b_0 x_4 + b_0 x_5 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 \\ & + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{15} x_1 x_5 + b_{23} x_2 x_3 + b_{24} x_2 x_4 \\ & + b_{25} x_2 x_5 + b_{34} x_3 x_4 + b_{35} x_3 x_5 + b_{45} x_4 x_5 + b_{11} x_1 - b_{11} x_1 x_2 \\ & - b_{11} x_1 x_3 - b_{11} x_1 x_4 - b_{11} x_1 x_5 + b_{22} x_2 - b_{22} x_1 x_2 - b_{22} x_2 x_3 \\ & - b_{22} x_2 x_4 - b_{22} x_2 x_5 - b_{33} x_3 + b_{33} x_1 x_3 - b_{33} x_2 x_3 - b_{33} x_3 x_4 \\ & - b_{33} x_3 x_5 + b_{44} x_4 - b_{44} x_1 x_4 - b_{44} x_2 x_4 - b_{44} x_3 x_4 - b_{44} x_4 x_5 \\ & + b_{55} x_5 - b_{55} x_1 x_5 - b_{55} x_2 x_5 - b_{55} x_3 x_5 - b_{55} x_4 x_5 + e \end{aligned}$$

Collecting like terms together gives:

$$\begin{aligned} Y = & X_1 (b_0 + b_1 + b_{11}) + X_2 (b_0 + b_2 + b_{22}) + X_3 (b_0 + b_3 + b_{33}) \\ & + X_4 (b_0 + b_4 + b_{44}) + X_5 (b_0 + b_5 + b_{55}) + X_1 X_2 (b_{12} - b_{11} - b_{22}) \\ & + X_1 X_3 (b_{13} - b_{11} - b_{33}) + X_1 X_4 (b_{14} - b_{11} - b_{44}) + X_1 X_5 (b_{15} - b_{11} - b_{55}) \\ & + X_2 X_3 (b_{23} - b_{22} - b_{33}) + X_2 X_4 (b_{24} - b_{22} - b_{44}) + X_2 X_5 (b_{25} - b_{22} - b_{55}) \\ & + X_3 X_4 (b_{34} - b_{33} - b_{44}) + X_3 X_5 (b_{35} - b_{33} - b_{55}) + X_4 X_5 (b_{45} - b_{44} - b_{55}) + e \end{aligned} \quad (3.33)$$

$$\begin{cases}
 a_1 = b_0 + b_1 + b_{11} \\
 a_2 = b_0 + b_2 + b_{22} \\
 a_3 = b_0 + b_3 + b_{33} \\
 a_4 = b_0 + b_4 + b_{44} \\
 a_5 = b_0 + b_5 + b_{55} \\
 a_{12} = b_{12} + b_{11} + b_{22} \\
 a_{13} = b_{13} + b_{11} + b_{33} \\
 a_{14} = b_{14} + b_{11} + b_{44} \\
 a_{15} = b_{15} + b_{11} + b_{55} \\
 a_{23} = b_{23} + b_{22} + b_{33} \\
 a_{24} = b_{24} + b_{22} + b_{44} \\
 a_{25} = b_{25} + b_{22} + b_{55} \\
 a_{34} = b_{34} + b_{33} + b_{44} \\
 a_{35} = b_{35} + b_{33} + b_{55} \\
 a_{45} = b_{45} + b_{44} + b_{55}
 \end{cases} \tag{3.34}$$

Substituting Eqn. (3.34) into Eqn. (3.33) yields:

$$Y = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{14}x_1x_4 + a_{15}x_1x_5 + a_{23}x_2x_3 + a_{24}x_2x_4 + a_{25}x_2x_5 + a_{34}x_3x_4 + a_{35}x_3x_5$$

The response,  $Y_i$  shows a total of 15 terms with 15 different coefficients.

### Response Coefficients

Fifteen practical results of response must be gotten from the laboratory in order to get the coefficient of Eqn (3.35). The result needed here are the practical responses when N is at point 1, 2, 3, 4, 5, 12, 13, 14, 15, 23, 24, 25, 34, 35 and 45. Assume that the practical responses at any point on the pentahedron be designated as  $\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4, \tilde{y}_5, \tilde{y}_{12}, \tilde{y}_{13}, \tilde{y}_{14}, \tilde{y}_{15}, \tilde{y}_{23}, \tilde{y}_{24}, \tilde{y}_{25}, \tilde{y}_{34}, \tilde{y}_{35}$  and  $\tilde{y}_{45}$ .

By using Table 3.6 and Eqn. (3.35)  $\tilde{y}_1$  can be expressed;

When  $N=1, x_1 = 1, x_2 = x_3 = x_4 = x_5 = 0$

$\tilde{y}_1 = a_1x_1 + 0 + 0 + 0 + 0 \dots \dots \dots + 0$  Therefore:

$$Y_1 = \alpha_1 \tag{3.36}$$

Similarly,

$$Y_2 = \alpha_2 \tag{3.37}$$

$$Y_3 = \alpha_3 \tag{3.38}$$

$$Y_4 = \alpha_4 \tag{3.39}$$

$$Y_5 = \alpha_2 \tag{3.40}$$

$$Y_{12} = \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_2 + \frac{1}{4} \alpha_{12} \tag{3.41}$$

$$Y_{13} = \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_3 + \frac{1}{4} \alpha_{13} \tag{3.42}$$

$$Y_{14} = \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_4 + \frac{1}{4} \alpha_{14} \tag{3.43}$$

$$Y_{15} = \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_5 + \frac{1}{4} \alpha_{15} \tag{3.44}$$

$$Y_{23} = \frac{1}{2} \alpha_2 + \frac{1}{2} \alpha_3 + \frac{1}{4} \alpha_{23} \tag{3.45}$$

$$Y_{24} = \frac{1}{2} \alpha_2 + \frac{1}{2} \alpha_4 + \frac{1}{4} \alpha_{24} \tag{3.46}$$

$$Y_{25} = \frac{1}{2} \alpha_2 + \frac{1}{2} \alpha_5 + \frac{1}{4} \alpha_{25} \tag{3.47}$$

$$Y_{34} = \frac{1}{2} \alpha_3 + \frac{1}{2} \alpha_4 + \frac{1}{4} \alpha_{34} \tag{3.48}$$

$$Y_{45} = \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_5 + \frac{1}{4} \alpha_{45} \tag{3.50}$$

At this point, it will be required to carry out compressive strength tests to determine the values of  $Y_1$  through  $Y_{45}$  to enable the coefficients to be computed.

Substituting Eqns (3.36) - (3.50) into Eqn (3.35) gives

$$\begin{aligned}
 Y = & y_1x_1 + y_2x_2 + y_3x_3 + y_4x_4 + y_5x_5 + (4y_{12} - 2y_1 - 2y_2)x_1x_2 + (4y_{13} - 2y_1 - 2y_3)x_1x_3 \\
 & (4y_{14} - 2y_1 - 2y_4)x_1x_4 + (4y_{15} - 2y_1 - 2y_5)x_1x_5 + (4y_{23} - 2y_2 - 2y_3)x_2x_3 + (4y_{24} - 2y_2 - 2y_4)x_2x_4 + \\
 & (4y_{25} - 2y_2 - 2y_5)x_2x_5 + (4y_{34} - 2y_3 - 2y_4)x_3x_4 + (4y_{35} - 2y_3 - 2y_5)x_3x_5 + (4y_{45} - 2y_4 - 2y_5) + \\
 & e \tag{3.51}
 \end{aligned}$$

The Eqn (3.51) is the optimization model. The final form of the model will be given subsequently.



It was realize that the coefficients of regression were too sensitive to handle when  $\sum Z = 1$ . To this effect, there is need to correct the shortcoming in order to obtain a meaningful results. Thus  $\sum Z = 10$  is used.

Hence, multiplying eqn (3.55b) by 10 gives:

$$10Z_1 + 10Z_2 + 10Z_3 + 10Z_4 + 10Z_5 = 10 \tag{3.55c}$$

Let

$$10Z_i = z_i$$

Therefore,

$$z_1 + z_2 + z_3 + z_4 + z_5 = 10 \tag{3.55e}$$

In the formulation of the regression equation,  $Z^{(0)}$  is chosen as the origin i.e.  $Z^{(0)} = 0$ . This means that

$$Z_1^{(0)} = 0, Z_2^{(0)} = 0, Z_3^{(0)} = 0, Z_4^{(0)} \text{ and } Z_5^{(0)} = 0.$$

Let

$$b_0 = F(0) \tag{3.56}$$

$$b_1 = \delta F(0) / \delta Z_i \tag{3.57}$$

$$b_{ij} = \delta^2 F(0) / \delta Z_i \delta Z_j \tag{3.58}$$

$$b_{ii} = \delta^2 F(0) / \delta Z_i^2 \tag{3.59}$$

Substituting Eqns (3.56) - (3.59) into Eqn (3.52) yields:

$$F(Z) = b_0 + \sum b_i Z_i + \sum b_{ij} Z_i Z_j + \sum b_{ii} Z_i^2 \tag{3.60a}$$

where,  $1 \leq i \leq 5, 1 \leq j \leq 5, 1 \leq i \leq 5, 1 \leq j \leq 5$  respectively.

Multiplying Eqn. (3.55) by  $b_0$  gives:

$$b_0 = b_0 Z_1 + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 + b_0 Z_5 \tag{3.60a}$$

Multiplying Eqn. (3.55b) by  $Z_i$  yields the following expression:

$$Z_1 = Z_1^2 + Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_1 Z_5 \tag{3.61}$$

$$Z_2 = Z_1 Z_2 + Z_2^2 + Z_2 Z_3 + Z_2 Z_4 + Z_2 Z_5 \tag{3.62}$$

$$Z_3 = Z_1 Z_3 + Z_2 Z_3 + Z_3^2 + Z_3 Z_4 + Z_3 Z_5 \tag{3.63}$$

$$Z_4 = Z_1 Z_4 + Z_2 Z_4 + Z_3 Z_4 + Z_4^2 + Z_4 Z_5 \tag{3.64}$$

$$Z_5 = Z_1 Z_5 + Z_2 Z_5 + Z_3 Z_5 + Z_4 Z_5 + Z_5^2 \tag{3.65}$$

On rearranging Eqn.(3.61) - Eqn.(3.65),  $+Z_1^2$  will becomes:

$$Z_1^2 = Z_1 - Z_1 Z_2 - Z_1 Z_3 - Z_1 Z_4 - Z_1 Z_5 \tag{3.66}$$

$$Z_2^2 = Z_2 - Z_1 Z_2 - Z_2 Z_3 - Z_2 Z_4 - Z_2 Z_5 \tag{3.67}$$

$$Z_3^2 = Z_3 - Z_1 Z_3 - Z_2 Z_3 - Z_3 Z_4 - Z_3 Z_5 \tag{3.68}$$

$$Z_4^2 = Z_4 - Z_1 Z_4 - Z_2 Z_4 - Z_3 Z_4 - Z_4 Z_5 \tag{3.69}$$

$$Z_5^2 = Z_5 - Z_1 Z_5 - Z_2 Z_5 - Z_3 Z_5 - Z_4 Z_5 \tag{3.70}$$

Substituting Eqns. (3.60b) and (3.66) - (3.70) into Eqn (3.60a) gives the expanded form of the expression for  $F(Z)$ .

$$\begin{aligned} &= b_0 Z_i + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 + b_0 Z_5 + b_1 Z_1 + b_2 Z_2 + b_3 Z_3 + b_4 Z_4 + b_5 Z_5 + b_{12} Z_1 Z_2 + b_{13} Z_1 Z_3 + b_{14} Z_1 Z_4 + b_{15} Z_1 Z_5 + \\ &b_{23} Z_2 Z_3 + b_{24} Z_2 Z_4 + b_{25} Z_2 Z_5 + b_{34} Z_3 Z_4 + b_{15} Z_3 Z_5 + b_{45} Z_4 Z_5 + b_{11} (Z_1 - Z_1 Z_2 - Z_1 Z_3 + Z_1 Z_4 - Z_1 Z_5) + b_{22} (Z_2 - Z_1 Z_2 - \\ &Z_2 Z_3 - Z_2 Z_4 - Z_2 Z_5) + b_{33} (Z_3 - Z_1 Z_3 - Z_2 Z_3 - Z_3 Z_4 - Z_3 Z_5) + b_{44} (Z_4 - Z_1 Z_4 - Z_2 Z_4 - Z_3 Z_4 - Z_4 Z_5) + b_{55} (Z_5 - Z_1 \\ &Z_5 - Z_2 Z_5 - Z_3 Z_5 - Z_4 Z_5) \end{aligned} \tag{3.71}$$

Factorizing Eqn.( 3.71) gives;

$$\begin{aligned} Y &= (b_0 + b_1 + b_{11})Z_1 + (b_0 + b_2 + b_{22})Z_2 + (b_0 + b_3 + b_{33})Z_3 + (b_0 + b_4 + b_{44})Z_4 + (b_0 + b_5 + b_{55}) Z_5 + (b_{12} - b_{11} \\ &- b_{22}) Z_1 Z_2 + (b_{13} - b_{11} - b_{33}) Z_1 Z_3 + (b_{14} - b_{11} - b_{44}) Z_1 Z_4 + (b_{15} - b_{11} - b_{55}) Z_1 Z_5 + (b_{23} - b_{22} - b_{33}) + (b_{24} - \\ &b_{22} - b_{44}) Z_2 Z_4 + (b_{25} - b_{22} - b_{55}) Z_2 Z_5 + (b_{34} - b_{33} - b_{44}) Z_3 Z_4 + (b_{35} - b_{33} - b_{55}) Z_3 Z_5 + (b_{45} - b_{44} - b_{55}) Z_3 Z_5 \end{aligned} \tag{3.72}$$

It is common sense that the summation of constants gives another constant.

$$\text{Thus, let } b_0 + b_i + b_{ii} = \alpha_1 \tag{3.73}$$

$$\text{and } b_{ij} + b_i + b_j = \alpha_{1j} \tag{3.74}$$

$$\begin{aligned} \text{Substituting Eqns. (3.73) and (3.74) into Eqn. (3.72) yields; } Y &= \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3 + \alpha_4 Z_4 + \alpha_5 Z_5 \\ &+ \alpha_{12} Z_1 Z_2 + \alpha_{13} Z_1 Z_3 + \alpha_{14} Z_1 Z_4 + \alpha_{15} \\ &Z_1 Z_5 + \alpha_{23} Z_2 Z_3 + \alpha_{24} Z_2 Z_4 + \alpha_{25} Z_2 Z_5 + \alpha_{34} Z_3 Z_4 + \alpha_{35} Z_3 Z_5 + \\ &\alpha_{45} Z_4 Z_5 \end{aligned} \tag{3.75}$$

Putting Eqn (3.75) in a compact form gives

$$Y = \sum \alpha_i Z_i \sum \alpha_{ij} Z_i Z_j \tag{3.76}$$

Where  $1 \leq i \leq j \leq 5$

The  $Y$  is the response function at any point of observation;  $\alpha_i$  and  $\alpha_{ij}$  are the coefficients of the regression equation; while  $Z_i$  and  $Z_{ij}$  are the predictors. For Eqn. (3.75) to be complete, the values of the coefficients must be determined.

**The Coefficients Of The Regression Model**

Different points of observation have different responses with different predictors at constant coefficients. At the n<sup>th</sup> observation point, the response function, Y<sup>(n)</sup>, corresponds with the predictor, Z<sub>i</sub><sup>(n)</sup>. That is to say

$$Y^{(n)} = \sum \alpha_i Z_i^{(n)} \sum \alpha_{ij} Z_i Z_j^{(n)} \tag{3.77}$$

Where, 1 ≤ I ≤ j ≤ 5 and n = 1,2,3, .....15

The eqn. (3.77) can be put in matrix form as:

$$[Y^{(n)}] = [Z^{(n)}][\alpha] \tag{3.78}$$

$$\begin{bmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ \vdots \\ Y^{(15)} \end{bmatrix} = \begin{bmatrix} Z_1^{(1)} & Z_1^{(1)} & Z_3^{(1)} & Z_4^{(1)} & \dots & Z_5^{(1)} \\ Z_1^{(2)} & Z_2^{(2)} & Z_3^{(2)} & Z_4^{(2)} & \dots & Z_5^{(2)} \\ Z_1^{(3)} & Z_2^{(3)} & Z_3^{(3)} & Z_4^{(3)} & \dots & Z_5^{(3)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ Z_1^{(15)} & Z_2^{(15)} & Z_3^{(15)} & Z_4^{(15)} & \dots & Z_5^{(15)} \end{bmatrix} * \begin{bmatrix} \alpha_1^1 \\ \alpha_2^2 \\ \alpha_3^3 \\ \vdots \\ \alpha_{45}^{15} \end{bmatrix} \tag{3.79}$$

The Table 3.9 shows the actual mixture proportions and their corresponding fractional portions for a system of submission of ΣZ = 1- The values of responses, Y<sup>(n)</sup> of Eqn (3.79) are the ones to be determined by carrying out the practical tests whose results are presented in the next chapter.

With values of y<sup>(n)</sup> and Z<sup>(n)</sup> known, it becomes easier to determine the values of the constant coefficient and from Eqn(3.79). Making [α] the subject of the equation, then Eqn. (3.78) give

$$[\alpha] = [Z(n)]^{-1} [y^{(n)}] \tag{3.80}$$

Expanding Eqn (3.80) yields.

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \vdots \\ \alpha_{15} \end{bmatrix} = * \begin{bmatrix} Y^{(1)} \\ Y^{(2)} \\ Y^{(3)} \\ Y^{(4)} \\ Y^{(5)} \\ \vdots \\ Y^{(15)} \end{bmatrix} \tag{3.81}$$

**Table 3.9: Values of actual mix proportions and their corresponding fractional portion of a system when ΣZ = 1**

N	S <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	S <sub>5</sub>	Response	Z <sub>1</sub>	Z <sub>2</sub>	Z <sub>3</sub>	Z <sub>4</sub>	Z <sub>5</sub>
1	0.51	1	2.25	0.25	4	Y <sub>1</sub>	0.0637	0.1248	0.2809	0.0312	0.4994
2	0.53	1	1.6	0.4	4	Y <sub>2</sub>	0.0704	0.1328	0.2125	0.0531	0.5312
3	0.54	1	2.1	0.9	6	Y <sub>3</sub>	0.0512	0.0949	0.1992	0.0854	0.5693
4	0.56	1	0.9	0.6	3	Y <sub>4</sub>	0.0924	0.1650	0.1485	0.0990	0.4950
5	0.57	1	1.25	1.25	3	Y <sub>5</sub>	0.0806	0.1414	0.1768	0.1768	0.4243
12	0.52	1	1.9	0.325	4	Y <sub>12</sub>	0.0671	0.1291	0.2453	0.0420	0.5165
13	0.525	1	2.15	0.575	5	Y <sub>13</sub>	0.0568	0.1081	0.2324	0.0622	0.5405
14	0.535	1	1.55	0.425	3.5	Y <sub>14</sub>	0.0763	0.1427	0.2211	0.0606	0.4993
15	0.54	1	1.725	0.75	3.5	Y <sub>15</sub>	0.0719	0.1331	0.2295	0.0998	0.4657
23	0.535	1	1.85	0.65	5	Y <sub>23</sub>	0.0592	0.1107	0.2048	0.0719	0.5534
24	0.545	1	1.25	0.5	3.5	Y <sub>24</sub>	0.0802	0.1472	0.1840	0.0736	0.5151
25	0.55	1	1.425	0.825	3.5	Y <sub>25</sub>	0.0753	0.1370	0.1952	0.1130	0.4795
34	0.55	1	1.5	0.75	4.5	Y <sub>34</sub>	0.0663	0.1205	0.1807	0.0904	0.5422
35	0.535	1	1.675	1.075	4.5	Y <sub>35</sub>	0.0609	0.1138	0.1907	0.1224	0.5122
45	0.565	1	1.075	0.925	3	Y <sub>45</sub>	0.0861	0.1523	0.1637	0.1409	0.4570

Where,  
S<sub>1</sub>, S<sub>2</sub>, S<sub>3</sub>, S<sub>4</sub> and S<sub>5</sub> are the actual mix proportions,

and  $Z_1, Z_2, Z_3, Z_4$  and  $Z_5$  are the fractional proportion.

**Table 3.10: Values of matrix of actual mix proportions and control mix proportions when  $\sum Z = 1$**

S/N	$Z_1$	$Z_2$	$Z_3$	$Z_4$	$Z_5$	$Z_1Z_2$	$Z_1Z_3$	$Z_1Z_4$	$Z_1Z_5$	$Z_2Z_5$	$Z_2Z_4$	$Z_2Z_5$	$Z_3Z_4$	$Z_3Z_5$	$Z_4Z_5$
1	0.0637	0.1248	0.2809	0.0312	0.4994	0.0079	0.0179	0.0020	0.0318	0.0351	0.0039	0.0623	0.0088	0.1403	0.0156
2	0.0704	0.1328	0.2125	0.0531	0.5312	0.0093	0.0150	0.0037	0.0374	0.0282	0.0071	0.0705	0.0113	0.1129	0.0282
3	0.0512	0.0949	0.1992	0.0854	0.5693	0.0049	0.0102	0.00444	0.0292	0.0189	0.0081	0.0540	0.0170	0.1134	0.0486
4	0.0924	0.1650	0.1485	0.0990	0.4950	0.0152	0.0137	0.0091	0.0457	0.0245	0.0163	0.0817	0.0147	0.0735	0.0490
5	0.0806	0.1414	0.1768	0.1768	0.4243	0.0114	0.0143	0.0143	0.0342	0.0250	0.0250	0.600	0.0313	0.0750	0.0750
6	0.0671	0.1291	0.2453	0.0420	0.5165	0.0087	0.0165	0.0028	0.0347	0.0317	0.0054	0.0667	0.0103	0.1267	0.0217
7	0.0568	0.1081	0.2324	0.0622	0.5405	0.0061	0.0132	0.0035	0.0307	0.0251	0.0067	0.0584	0.0144	0.1256	0.0336
8	0.0763	0.1427	0.2211	0.0606	0.4993	0.0109	0.0169	0.0046	0.0381	0.0315	0.0086	0.0712	0.0134	0.1104	0.0303
9	0.0719	0.1331	0.2295	0.0998	0.4657	0.0096	0.0165	0.0072	0.0335	0.0305	0.0133	0.0620	0.0229	0.1069	0.0465
10	0.0592	0.1107	0.2048	0.0719	0.5534	0.0066	0.0121	0.0043	0.0328	0.0227	0.0080	0.0613	0.0147	0.1133	0.0398
11	0.0802	0.1472	0.1840	0.0736	0.5151	0.0118	0.0148	0.0059	0.0413	0.0271	0.0108	0.0758	0.0135	0.0948	0.0379
12	0.0753	0.1370	0.1952	0.1130	0.4795	0.0103	0.0147	0.0085	0.0361	2.0267	0.0155	0.0657	0.0221	0.0936	0.0542
13	0.0663	0.1205	0.1807	0.0904	0.5422	0.0080	0.0120	0.0060	0.0359	0.0218	0.0109	0.0653	0.0163	0.0980	0.0490
14	0.0609	0.1138	0.1907	0.1224	0.5122	0.0069	0.0116	0.0075	0.0312	0.0217	0.0139	0.0583	0.0233	0.0977	0.0627
15	0.0861	0.1523	0.1637	0.1409	0.4570	0.0131	0.0141	0.0121	0.0393	0.0249	0.0215	0.0696	0.0231	0.0748	0.0644
16	0.0620	0.1167	0.2333	0.1143	0.4737	0.0072	0.0145	0.0071	0.0293	0.0272	0.0133	0.0553	0.0267	0.1105	0.0542
17	0.0789	0.1456	0.1769	0.0757	0.5228	0.0115	0.0140	0.0060	0.0413	0.0258	0.0110	0.0761	0.013	0.0925	0.0396
18	0.0615	0.1111	0.1844	0.1222	0.5209	0.0068	0.0113	0.0075	0.0320	0.0205	0.0136	0.0578	0.0225	0.0960	0.0636
19	0.0738	0.1398	0.1678	0.0832	0.5354	0.0103	0.0124	0.0061	0.0395	0.0235	0.0116	0.0749	0.0140	0.0898	0.0445
20	0.0546	0.0940	0.2067	0.1334	0.5112	0.0051	0.0113	0.0073	0.0279	0.0194	0.0125	0.0480	0.0276	0.1057	0.0682
21	0.0648	0.1154	0.2135	0.0906	0.5157	0.0075	0.0138	0.0059	0.0334	0.0246	0.0105	0.0595	0.0193	0.1101	0.0467
22	0.0574	0.1042	0.2344	0.1562	0.4479	0.0060	0.0134	0.0090	0.0257	0.0244	0.0163	0.0466	0.0366	0.1050	0.0700
23	0.0728	0.1262	0.2712	0.0833	0.4466	0.0092	0.0197	0.0061	0.0325	0.0342	0.0105	0.0563	0.0226	0.1211	0.0372
24	0.0661	0.1140	0.2257	0.0897	0.5056	0.0075	0.0149	0.0059	0.0334	0.0255	0.0101	0.0571	0.0203	0.1141	0.0454
25	0.0637	0.1140	0.1749	0.1504	0.4969	0.0073	0.0111	0.0096	0.0317	0.0199	0.0171	0.0566	0.0263	0.0869	0.0748
26	0.0601	0.1045	0.2049	0.0755	0.5551	0.0063	0.0123	0.0045	0.0334	0.0214	0.0079	0.0580	0.0154	0.1138	0.0418
27	0.0690	0.1293	0.1746	0.0737	0.5534	0.0089	0.0121	0.0067	0.0382	0.0226	0.0095	0.0716	0.0129	0.0966	0.0408
28	0.0648	0.1153	0.2156	0.1038	0.5005	0.0075	0.0140	0.0067	0.0324	0.0249	0.0120	0.0577	0.0224	0.0107	0.0519
29	0.0778	0.1415	0.1677	0.0736	0.5393	0.0110	0.0131	0.0057	0.0420	0.0237	0.0104	0.0763	0.0123	0.0905	0.0397
30	0.0695	0.1270	0.2337	0.1061	0.4637	0.0088	0.0162	0.0074	0.0322	0.0297	0.0135	0.0589	0.0248	0.1084	0.0492

**Table 3.11: Values of inverse matrix of actual proportions when  $\sum Z = 1$**

$Z_1$	$Z_2$	$Z_3$	$Z_4$	$Z_5$	$Z_1Z_2$	$Z_1Z_3$	$Z_1Z_4$	$Z_1Z_5$	$Z_2Z_3$	$Z_2Z_4$	$Z_2Z_5$	$Z_3Z_4$	$Z_3Z_5$	$Z_4Z_5$
-49050.26	-	14310.	-	-	10085	80072.	-	98315.	-	19862	24729	71294.	25324.16	116157.27
	96940.8	44	8594	19578.	1.24	46	58612	91	14752	8.18	6.14	22		
	0		7.84	85			.49		7.41					

-16895.20	-	5727.4	-	-	38873	26389.	-	33351.	-	76425.	-	22119.	8231.03	40029.40
	39239.0	9	3159	6767.9	.80	42	24261	51	48947.	73	83443	25		
	6		3.34	6			.22		12		.64			
68.00	287.43	27.00	-	-1481	-	100.51	57.26	32.17	-	-82.82	-89.43	58.47	11.76	58.66
			19.5		290.8				203.77					
			2		2									
-113.38	-636.91	-25.94	-	3.14	449.2	39.63	-	119.22	57.95	664.28	-	5.85	44.22	140.02
			189.		8		171.7				386.1			
			28				9				9			
-1.25	-19.18	0.71	-7.11	-0.69	12.14	-8.83	-8.72	9.37	14.69	24.31	-18.56	-5.32	0.78	7.76
123600.76	260643.	-	2221	49403.	-	-	15908	-	36644	-	61830	-	-	-292756.67
	08	38195.	54.6	71	26569	19844	4.38	24630	8.87	52287	0.05	17292	62438.59	
		42	4		8.74	1.69		9.51		1.10	3.78			
49681.05	104558.	-	8288	18488.	-	-	59972	-	13600	-	23790	-	-	-110848.58
	40	12993.	0.59	66	10819	73971.	.38	95077.	3.85	19694	3.12	67235.	24222.37	
		44			4.65	65		28		5.07		01		
54628.81	122268.	-	9558	20665.	-	-	66978	-	14984	-	26887	-	27618.32	-125558.70
	17	13805.	2.34	07	11979	83878.	.88	10618	4.40	22840	7.03	73597.		
		01			6.03	15		3.40		7.57		52		
46712.38	84887.5	14297.	8272	19396.	-	-	53769	-	14896	-	24238	-	-	-114022.93
	4	43	4.68	18	90747	80353.	.25	96265.	4.10	18639	6.31	71719.	25034.96	
					.88	43		54		9.15		34		
16486.02	34804.7	-	3350	7421.4	-	-	23681	-	55941.	-	89037	-	-8875.38	-43209.92
	7	6570.8	7.29	4	34547	30092.	.02	35301.	68	77796.	.94	24485.		
		0			.65	48		62		47		83		
13777.19	25042.4	-	2604	6084.0	-	-	19285	-	47565.	-	71305	-	-6981.90	-34584.18
	1	59941.	2.11	3	28176	24262.	.21	28851.	64	59411.	.97	20892.		
		74			.66	78		55		42		36		
18218.66	46096.2	-	3343	6877.1	-	-	26996	-	48068.	-	86269	-	-8396.82	-41267.40
	8	5749.5	0.64	3	44607	26184.	.14	34533.	68	83377.	.28	21839.		
		4			.49	95		75		11		75		
74.00	593.10	-8.10	418.	52.23	-	-	172.5	-	325.33	-	890.0	-	-108.41	-435.11
			51		234.6	290.85	6	626.29		1050.5	1	135.71		
					8					9				
-135.25	-565.21	-26.59	-	11.19	539.6	-76.51	-	-10.63	200.52	338.67	13.68	-61.26	-6.07	-20.33
			46.9		9		154.9							
			1				8							
213.21	1155 75	34.16	339.	3.09	-	-22.44		235.72	-	-	653.3	23.00	-58.11	-250.48
			52		873.2				151.43	1221.4	7			
					8		390.7			3				

**Tests For Adequacy Of The Models**

Two methods were used: Students’ t-test and fishers’ f-test:

**Student T-Test Method**

This is one of the statistical tools used in checking the adequacy of the model.

The equation of the estimated variance is given as:

$$S^2 = S_Y^2 [\sum a_i^2/n_i + \sum a_{ij}^2/n_{ij}] \tag{3.82}$$

Where:  $1 \leq i \leq q$  and  $1 \leq j \leq q$  respectively

In this work, the number of replicates is the same and  $n_i = n_{ij} = n$ . Thus, Eqn (3.82) can be rewritten as:

$$S^2 = S_Y^2 [\sum a_i^2/n + \sum a_{ij}^2/n] \tag{3.83}$$

$$= \frac{S_Y^2}{n} * [\sum a_i^2 + \sum a_{ij}^2]$$

Where:  $1 \leq i \leq q$  and  $1 \leq j \leq q$

$$\text{Taking } \epsilon = [\sum a_i^2 + \sum a_{ij}^2] \tag{3.84}$$

Where:  $1 \leq i \leq q$  and  $1 \leq j \leq q$

$$\text{Then, } S^2 = S_Y^2 * \frac{\epsilon}{n} \tag{3.85}$$

Eqn (3.51) can be summarized as:

$$Y = (\sum a_i)n_i + (\sum a_{ij})n_{ij} + e \tag{3.86}$$

$$\text{Where } Y = x_1\{2x_1 - 1\} \tag{3.87}$$

$$a_{ij} = 4x_i x_j \tag{3.88}$$

Substituting Eqn (3.87) and (3.88) into Eqn (3.84) gives:

$$\epsilon = [x_i(x_i - 1)]^2 + [4x_i x_j]^2 \tag{3.89}$$

1:  $1 \leq i \leq q$  and  $1 \leq j \leq q$

Paradine and Rivelt (1970) gave the equation for calculating t for a t- test statistics as:

$$t = \frac{\Delta y \sqrt{n}}{S_y \sqrt{(1+\epsilon)}} \tag{3.90}$$

Where  $\Delta y = Y_e - Y_m$

$Y_e$  = experimental test response

$Y_m$  = theoretical response from model

n = number of replicates at any arbitrary point

$S_y$  is as given in Eqns (3.95) and (3.83) and  $\epsilon$  is as given in Eqn (3.84) or Eqn(3.89)

This calculated t- values would be compared with a t value from the table.

The t- value from statistical table is given as  $t_{\alpha/N} (V)$

Where  $\alpha$  is the significant level = 0.05

N is the number of points of observation

V is the degree of freedom equal to N - 1

According to Nwaogazie (2006), the t-test method used in checking the adequacy of a model, is performed after the mean and standard deviation results are obtained. It was observed that experiment involving simplex design is usually saturated, so additional points of observation (the control points) were required to test the adequacy of the models.

**F-Statistical Test Method**

The Fisher’s test, otherwise called is a statistical tool used in the analysis of variance. It is employed in determining the equality of variance among variables. In F-statistic, the variance from the model response,  $s_c$  is compared with variances obtained from experimental responses. The equation for fisher test is given as:

$$F = \frac{S_1^2}{S_2^2} \tag{3.91}$$

$$S^2 = \frac{[\sum(y_1 - \bar{y})^2]}{(n-1)} \tag{3.92}$$

Where  $S_1$ = Variance for model response

$S_2$ = Variance for experimental response

The Null hypothesis ( $H_0$ ) should be accepted if  $S_1^2/S_2^2 < F_{table}$ . If this condition is not satisfied, then the alternative hypothesis ( $H_i$ ) should be accepted. Therefore, the accepted range of values for  $S_1^2/S_2^2$  is given as:

$$\frac{1}{F_\alpha} (V_1 V_2) S_1^2 / S_2^2 \leq F_\alpha (V_1 V_1) \tag{3.93}$$

Where  $\alpha$  = Level of significance

V = number of degrees of freedom (i.e. n - 1)

F = critical value obtained from F - Statistic table.

**Error Of The Replicates**

Several errors are usually introduced during the experimental test due to human inconsistency, variation in test tools and equipment. It could also have occurred due to conditional variations like humidity, pressure, temperature etc. These errors are described as random errors. At any arbitrary point of observation, replicate values, are obtained and used to get the mean value of response at that point. The mean value is used because the replicate values vary from each other due to the inconsistencies mentioned above. The variance of the replicates at the arbitrary point of observation, is designated as  $S_i$ . Cranmer (1946) gave the equation of variance as:

$$S_y^2 = \frac{\sum(y_1 - \bar{y})^2}{n-1} \tag{3.94}$$

where  $1 \leq i \leq n$

$$y = \sum \frac{Y_i}{n} \tag{3.95}$$

Expanding Eqn (3.84) gives:

$$S_y^2 = \frac{(\sum Y_i^2 - (\sum Y_i)^2/n)}{n-1} \tag{3.96}$$

Thus, the equation of variance of the replicates at any arbitrary point of observation will be given as:

$$S_y^2 = \frac{(\sum Y_i^2 - (\sum Y_i)^2/n)}{n_i-1} \tag{3.97}$$

where  $1 \leq i \leq n$

Hence, the variance of the replicates at all the points of observation, is the summation of the individual variances divided by the number of points of observations.

$$S_y^2 = \frac{\sum S_i^2}{v} \tag{3.98}$$

where  $1 \leq i \leq n$

V is the degree of freedom, which is equal to N - 1. Therefore, the random error or standard deviation becomes:  

$$S_y = \sqrt{(S_y^2)} \tag{3.99}$$

**Computer Programs For Optimization Of The Compressive And Flexural Strength Of Sand- Laterite Concrete.**

In this work, computer programs written in visual basic language, were developed for Scheffe's simplex and Osadebe's regression models. In the programs, actual concrete mix proportion were specified as inputs and the computer print out the corresponding crushing strengths and flexural strengths for the mix proportions. The computer programs for the Scheffe's and Osadebe's models, were written in Visual Basic 6.0 and presented as appendices A,B,C and D.

Appendix A and C contains programs based on Scheffe's simplex model and appendix B and D contains programs based on Osadebe's regression model.

**IV. Results And Discussion**

**Presentation Of Results**

The results of the various tests carried out in the research work, are presented in fit section. The tests include:

- (a) Physical Properties of Ibeto cement
- (b) Chemical properties of Ibeto cement
- (c) Grain size distribution of the sand
- (d) Initial and final setting time measurement of concrete Slump test result Compressive strength tests - Flexural strength test

The physical properties of Ibeto cement are presented in Table 4.1

**Table 4.1:** Physical Properties of Ibeto cement

S/N	Properties	Values
1	Moisture content	0.003
2	Specific gravity	3.16
3	Finess	190 plus
4	PH	9.2

The result of chemical tests are given in Table 4.2

**Table 4.2:** Chemical properties of Ibeto cement

S/N	Oxides	Mass fraction %
1.	Alumina (Al <sub>2</sub> O <sub>3</sub> )	20.60
2.	Iron oxide (Fe <sub>2</sub> O <sub>3</sub> )	6.408
3.	Lime (CaO)	11.40
4.	Loss on Ignition (L01)	11.40
5.	Magnesium oxide (MgO)	0.095
6.	Potassium oxide (K <sub>2</sub> O)	2.68
7.	Silicate (SiO <sub>2</sub> )	51.50
8.	Sodium oxide (Na <sub>2</sub> O)	2.60
9.	Titanium Oxide (TiO <sub>2</sub> )	0.56

**(c) Grain size distribution of sharp sand**

The sieve analysis of the sharp sand yielded the results shown in Table 4.3

**Table 4.3:** Grain size distribution of sharp river sand

Sieve size	Material weight	% Retained	% Passing
4.76mm	0	0	100
2.40mm	1.8	0.9	99.1
1.20mm	5.0	2.5	96.6
600µm	27.4	13.7	82.9
300 µm	98.0	49.0	33.9
150 µm	60.4	30.2	3.7
Passing 100 µm	7.6	3.7	0
Total	200	100	

**(d) Measurement of the initial and final setting times of concrete**

The initial and final setting times of the various mixes, are given in Table 4.4

**Table 4.4:** Setting time results of cement paste (minutes)

S/N	Points of observation	Initial setting time(minutes)	Final setting time(Unit)
1	N <sub>1</sub>	155.6	215
2	N <sub>2</sub>	163	231
3	N <sub>3</sub>	170.5	238
4	N <sub>4</sub>	180.7	251.7
5	N <sub>5</sub>	156	223.6
6	N <sub>6</sub>	163	227.9
7	N <sub>7</sub>	168	234
8	N <sub>8</sub>	172.4	235.9
9	N <sub>9</sub>	173	239
10	N <sub>10</sub>	176.3	244
11	N <sub>11</sub>	167.5	233
12	N <sub>12</sub>	164.4	228
13	N <sub>13</sub>	170	237
14	N <sub>14</sub>	166.6	231
15	N <sub>15</sub>	164.7	229.2
16	C <sub>1</sub>	174.4	242.6
17	C <sub>2</sub>	173.5	241
18	C <sub>3</sub>	160.2	222.6
19	C <sub>4</sub>	161.7	224.4
20	C <sub>5</sub>	170.6	237.5
21	C <sub>6</sub>	160.0	222
22	C <sub>7</sub>	161.6	238.4
23	C <sub>8</sub>	176	242.5
24	C <sub>9</sub>	185.6	261
25	C <sub>10</sub>	164.8	231.8
26	C <sub>11</sub>	168	234
27	C <sub>12</sub>	172.8	237
28	C <sub>13</sub>	173.0	339
29	C <sub>14</sub>	177.6	248.4
30	C <sub>15</sub>	180.8	251.5

**Results of the slump tests of various concrete mix proportions**

The results of the slump tests are given in Table 4.5

**Table 4.5:** The results of the slump tests of concrete mixes.

S/N	Points of observation	Slump (mm)
1	N <sub>1</sub>	4.5
2	N <sub>2</sub>	4.0
3	N <sub>3</sub>	2.5
4	N <sub>4</sub>	10.2
5	N <sub>5</sub>	7.5
6	N <sub>6</sub>	6.0
7	N <sub>7</sub>	7.0
8	N <sub>8</sub>	9.5
9	N <sub>9</sub>	6.5
10	N <sub>10</sub>	9
11	N <sub>11</sub>	10.0
12	N <sub>12</sub>	11.0
13	N <sub>13</sub>	7.5
14	N <sub>14</sub>	8.5
15	N <sub>15</sub>	10.0
16	C <sub>1</sub>	7.0
17	C <sub>2</sub>	8.0
18	C <sub>3</sub>	6.8
19	C <sub>4</sub>	11.5
20	C <sub>5</sub>	6.0
21	C <sub>6</sub>	7.0

22	C <sub>7</sub>	8.5
23	C <sub>8</sub>	6.5
24	C <sub>9</sub>	5.3
25	C <sub>10</sub>	10.5
26	C <sub>11</sub>	3.5
27	C <sub>12</sub>	6.6
28	C <sub>13</sub>	10.5
29	C <sub>14</sub>	8.4
30	C <sub>15</sub>	6.5

**(f) Compressive strength test results**

The concrete cubes made with partial replacement of sand with laterite, were cured and crashed on the 28th day and the results presented in Table 4.6

**Table 4.6: Concrete compressive strength results**

S/N	Points of observation	Replicate 1	Replicate 2	Replicate 3	Mean cube strength (N/mm <sup>2</sup> )
1	N <sub>1</sub>	19.11	19.02	19.33	19.15
2	N <sub>2</sub>	18.22	18.67	18.44	18.44
3	N <sub>3</sub>	16.89	17.11	17.56	17.19
4	N <sub>4</sub>	14.22	14.67	14.44	14.44
5	N <sub>5</sub>	14.89	14.00	14.22	14.37
6	N <sub>6</sub>	13.56	13.87	13.91	13.78
7	N <sub>7</sub>	13.33	13.11	14.27	13.57
8	N <sub>8</sub>	13.96	14.44	14.00	14.13
9	N <sub>9</sub>	14.58	15.16	14.67	14.80
10	N <sub>10</sub>	13.56	14.58	14.00	14.04
11	N <sub>11</sub>	15.33	15.56	15.33	15.41
12	N <sub>12</sub>	15.11	15.02	15.56	15.23
13	N <sub>13</sub>	16.89	16.71	16.67	16.76
14	N <sub>14</sub>	13.78	14.00	13.56	13.78
15	N <sub>15</sub>	15.56	15.33	15.47	15.45
16	C <sub>1</sub>	12.07	12.10	12.08	12.083
17	C <sub>2</sub>	13.65	13.61	13.84	13.701
18	C <sub>3</sub>	12.48	12.39	12.36	12.41
19	C <sub>4</sub>	13.38	13.45	13.52	13.45
20	C <sub>5</sub>	14.08	14.13	14.15	14.12
21	C <sub>6</sub>	14.22	13.91	14.44	14.19
22	C <sub>7</sub>	13.91	14.01	13.99	13.97
23	C <sub>8</sub>	14.48	14.46	14.50	14.48
24	C <sub>9</sub>	14.40	14.33	14.41	14.38
25	C <sub>10</sub>	14.82	14.59	14.42	14.61
26	C <sub>11</sub>	13.83	13.76	13.84	13.81
27	C <sub>12</sub>	13.48	13.68	13.64	13.60
28	C <sub>13</sub>	13.95	13.96	14.03	13.98
29	C <sub>14</sub>	14.80	14.91	14.84	14.85
30	C <sub>15</sub>	12.97	12.74	12.93	12.88

**(g) Flexural strength test result**

The concrete beams made with sand partially replaced with laterite were cured and crushed on the 28th day, and result presented in Table 4.7

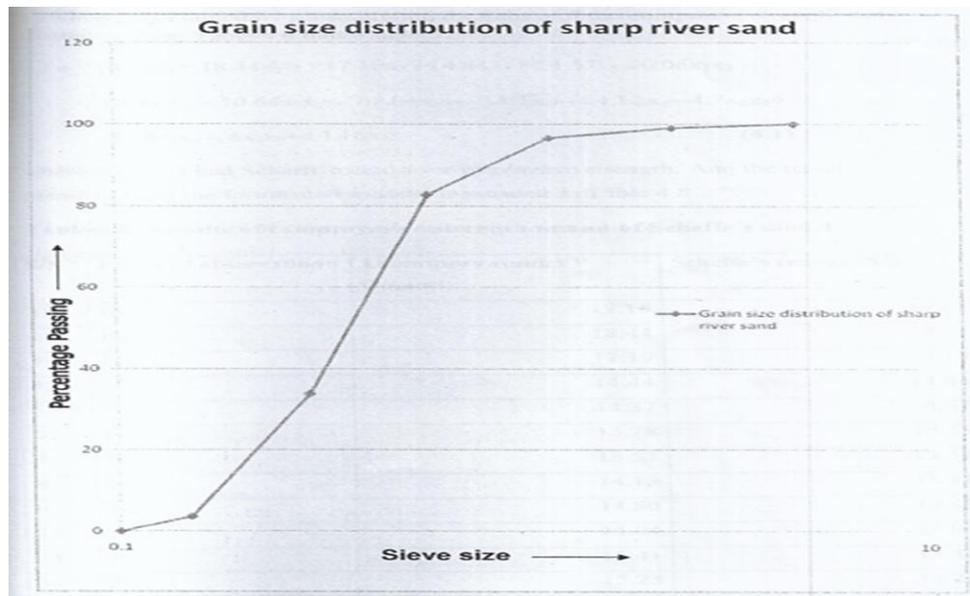
**Table 4.7: Flexural strength results of concrete beams**

S/N	Points of observation	Replicatel	Replicate2	Replicate3	Mean beam strength (N/mm <sup>2</sup> )
1	N <sub>1</sub>	2.22	2.40	2.13	2.25
2	N <sub>2</sub>	2.22	2.04	2.22	2.16
3	N <sub>3</sub>	2.67	2.31	1.96	2.31
4	N <sub>4</sub>	1.87	1.78	2.13	1.93

5	N <sub>5</sub>	1.51	1.60	1.33	1.48
6	N <sub>6</sub>	1.96	2.22	2.04	2.07
7	N <sub>7</sub>	1.33	1.60	1.24	1.39
8	N <sub>8</sub>	1.60	1.78	1.87	1.75
9	N <sub>9</sub>	2.22	1.96	2.13	2.10
10	N <sub>10</sub>	2.49	2.22	2.40	2.37
11	N <sub>11</sub>	1.96	2.22	2.04	2.07
12	N <sub>12</sub>	1.96	1.78	1.87	1.87
13	N <sub>13</sub>	2.22	2.13	2.04	2.13
14	N <sub>14</sub>	2.22	1.78	1.96	1.99
15	N <sub>15</sub>	1.78	1.87	2.04	1.90
16	C <sub>1</sub>	1.96	2.04	1.78	1.93
17	C <sub>2</sub>	1.69	1.78	1.78	1.75
18	C <sub>3</sub>	1.69	1.60	1.87	1.72
19	C <sub>4</sub>	1.78	1.87	1.96	1.87
20	C <sub>5</sub>	1.96	1.87	1.87	1.90
21	C <sub>6</sub>	2.04	2.13	1.96	2.04
22	C <sub>7</sub>	1.87	1.96	1.78	1.87
23	C <sub>8</sub>	2.04	2.13	2.04	2.07
24	N <sub>6</sub>	1.69	2.04	1.87	1.87
25	N <sub>7</sub>	1.96	2.22	2.04	2.07
26	N <sub>8</sub>	1.96	1.69	1.78	1.81
27	N <sub>9</sub>	1.87	2.04	1.96	1.96
28	N <sub>10</sub>	1.96	2.13	1.87	1.99
29	N <sub>11</sub>	2.04	1.96	2.04	2.01
30	N <sub>12</sub>	1.78	2.13	2.04	1.99

**Discussion Of Results**

Fig. 4.1 Below shows the graph of grain size distribution of sharp river sand



**Fig. 4.1:** Graph representing the Grain size distribution of sharp river sand.

**Determination of Scheffe’s simplex models for compressive strength.**

The final scheffe’s model for concrete made with partial replacement of sand with laterite, is obtained by substituting the values of the compressive strength test results from Table 4.6 into Eqn (3.51)

$$Y_{sc} = 19.15x_1 + 18.44x_2 + 17.19x_3 + 14.44x_4 + 14.37x_5 - 20.06x_1x_2 - 18.4x_1x_3 - 10.66x_1x_4 - 7.84x_1x_5 - 15.1x_2x_3 - 4.12x_2x_4 - 4.7x_2x_5 + 3.78x_3x_4 - 8x_3x_5 + 4.18x_4x_5 \tag{4.1}$$

where  $Y_{sc}$  = Final Scheffe model for compressive strength. And the result obtained from the formulated model is presented in Table 4.8

**Table 4.8:** Results of compressive strength test and of Scheffe’s model

S/N	Points of observation	Laboratory results $\bar{Y}$ (N/mm <sup>2</sup> )	Scheffe’s results (N/mm <sup>2</sup> )
1	N <sub>1</sub>	19.15	19.15

2	N <sub>2</sub>	18.44	18.44
3	N <sub>3</sub>	17.19	17.19
4	N <sub>4</sub>	14.44	14.44
5	N <sub>5</sub>	14.37	14.37
6	N <sub>6</sub>	13.78	13.78
7	N <sub>7</sub>	13.57	13.57
8	N <sub>8</sub>	14.13	14.13
9	N <sub>9</sub>	14.80	14.80
10	N <sub>10</sub>	14.04	14.04
11	N <sub>11</sub>	15.41	15.41
12	N <sub>12</sub>	15.23	15.23
13	N <sub>13</sub>	16.76	16.76
14	N <sub>14</sub>	13.78	13.78
15	N <sub>15</sub>	15.45	15.45
16	C <sub>1</sub>	12.083	13.50
17	C <sub>2</sub>	13.701	12.67
18	C <sub>3</sub>	12.41	13.44
19	C <sub>4</sub>	13.45	14.87
20	C <sub>5</sub>	14.12	13.00
21	C <sub>6</sub>	14.19	14.41
22	C <sub>7</sub>	13.97	13.75
23	C <sub>8</sub>	14.48	13.52
24	N <sub>6</sub>	14.38	13.08
25	N <sub>7</sub>	14.61	14.79
26	N <sub>8</sub>	13.81	12.88
27	N <sub>9</sub>	13.6	13.29
28	N <sub>10</sub>	13.98	14.25
29	N <sub>11</sub>	14.85	15.42
30	N <sub>12</sub>	12.88	13.08

**Test of goodness of fit of Scheffe’s model for compressive strength.**

The results obtained were tested for goodness of fit to ensure the adequacy of the model developed.

The test of goodness of fit of Scheffe’s model was carried out using statistical students’ T-test and fisher test (F-test). In both tests, it was assumed that the results predicted by the model will be 95% accurate.

**Table 4.9:** Computations of standard error of replicates of compressive strength of concrete.

S/No	Replicates (y <sub>i</sub> )			Y <sub>i</sub>	y <sub>i</sub> <sup>2</sup>	Σy <sub>i</sub>	Σ y <sub>i</sub> <sup>2</sup>	(Σ y <sub>i</sub> ) <sup>2</sup>	S <sub>i</sub> <sup>2</sup>
	1	2	3						
1	19.02	19.02	19.33	19.15	366.7	57.46	1100.6	3301.7	0.02
2	18.22	18.67	18.44	18.44	340.03	55.33	1020.57	3061.4	0.05
3	16.89	17.11	17.56	17.19	295.5	51.56	886.38	2658.4	0.12
4	14.22	14.67	14.44	14.44	208.51	43.33	625.93	1877.5	3.22
5	14.89	14.00	14.22	14.37	206.5	43.11	619.92	1858.5	0.22
6	13.56	13.87	13.91	13.78	189.89	41.34	569.74	1709	0.04
7	13.33	13.11	14.27	13.57	184.15	40.71	553.19	1657.3	0.38
8	13.96	14.44	14.00	14.13	199.66	42.0	599.4	1797.8	0.08
9	14.58	15.16	14.67	14.8	219.04	44.41	657.61	1972.2	0.1
10	13.56	14.58	14.00	14.04	197.12	42.14	592.45	1775.8	0.28
11	15.33	15.56	15.33	15.41	237.47	46.22	712.13	2136.3	0.02
12	15.11	15.02	15.56	15.23	231.95	45.69	696.03	2087.6	0.09
13	16.89	16.71	16.67	16.76	280.9	50.27	842.39	2527.1	0.02
14	13.78	14.00	13.56	13.78	189.89	41.34	569.76	1709	0.05
15	15.56	15.33	15.47	15.45	238.7	46.36	716.44	2149.2	0.01
16	11.13	13.09	12.03	12.08	145.9	56.49	439.95	1314.1	0.96
17	12.90	13.40	14.80	13.70	187.7	60.05	565.06	1689.4	0.97
18	12.22	12.44	12.57	12.41	154.0	61.24	462.09	1386.1	0.04
19	13.45	13.45	13.45	13.45	180.9	52.35	542.71	1628.1	0.00
20	13.78	14.31	14.27	14.12	199.4	54.36	598.3	1794.4	0.09
21	14.56	14.12	13.89	14.19	201.4	57.56	604.3	1812.2	0.12
22	13.99	13.95	13.97	13.97	195.2	54.36	585.48	1756.5	0.00
23	15.22	13.22	15.00	14.48	209.7	58.44	631.42	1887.0	0.21
24	14.56	14.02	14.56	14.38	206.8	58.14	620.55	1861.0	0.10
25	14.77	14.55	14.51	14.61	213.5	52.85	640.4	1921.1	0.02
26	14.26	13.73	13.44	13.81	190.7	53.42	572.49	1716.5	0.17
27	13.78	13.29	13.73	13.60	189.9	52.8	555.03	1664.6	0.08
28	13.36	14.32	14.26	13.98	195.4	53.33	586.73	1758.5	0.29
29	15.40	14.82	14.33	14.85	220.5	53.55	662.14	1984.7	0.40
30	13.22	12.73	12.69	12.88	165.9	53.64	497.86	1493.1	0.09

$$SY^2 = \frac{12.05}{29} = 0.42$$

$$sy = \sqrt{0.42} = 0.65$$

**Table 4.10:** T-test computations for Scheffe’s compressive strength model.

S/N	Y <sub>c</sub>	Y <sub>m</sub>	D <sub>i</sub> =Y <sub>c</sub> -Y <sub>m</sub>	D <sub>A</sub> - D <sub>i</sub>	(D <sub>A</sub> - D <sub>i</sub> ) <sup>2</sup>
C <sub>1</sub>	12.083	13.50	-1.417	1.454	2.115
C <sub>2</sub>	13.701	12.67	1.031	-0.994	0.987
C <sub>3</sub>	12.41	13.44	-1.030	1.067	1.139
C <sub>4</sub>	13.45	14.87	-0.420	1.457	2.124
C <sub>5</sub>	14.12	13.00	0.120	-1.083	1.172
C <sub>6</sub>	14.19	14.41	-0.220	0.257	0.066
C <sub>7</sub>	13.97	13.75	0.220	-0.183	0.033
C <sub>8</sub>	14.48	13.52	-0.960	-0.923	0.851
C <sub>9</sub>	14.38	13.08	1.300	-1.263	1.594
C <sub>10</sub>	14.61	14.79	-0.180	0.217	0.047
C <sub>11</sub>	13.81	12.88	0.930	-0.893	0.797
C <sub>12</sub>	13.6	13.29	0.310	-0.273	0.074
C <sub>13</sub>	13.98	14.25	-0.272	0.309	0.096
C <sub>14</sub>	14.85	15.42	-0.570	0.6.7	0.369
C <sub>15</sub>	12.88	13.08	-0.200	0.237	0.056
<b>Σ</b>			<b>0.562</b>		11.522

DA = Di/N = -0.037

Σ(D<sub>A</sub>-D<sub>i</sub>)<sup>2</sup> - 11.522

S<sup>2</sup> = Σ(D<sub>A</sub> - D<sub>i</sub>)<sup>2</sup>/(N-1) =0.823

S = √S<sup>2</sup> = 0.907

T = D<sub>A</sub>\*(N)<sup>0.5</sup>/S =0.16

T - value from standard statistical table is given as t<sub>α(v)</sub>=t<sub>0.05(14)</sub>= 1.76.

Since the T value from standard statistical table is greater than T calculated, the differences between laboratory results and model results, are not significant.

**Table 4.11:** F - test computations of Scheffe’s compressive strength of concrete

Control point	Y <sub>c</sub>	Y <sub>m</sub>	Y <sub>c</sub> -Y <sub>c</sub>	Y <sub>c</sub> -Y <sub>M</sub>	(Y <sub>c</sub> -Y <sub>c</sub> ) <sup>2</sup>	(Y <sub>c</sub> -Y <sub>M</sub> ) <sup>2</sup>
C <sub>1</sub>	12.083	13.50	-1.68447	-0.23	2.837428	0.0529
C <sub>2</sub>	13.701	12.67	-0.06647	-1.06	0.004418	1.1236
C <sub>3</sub>	12.41	13.44	-0.35747	-0.29	1.842716	0.0841
C <sub>4</sub>	13.45	14.87	-0.31747	1.14	0.100785	1.2996
C <sub>5</sub>	14.12	13.00	0.352533	-0.73	0.124280	0.53
C <sub>6</sub>	14.19	14.41	0.422433	0.68	0.178534	0.4624
C <sub>7</sub>	13.97	13.75	0.202533	0.02	0.041020	0.0004
C <sub>8</sub>	14.48	13.52	0.712533	-0.21	0.507704	0.04441
C <sub>9</sub>	14.38	13.08	0.612533	-0.65	0.315197	0.4225
C <sub>10</sub>	14.61	14.79	0.842533	1.06	0.709862	1.1236
C <sub>11</sub>	13.81	12.88	0.042533	-0.89	0.001809	0.7225
C <sub>12</sub>	13.6	13.29	-0.16747	0.44	0.028045	0.2704
C <sub>13</sub>	13.98	14.25	0.210533	0.52	0.044324	0.2704
C <sub>14</sub>	14.85	15.42	1.082533	1.69	1.71878	0.4225
C <sub>15</sub>	12.88	13.08	-0.88747	-0.65	0.787597	0.4225
<b>TOTAL</b>	<b>206.512</b>	<b>205.946</b>			<b>8.755598</b>	<b>9.6112</b>
<b>MEAN</b>	<b>13.76747</b>	<b>13.72973</b>				

S<sub>e</sub><sup>2</sup> = Σ(Y<sub>c</sub> - Y<sub>c</sub>)<sup>2</sup> / (N - 1) = 8.755598 / (15- 1) = 0.63

S<sub>m</sub><sup>2</sup> = (Y<sub>m</sub> - Y<sub>m</sub>)<sup>2</sup> / (N - 1) = 9.6112/(15 - 1) = 0.69

Therefore,  $S_1^2 = 0.69$  and  $S_2^2 = 0.63$ .

$F_{\text{calculated}} = S_1^2 / S_2^2 = 0.69 / 0.63 = 1.10$

From Statistic tables,  $F_{0.05} (14,14) = 2.48$

Therefore,

$1/F = 0.40$

Thus, the condition  $1/F < S_1^2 / S_2^2 < F = 0.40 < 1.10 < 2.48$  has been satisfied.

And so, the differences between laboratory results and model results are not significant

**Determination of Osadebe’s Regression Model For Compressive Strength**

The Osadebe’s regression coefficients are determined from Eqn. 3.75 by substituting the values of the responses, (i.e compressive strength) given in Table 4.4 yielded the following coefficients

$\alpha_1 = 28301622.66$ ;  $\alpha_2 = 9003866.086$ ;  $\alpha_3 = 29271.62268$   
 $\alpha_4 = 29245.7358$   $\alpha_5 = 386.316040469$   $\alpha_6 = 232814.82$   
 $\alpha_7 = -26533480.34$   $\alpha_8 = 30157825.49$   $\alpha_9 = -28403570.7$   
 $\alpha_{10} = -10045046.92$   $\alpha_{11} = -8001414.089$   $\alpha_{12} = -8947546.731$   
 $\alpha_{13} = -117217.9105$   $\alpha_{14} = -33866.22456$   $\alpha_{15} = -26248.39497$

Substituting these coefficients into Eqn. (3.75) gives the final Osadebe’s regression model for compressive strength, (i.e. Eqn (4.2).

$$Y = 28301622.66Z_1 + 9003866.08Z_2 + 29271.62268Z_3 + 29245.7358Z_4 + 386.3160404Z_5 - 69232814.82Z_1Z_2 - 26533480.34Z_1Z_3 - 30157825.49Z_1Z_4 - 28403570.7Z_1Z_5 - 10045046.92Z_2Z_3 - 8001414.089Z_2Z_4 - 8947546.731Z_2Z_5 - 117217.9105Z_3Z_4 - 33866.22456Z_3Z_5 - 26248.39497Z_4Z_5 \tag{4.2}$$

where  $Y_{oc}$  = Final Osadabe’s regression model for compressive strength.

The results obtained from the formulated model is presented in Table (4.12).

**Table 4.12:** Results of compressive strength test and those of Osadebe regression model.

S/N	Points of Observation	Laboratory results Y(N/mm <sup>2</sup> )	Osadebe’s results (N/mm <sup>2</sup> )
1	N <sub>1</sub>	19.15	19.15
2	N <sub>2</sub>	18.44	18.44
3	N <sub>3</sub>	17.19	17.19
4	N <sub>4</sub>	14.44	14.44
5	N <sub>5</sub>	14.37	14.37
6	N <sub>6</sub>	13.78	13.78
7	N <sub>7</sub>	13.57	13.57
8	N <sub>8</sub>	14.13	14.13
9	N <sub>9</sub>	14.80	14.80
10	N <sub>10</sub>	14.04	14.04
11	N <sub>11</sub>	15.41	15.41
12	N <sub>12</sub>	15.23	15.23
13	N <sub>13</sub>	16.76	16.76
14	N <sub>14</sub>	13.78	13.78
15	N <sub>15</sub>	15.45	15.45
16	C <sub>1</sub>	12.083	13.09
17	C <sub>2</sub>	13.701	12.71
18	C <sub>3</sub>	12.41	13.07
19	C <sub>4</sub>	13.45	14.86
20	C <sub>5</sub>	14.12	12.69
21	C <sub>6</sub>	14.19	14.14
22	C <sub>7</sub>	13.97	13.36
23	C <sub>8</sub>	14.48	13.41
24	C <sub>9</sub>	14.38	12.91
25	C <sub>10</sub>	14.61	14.62
26	C <sub>11</sub>	13.81	12.64
27	C <sub>12</sub>	13.60	12.92
28	C <sub>13</sub>	13.98	13.97
29	C <sub>14</sub>	14.85	15.45
30	C <sub>15</sub>	12.88	12.85

**Test of adequacy of Osadebe’s model for compressive strength**

The test of adequacy of Osadebe’s model was carried out to ensure its goodness using statistical

student's-test and fisher's test. In both tests, it was assumed that the model will be 95% accurate. The model equation was tested to see if the predicted results from the model agree with experimental results.

**Table 4.13: T-test computations for Osadebe's compressive strength model**

S/N	Y <sub>E</sub>	Y <sub>m</sub>	D <sub>i</sub> =Y <sub>E</sub> -Y <sub>m</sub>	D <sub>A</sub> -D <sub>i</sub>	(D <sub>A</sub> -D <sub>i</sub> ) <sup>2</sup>
C <sub>1</sub>	12.083	13.09	-1.007	1.262	1.593
C <sub>2</sub>	13.701	12.71	0.991	-0.736	0.542
C <sub>3</sub>	12.41	13.07	-0.66	0.915	0.837
C <sub>4</sub>	13.45	14.86	-1.41	1.665	2.772
C <sub>5</sub>	14.12	12.69	1.43	- 1.175	1.381
C <sub>6</sub>	14.19	14.14	0.05	0.205	0.042
C <sub>7</sub>	13.97	13.36	0.61	-0.355	0.126
C <sub>8</sub>	14.48	13.41	1.07	-0.815	0.664
C <sub>9</sub>	14.38	12.91	1.47	-1.215	1.476

**Table 4.13 continued**

S/N	Y <sub>E</sub>	Y <sub>m</sub>	D <sub>i</sub> =Y <sub>E</sub> -Y <sub>m</sub>	D <sub>A</sub> -D <sub>i</sub>	(D <sub>A</sub> -D <sub>i</sub> ) <sup>2</sup>
C <sub>10</sub>	14.61	14.62	-0.01	0.265	0.070
C <sub>11</sub>	13.81	12.64	1.17	-0.915	0.837
C <sub>12</sub>	13.60	12.92	0.68	-0.425	0.181
C <sub>13</sub>	13.98	13.97	0.008	0.247	0.061
C <sub>14</sub>	14.85	15.45	-0.60	0.855	0.731
C <sub>15</sub>	12.88	12.85	0.03	0.225	0.051
$\Sigma$			<b>3.82</b>		<b>11.364</b>

$DA = \Sigma Di/N = 0.255$

$(D_A - D_i)^2 = 11.364$

$S^2 = (D_A - D_i)^2 / (N - 1) = 0.812$

$S = \sqrt{S^2} = 0.90$

$T = Da * (N)^{0.5} / S = 1.097$

T value from standard statistical table, is given as  $t_{\alpha(v)} = t_{0.05(14)} = 1.76$ .

Since the T value from standard statistical table is greater than T calculated, the differences between laboratory results and model results are not significant. And so the model can be used to predict accurately the compressive strength of sand-laterite concrete.

**Table 4.14: F-test computations of Osadebe's compressive strength of concrete**

Control Point	Y <sub>e</sub>	Y <sub>m</sub>	Y <sub>e</sub> -Y <sub>e</sub>	Y <sub>m</sub> -Y <sub>m</sub>	(Y <sub>e</sub> -Y <sub>e</sub> ) <sup>2</sup>	(Y <sub>m</sub> -Y <sub>m</sub> ) <sup>2</sup>
C <sub>1</sub>	12.083	13.09	-0.53793	-0.4236	0.289372	0.179437
C <sub>2</sub>	13.701	12.71	-0.91993	-0.8036	0.846277	0.645773
C <sub>3</sub>	12.41	13.07	-0.21093	-0.4396	0.044493	0.193248
C <sub>4</sub>	13.45	14.86	0.829067	1.3424	0.687352	1.802038
C <sub>5</sub>	13.12	12.69	-0.50093	-0.8216	0.250934	0.675027
C <sub>6</sub>	14.19	14.14	0.569067	0.6304	0.323837	0.397404
C <sub>7</sub>	14.97	13.36	0.349067	-0.1536	0.121848	0.023593
C <sub>8</sub>	14.48	13.41	-0.14093	-0.1056	0.019862	0.011151
C <sub>9</sub>	13.38	12.91	-0.24093	-0.5996	0.058049	0.35952
C <sub>10</sub>	13.61	14.62	0.989067	1.1084	0.978253	1.228551
C <sub>11</sub>	12.81	12.64	-0.81093	-0.8686	0.657613	0.754466
C <sub>12</sub>	13.60	12.92	-0.02093	-0.5916	0.000438	0.349991
C <sub>13</sub>	13.98	13.97	0.159067	0.4574	0.025302	0.209215
C <sub>14</sub>	14.85	15.45	1.229067	1.9344	1.510605	3.741903
C <sub>15</sub>	12.88	12.85	-0.74093	-0.6656	0.548982	0.443023
<b>TOTAL</b>	<b>206.512</b>	<b>202.689</b>			<b>6.363217</b>	<b>11.01434</b>
<b>MEAN</b>	<b>13.76947</b>	<b>13.5126</b>				

where, Y<sub>e</sub> = Experimental strength

$Y_m$  = Model strengths

$\bar{Y}_e$  = Mean of experimental strength

$\bar{Y}_m$  = Mean of model strengths

$$S^2 = \sum(Y_e - \bar{Y}_e)^2 / (N - 1) = 6.363217 / (15 - 1) = 0.45$$

$$SM^2 = \sum(Y_m - \bar{Y}_m)^2 / (N - 1) = 11.01434 / (15 - 1) = 0.79$$

Therefore,  $S_1^2 = 0.79$  and  $S_2^2 = 0.45$ .

$$F_{\text{calculated}} = S_1^2 / S_2^2 = 0.79 / 0.45 = 1.73$$

From Statistic tables,  $F_{0.05}(14,14) = 2.48$

$$1/F = 0.40$$

Therefore, the condition,  $1/F < S_1^2 / S_2^2 < F = 0.4 < 1.73 < 2.48$ . Thus, the condition,  $1/F < S_1^2 / S_2^2 < F$ , has been satisfied. And so, the differences between laboratory results and model results are not significant. Hence the Osadebe model can be used to predict accurately, the compressive strength of sand-lateritic concrete.

**Comparison of experimental, Scheffe's and Osadebe's compressive strengths of concrete.**

The compressive strengths obtained from the experiments were compared with compressive strengths predicted by Scheffe and Osadebe's models as given in Table 4.15

**Table 4.15:** Comparisons of Experimental, Scheffe's and Osadebe's compressive strengths of sand-clay concrete.

Points of observation	$Y_e$	$Y_s$	$Y_0$	A	B %	C	D%	E	F%
C <sub>1</sub>	12.083	13.50	13.09	0.41	3.07	-0.42	-3.22	-0.01	-0.05
C <sub>2</sub>	13.701	12.67	12.71	-0.04	-0.30	0.03	0.24	-0.01	-0.06
C <sub>3</sub>	12.41	13.44	13.07	0.37	2.75	-0.03	-0.25	0.34	2.51
C <sub>4</sub>	13.45	14.87	14.86	0.01	0.10	-0.42	-2.91	-0.41	-2.80
C <sub>5</sub>	14.12	13.00	12.69	0.30	2.34	0.13	0.95	0.43	3.27
C <sub>6</sub>	14.19	14.41	14.14	0.26	1.83	-0.22	-1.53	0.05	0.33
C <sub>7</sub>	13.97	13.75	13.36	0.40	2.87	0.22	1.55	0.61	4.37
C <sub>8</sub>	14.48	13.52	13.41	0.12	0.87	-0.04	-0.33	0.07	0.54
C <sub>9</sub>	14.38	13.08	12.91	0.16	1.24	0.31	2.28	0.47	3.49
C <sub>10</sub>	14.61	14.79	14.62	0.17	1.16	-0.18	-1.25	-0.01	-0.08
C <sub>11</sub>	13.81	12.88	12.64	0.24	1.83	-0.07	-0.55	0.17	1.30
C <sub>12</sub>	13.60	13.29	12.92	0.37	2.75	0.31	2.30	0.68	4.99
C <sub>13</sub>	13.98	14.25	13.97	0.28	1.99	-0.47	-3.43	-0.19	-1.38
C <sub>14</sub>	14.85	15.42	15.45	-0.03	-0.21	-0.57	-3.80	-0.60	-4.02
C <sub>15</sub>	12.88	13.08	12.85	0.23	1.75	-0.20	-1.52	0.03	0.26

where

$Y_e$  = Experimental strengths result

$Y_s$  = Scheffe's strengths result

$Y_0$  = Osadebe's strengths result

A = Difference between results obtained from experimental test and Scheffe's models.

B = Percentage difference between results obtained from experimental test and Scheffe's model.

C = Difference between results obtained from experimental test and Osadebe's models.

D = Percentage difference between results obtained from experimental test and Osadebe's model.

E = Difference between results obtained from Scheffe's model and Osadebe's model.

F = Percentage difference between results obtained from Scheffe's model and Osadebe's model

From the comparisons in Table 4.15, the experimental results of the compressive strength tests agreed closely with the results predicted by Scheffe's and Osadebe's models for predicting the compressive strength of sand-clay concrete. The maximum percentage difference between results predicted by Scheffe's and Osadebe's models, is 3.07% while the maximum percentage difference between results obtained from experimental investigation and those predicted by Scheffe's model, is 2.30%. And, the maximum percentage difference between results obtained from experimental investigation and those from Osadebe's model, is 4.99%. In all, the percentage differences are negligible.

**Determination of final Scheffe's simplex model for flexural strength**

The final scheffe's model for concrete made with sand partially replaced with laterite, is also obtained by substituting the values of the flexural strength, Y (given in Table 4.7) into Eqn (3.51).

$$Y_{sf} = 2.25x_1 + 2.16x_2 + 2.31x_3 + 1.93x_4 + 1.48x_5 - 0.54 X_1X_2 - 3.56x_1x_3 - 1.36 x_1x_4 + 0.94 x_1x_5 + 0.54x_2x_3 + 0.1x_2x_4 + 1.26x_2x_5 + 0.04x_3x_4 + 0.38x_3x_5 + 0.78x_4x_5 \quad (4.3)$$

where  $Y_{sf}$  = Final Scheffe's model for flexural strength and the result obtained from the formulated model is presented in Table 4.16.

**Table 4.16:** Results obtained from flexural strength tests and of Scheffe's model.

S/N	Points of observation	Laboratory results Y (N/mm <sup>2</sup> )	Scheffe's results (Y) (N/mm <sup>2</sup> )
1	N <sub>1</sub>	2.25	2.25
2	N <sub>2</sub>	2.16	2.16
3	N <sub>3</sub>	2.31	2.31
4	N <sub>4</sub>	1.93	1.93
5	N <sub>5</sub>	1.48	1.48
6	N <sub>6</sub>	2.07	2.07
7	N <sub>7</sub>	1.39	1.39
8	N <sub>8</sub>	1.75	1.75
9	N <sub>9</sub>	2.10	2.10
10	N <sub>10</sub>	2.37	2.37
11	N <sub>11</sub>	2.07	2.07
12	N <sub>12</sub>	1.87	1.87
13	N <sub>13</sub>	2.13	2.13
14	N <sub>14</sub>	1.99	1.99
15	N <sub>15</sub>	1.90	1.90
16	C <sub>1</sub>	1.93	1.93
17	C <sub>2</sub>	1.75	1.78
18	C <sub>3</sub>	1.72	1.76
19	C <sub>4</sub>	1.87	1.80
20	C <sub>5</sub>	1.90	1.92
21	C <sub>6</sub>	2.04	2.01
22	C <sub>7</sub>	1.87	1.83
23	C <sub>8</sub>	2.07	2.14
24	C <sub>9</sub>	1.87	1.84
25	C <sub>10</sub>	2.07	2.00
26	C <sub>11</sub>	1.81	1.83
27	C <sub>12</sub>	1.96	1.96
28	C <sub>13</sub>	1.99	1.90
29	C <sub>14</sub>	2.01	1.93
30	C <sub>15</sub>	1.98	1.96

**Test of goodness of fit of scheffe's model for flexural strength.**

The test of goodness of fit of Scheffe's model for flexural strength were tested using statistical students'-test and fisher's test (F-test). In both tests, it was assumed that the results predicted by the model will be 95% accurate. The model predicted results were tested against the controlled experimental results to see if the predicted results agree with the experimental result.

**Table 4.17:** Computations of standard error of replicates of flexural strength of concrete

S/N	Replicates			Mean, (y <sub>i</sub> )	y <sup>2</sup> <sub>i</sub>	Σy <sub>i</sub>	Σy <sub>i</sub> <sup>2</sup>	(Σy <sub>i</sub> ) <sup>2</sup>	S <sup>2</sup> <sub>i</sub>
	1	2	3						
1	2.22	2.40	2.12	2.25	5.0625	6.74	15.2253	45.4276	0.02
2	2.22	2.04	2.22	2.16	4.6656	6.48	14.0184	41.9904	0.01
3	2.67	2.31	1.96	2.31	5.3361	6.94	16.3066	48.1636	0.13
4	2.67	2.31	1.96	1.93	3.7249	4.78	7.9422	22.8484	0.16
5	1.51	1.60	1.33	1.48	2.1904	4.44	6.6090	19.7136	0.02
6	1.96	2.22	2.04	2.07	4.2849	6.22	12.9316	38.6884	0.02
7	1.33	1.60	1.24	1.39	1.9321	4.17	5.8665	17.3889	0.04

8	1.60	1.78	1.87	1.75	3.0625	5.25	9.2253	27.5625	0.02
9	2.22	1.96	2.13	2.10	4.410	6.31	13.3069	39.8161	0.02
10	2.49	2.22	2.40	2.37	5.6169	7.11	16.8885	50.5521	0.02
11	1.96	2.22	2.04	2.07	4.2849	6.22	12.9316	38.6884	0.02
12	1.96	1.78	1.87	1.87	3.4969	5.61	10.5069	31.4721	0.01
13	2.22	2.13	2.04	2.13	4.5369	6.39	13.6269	40.8321	0.01
14	2.22	1.78	1.96	1.99	3.9601	5.96	11.9384	35.5216	0.05
15	1.78	1.87	2.04	1.90	3.610	5.69	10.8269	32.3761	0.02
16	1.96	2.04	1.78	1.93	3.7249	5.78	11.1716	33.4084	0.02
17	1.69	1.78	1.78	1.75	3.0625	5.25	9.1929	27.5625	0.00
18	1.69	1.60	1.87	1.72	2.9584	5.16	8.9130	26.6256	0.02
19	1.78	1.87	1.96	1.87	3.4969	5.61	10.5069	31.4721	0.01
20	1.96	1.87	1.87	1.90	3.6100	5.70	10.8354	32.4900	0.00
21	2.04	2.13	1.96	2.04	4.1616	6.13	12.5401	37.5769	0.01
22	1.87	1.96	1.78	1.87	3.4969	5.61	105069	31.4721	0.01
23	2.04	2.13	2.04	2.07	4.2849	6.21	12.8601	38.5641	0.01
24	1.69	2.04	1.87	1.87	3.4969	5.60	10.5146	31.3600	0.03
25	1.96	2.22	2.04	2.07	4.2849	6.22	12.9316	38.6884	0.02
26	1.96	1.69	1.78	1.81	3.2761	5.43	9.8661	29.4849	0.02
27	1.87	2.04	1.96	1.96	3.8416	5.87	11.5001	34.4569	0.01
28	1.96	2.13	1.87	1.99	3.9601	5.99	11.8754	35.8801	-0.00
29	2.04	1.96	2.04	2.01	4.0401	6.04	12.1648	36.4816	0.00
30	1.78	2.13	2.04	1.98	3.9204	5.95	11.8669	35.4025	0.03
								Σ	<b>0.76</b>

Using Eqns (4.1) and Eqn (4.2),  $S^2y$  and  $Sy$  can be calculated as follows:-

$$S_y^2 = \frac{\sum S_i^2}{N-1} = \frac{0.76}{15-1} = \frac{0.76}{14} = 0.054$$

$$S_y = \sqrt{S_y^2} = \sqrt{0.054} = 0.23$$

**Table 4.18:** T-test computations for Scheffe's model for flexural strength of concrete.

S/N	$Y_c$	$Y_m$	$D_i = Y_c - Y_m$	$D_A - D_i$	$(D_A - D_i)^2$
C <sub>1</sub>	1.93	1.93	0.00	0.017	0.00029
C <sub>2</sub>	1.75	1.78	-0.03	0.047	0.00221
C <sub>3</sub>	1.72	1.76	-0.04	0.057	0.00325
C <sub>4</sub>	1.87	1.80	0.07	-0.053	0.00281
C <sub>5</sub>	1.90	1.92	-0.02	0.037	0.00137
C <sub>6</sub>	2.04	2.01	0.03	-0.013	0.00017
C <sub>7</sub>	1.87	1.83	0.04	-0.023	0.00053
C <sub>8</sub>	2.07	2.14	-0.07	0.087	0.00757
C <sub>9</sub>	1.87	1.84	0.03	-0.013	0.00017
C <sub>10</sub>	2.07	2.00	0.07	-0.053	0.00281
C <sub>11</sub>	1.81	1.83	-0.02	0.037	0.00137
C <sub>12</sub>	1.96	1.96	0.00	0.017	0.000029
C <sub>13</sub>	1.99	1.90	0.09	-0.073	0.00533
C <sub>14</sub>	2.01	1.93	0.08	-0.063	0.00397
C <sub>15</sub>	1.98	1.96	0.02	-0.003	0.00001
			<b>ΣDi = 0.25</b>		<b>Σ((DA-Di)<sup>2</sup> = 0.03215</b>

$$D_A = \sum D_i / N = 0.25 / 15 = 0.017$$

$$S^2 = 0.0321 / 14 = 0.0023$$

$$S = \sqrt{0.0023} = 0.048,$$

Therefore,  $T_{cal} = \frac{0.017*}{0.048}(15)^{0.5} = 1.372$

$T_{table} 2.245 > T_{cal} 1.372$

And so, the differences between laboratory results and model results are not significant.

**Table 4.19:** F-test computations for scheffe's flexural strength of concrete

S/N	$Y_e$	$Y_m$	$Y_e - \check{Y}_e$	$(Y_e - \check{Y}_e)^2$	$(Y_m - \check{Y}_m)$	$(Y_m - \check{Y}_m)^2$
C <sub>1</sub>	1.93	1.93	0.007	0.000049	0.024	0.000576
C <sub>2</sub>	1.75	1.78	-0.173	0.029929	-0.126	0.015876
C <sub>3</sub>	1.72	1.76	-0.203	0.041209	-0.146	0.021316
C <sub>4</sub>	1.87	1.80	-0.053	0.002809	-0.106	0.011236
C <sub>5</sub>	1.90	1.92	-0.023	0.000529	0.014	0.000196
C <sub>6</sub>	2.04	2.01	0.117	0.013689	0.104	0.010816
C <sub>7</sub>	1.87	1.83	-0.053	0.002809	-0.076	0.005776
C <sub>8</sub>	2.07	2.14	0.147	0.021609	0.234	0.054756
C <sub>9</sub>	1.87	1.84	-0.053	0.002809	-0.026	0.000436
C <sub>10</sub>	2.07	2.00	0.147	0.021609	0.094	0.008836
C <sub>11</sub>	1.81	1.83	-0.113	0.012769	-0.076	0.005776
C <sub>12</sub>	1.96	1.96	0.037	0.001369	0.054	0.002916
C <sub>13</sub>	1.99	1.90	0.067	0.004489	-0.006	0.000036
C <sub>14</sub>	2.01	1.93	0.087	0.007569	0.024	0.000576
C <sub>15</sub>	1.98	1.96	0.057	0.003249	0.054	0.002916
	$\Sigma = 28.84$	<b>28.59</b>		<b>0.170916</b>		<b>0.14596</b>
<b>Mean</b>	<b>1.923</b>	<b>1.906</b>				

$$S^2_m = \frac{\sum(Y_m - \check{Y}_m)^2}{(N-1)} = \frac{0.14596}{14} = 0.0104$$

$$S^2_e = \frac{\sum(Y_e - \check{Y}_e)^2}{(N-1)} = \frac{0.14596}{14} = 0.0122$$

$$F_{cal} = S^2_1/S^2_2 = 0.85 < F_{0.05}(14,14) = 2.48$$

From standard statistics table,  $F_{0.05}(14,14) = 2.48$

$$1/F = 0.4$$

Therefore, the condition,  $1/F = S^2_1/S^2_2 = F = 0.4 < 0.85 < 2.48$ . Thus, the condition  $1/F = S^2_1/S^2_2 = F$  has been satisfied and so the difference between laboratory results and model results are not significant.

**Determination Of Osadebe's Regression Model For Flexural Strength**

The Osadebe's regression coefficients are determined from Eqns ( 3.77 ) and(3.79). Substituting the values of the responses,  $Y_e$ ( i.e. Flexural strength given in Table 4.5) into these equations, yielded the following coefficients

$$\begin{aligned} \alpha_1 &= -2.8549239.59, & \alpha_2 &= -9140532.44 \\ \alpha_3 &= -22129.1294, & \alpha_4 &= -37072.77906 \\ \alpha_5 &= 969.7840031, & \alpha_6 &= 7001663.45 \\ \alpha_7 &= 26915255.64, & \alpha_8 &= 30634809.7 \\ \alpha_9 &= 28494114.86, & \alpha_{10} &= 10104656.6 \\ \alpha_{11} &= 8018922.333, & \alpha_{11} &= 9167119.129 \\ \alpha_{13} &= -2.8549239.59, & \alpha_{14} &= 16938.85816 \\ \alpha_{15} &= 38442.81159 \end{aligned}$$

Then, substituting these coefficients into Eqn. (3.75), gives the final Osadebe's regression model for flexural strength, where  $Y_{of}$  = Final Osadebe's regression model for flexural strength, (i.e. Eqn (4.4) .

$$Y_{of} = -2.8549239.59Z_1 - 9140532.44Z_2 - 22129.1294Z_3 - 37072.77906Z_4 + 969.7840031Z_5 + 7001663.45Z_6 + 26915255.64Z_7 + 30634809.7Z_8 + 28494114.86Z_9 + 10104656.6Z_{10} + 8018922.333Z_{11} + 9167119.129Z_{12} - 2.8549239.59Z_{13} + 16938.85816Z_{14} + 38442.81159Z_{15} \tag{4.4}$$

And the results obtained from the formulated model are presented in Table 4.20

**Table 4.20: Result of flexural strength test and of Osadebe model**

S/N	Points of observation	Laboratory results $Y(N/mm^2)$	Osadebe's results $(N/mm^2)$
1	$N_1$	2.25	2.25
2	$N_2$	2.16	2.16

3	N <sub>3</sub>	2.31	2.31
4	N <sub>4</sub>	1.93	1.93
5	N <sub>5</sub>	1.48	1.48
6	N <sub>6</sub>	2.07	2.07
7	N <sub>7</sub>	1.39	1.39
8	N <sub>8</sub>	1.75	1.75
9	N <sub>9</sub>	2.10	2.10
10	N <sub>10</sub>	2.37	2.37
11	N <sub>11</sub>	2.07	2.07
12	N <sub>12</sub>	1.87	1.87
13	N <sub>13</sub>	2.13	2.13
14	N <sub>14</sub>	1.99	1.99
15	N <sub>15</sub>	1.90	1.90
16	C <sub>1</sub>	1.93	1.88
17	C <sub>2</sub>	1.75	1.74
18	C <sub>3</sub>	1.72	1.75
19	C <sub>4</sub>	1.87	1.80
20	C <sub>5</sub>	1.90	1.92
21	C <sub>6</sub>	2.04	1.98
22	C <sub>7</sub>	1.87	1.85
23	C <sub>8</sub>	2.07	2.14
24	C <sub>9</sub>	1.87	1.78
25	C <sub>10</sub>	2.07	2.05
26	C <sub>11</sub>	1.81	1.84
27	C <sub>12</sub>	1.96	1.98
28	C <sub>13</sub>	1.99	1.96
29	C <sub>14</sub>	2.01	2.03
30	C <sub>15</sub>	1.98	1.97

**Test of adequacy of Osadebe’s model for flexural strength.**

The test of adequacy of Osadebe’s model is done using statistical students’ T- test and Fisher’s test. In both tests, it was assumed that the results predicted by the model will be 95% accurate.

**Table 4.21: T -test Computations for Osadebe’s model of flexural strength of concrete**

S/N	Y <sub>e</sub>	Y <sub>m</sub>	D <sub>i</sub> = Y <sub>e</sub> -Y <sub>m</sub>	Da - Dj	(DA - Di) <sup>2</sup>
C <sub>1</sub>	1.93	1.88	0.05	-0.039	0.001521
C <sub>2</sub>	1.75	1.74	0.01	0.001	0.000001
C <sub>3</sub>	1.72	1.75	-0.03	0.041	0.001681
C <sub>4</sub>	1.87	1.80	0.07	-0.059	0.003481
C <sub>5</sub>	1.90	1.92	-0.02	0.031	0.000961
C <sub>6</sub>	2.04	1.98	0.06	-0.049	0.002401
C <sub>7</sub>	1.87	1.85	0.02	-0.009	0.000081
C <sub>8</sub>	2.07	2.14	-0.07	0.081	0.006561
C <sub>9</sub>	1.87	1.78	0.09	-0.889	0.790321
C <sub>10</sub>	2.07	2.05	0.02	-0.009	0.000081
C <sub>11</sub>	1.81	1.84	-0.03	0.041	0.001681
S/N	Y <sub>e</sub>	Y <sub>m</sub>	D <sub>i</sub> = Y <sub>e</sub> -Y <sub>m</sub>	Da - Dj	(DA - Di) <sup>2</sup>
C <sub>12</sub>	1.96	1.98	-0.02	0.031	0.000961
C <sub>13</sub>	1.99	1.96	0.03	-0.019	0.000361
C <sub>14</sub>	2.01	2.03	-0.02	0.031	0.000961
C <sub>15</sub>	1.98	1.97	0.01	0.001	0.000001
			∑D <sub>1</sub> = 0.17		∑(D <sub>A</sub> -D <sub>i</sub> ) <sup>2</sup> = 0.811055

$$DA = \frac{\sum Di}{N} = 0.17/15 = 0.011$$

$$S^2 = \frac{\sum (D_A - D_i)^2}{(N-1)} = 0.811055/14 = 0.058$$

$$S = \sqrt{0.058} = 0.24$$

$$T_{Cal} = \frac{DA * (N)05}{S} = \frac{0.011 * (15)^{0.5}}{0.24} = 0.178$$

Therefore,  $T_{table} 2.245 > T_{cal} 0.178$ . Hence, the differences between laboratory results and model results are not significant. And so, the Osadebe's model can be used to predict accurately the flexural strength of sand-clay concrete.

**Table 4.22:** F-test computations of Osadebe's flexural strength of concrete

S/N	Y <sub>e</sub>	Y <sub>m</sub>	Y <sub>e</sub> -Y <sub>e</sub>	(Y <sub>e</sub> -Y <sub>e</sub> ) <sup>2</sup>	(Y <sub>e</sub> -Y <sub>e</sub> )	(Y <sub>m</sub> -Y <sub>m</sub> ) <sup>2</sup>
C <sub>1</sub>	1.93	1.88	0.007	0.000049	-0.031	0.000961
C <sub>2</sub>	1.75	1.74	-0.173	0.029929	-0.171	0.029241
C <sub>3</sub>	1.72	1.75	-0.203	0.041209	-0.161	0.025921
C <sub>4</sub>	1.87	1.80	-0.053	0.002809	-0.111	0.012321
C <sub>5</sub>	1.90	1.92	-0.023	0.000529	0.009	0.000081
C <sub>6</sub>	2.04	Table	4.22 conti	nued )	0.069	0.004761
C <sub>7</sub>	1.87	1.85	-0.053	0.002809	-0.061	0.003721
C <sub>8</sub>	2.07	2.14	0.147	0.021609	0.229	0.052441
C <sub>9</sub>	1.87	1.78	-0.053	0.002809	-0.131	0.017161
C <sub>10</sub>	2.07	2.05	0.147	0.021609	0.139	0.019321
C <sub>11</sub>	1.81	1.84	-0.113	0.012769	-0.071	0.005041
C <sub>12</sub>	1.96	1.98	0.037	0.001369	0.069	0.004761
C <sub>13</sub>	1.99	1.96	0.067	0.004489	0.049	0.002401
C <sub>14</sub>	2.01	2.03	0.087	0.007569	0.119	0.014161
C <sub>15</sub>	1.98	1.97	0.057	0.003249	0.059	0.003481
Σ	<b>28.84</b>	<b>28.67</b>		<b>0.170916</b>		<b>0.195775</b>
Mean	<b>1.923</b>	<b>1.911</b>				

$$S^2_m = \frac{\sum (Y_m - \bar{Y}_m)^2}{(N-1)} = \frac{0.195775}{14} = 0.0140$$

$$S^2_e = \frac{\sum (Y_e - \bar{Y}_e)^2}{(N-1)} = \frac{0.170916}{14} = 0.0122$$

Therefore  $S^2_1 = 0.0140$  and  $S^2_2 = 0.0122$

$$F_{cal} = \frac{S^2_1}{S^2_2} = \frac{0.0140}{0.0122} = 1.15$$

From statistics table,  $F_{0.05}(14,14) = 2.48$

$$\text{Therefore, } 1/F = 0.40$$

Therefore, the condition  $1/F < S^2_1 / S^2_2 < F = 0.40 < 1.15 < 2.48$ . Thus, the condition  $1/F < S^2_1 / S^2_2 < F$ , has been satisfied, the differences between laboratory results and model results are not significant.

**Comparison of experimental, Scheffe's and Osadebe's results in flexural strength of concrete.**

The flexural strength results obtained from the experiment, were compared with Scheffe's and Osadebe's results for flexural strength results in Table 4.16 and Table 4.20 as shown in Table 4.23.

**Table 4.23:** Comparison of experimental, Scheffe's and Osadebe's flexural strength results of sand-lateritic concrete.

Points of observation	Y <sub>e</sub>	Y <sub>s</sub>	Y <sub>o</sub>	A	B%	C	D%	E	F%
C <sub>1</sub>	1.93	1.93	1.88	0.00	0.00	0.05	2.59	0.05	2.59
C <sub>2</sub>	1.75	1.78	1.74	-0.03	-1.71	0.01	0.57	0.04	2.25
C <sub>3</sub>	1.72	1.76	1.75	-0.04	-2.33	-0.03	-1.74	0.01	0.57
C <sub>4</sub>	1.87	1.80	1.84	0.07	1.60	0.07	3.74	-0.04	-2.22
C <sub>5</sub>	1.90	1.92	1.92	-0.02	-1.05	-0.02	-1.05	0.00	0.00
C <sub>6</sub>	2.04	2.01	1.98	0.03	1.47	0.06	2.94	0.03	1.49
C <sub>7</sub>	1.87	1.83	1.85	0.04	2.14	0.02	1.07	-0.02	1.09
C <sub>8</sub>	2.07	2.14	2.14	-0.07	-3.38	-0.07	-3.38	0.00	0.00
C <sub>9</sub>	1.87	1.84	1.81	0.03	1.60	0.06	3.21	-0.03	1.63
C <sub>10</sub>	2.07	2.00	2.05	0.07	3.38	0.02	0.97	-0.05	-2.50
C <sub>11</sub>	1.81	1.83	1.84	-0.02	-1.11	-0.02	-1.66	0.01	-0.55
C <sub>12</sub>	1.96	1.96	1.98	0.00	0.00	-0.02	-1.02	-0.02	-1.02
C <sub>13</sub>	1.99	1.90	1.96	0.09	2.51	0.03	1.51	-0.06	3.16
C <sub>14</sub>	2.01	1.93	2.03	0.08	2.98	-0.02	-0.10	-0.10	-5.18
C <sub>15</sub>	1.98	1.96	1.97	0.02	1.01	0.01	0.51	-0.01	-0.15

Where A = Difference between results obtained from experimental test and Scheffe’s model.  
 B = Percentage difference between results obtained from experimental test and Scheffe’s model.  
 C = Difference between results obtained from experimental test and Osadebe’s model.  
 D = Percentage difference between results obtained from experimental test and Osadebe’s model.  
 E = Difference between results obtained from Scheffe’s model and Osadebe’s model.  
 F = Percentage difference between results obtained from Scheffe’s model and Osadebe’s models.

From the comparisons in Table 4.23, the experimental flexural strength results agreed closely with those predicted by Scheffe’s and Osadebe’s models of sand- lateritic concrete. The maximum percentage difference between flexural strength results obtained from experimental tests and those from Scheffe’s model, are 3.38% while the maximum percentage differences between those obtained from experimental tests and Osadebe’s model are 3.74%. those the models based on Scheffe’s and Osadebe’s theories, can be used to predict accurately flexural strengths of sand-lateritic concrete.

**The optimum compressive and flexural strengths and the mix ratios.**

The optimum compressive and flexural strengths and the mix ratios that yielded them, are given in Table 4.24

**Table 4.24:** Optimum mix ratios and their corresponding optimum compressive and flexural strengths in sand-lateritic concrete

S/N	Strengths	Mix ratios	Optimum strength (N/mm <sup>2</sup> )
1	Compressive	0.51: 1 : 2.25:0.25 : 4	19.15
2.	Flexural	0.535 : 1 : 1.85:0.65 : 5	2.37

**V. Conclusions And Recommendations**

**Conclusions**

From the results of this study, several noteworthy observations emerge, and the following conclusions summarize the study’s key contributions; this study’s key contributions are:

- (i) Two mathematical models each based on Scheffe’s and Osadebe’s theories, were formulated for predicting both compressive and flexural strength of sand- lateritic concretes. The adequacy of the models were verify and confirmed using Student’s T-test and Fisher’s F-test.
- (ii) The optimum compressive strength of sand-lateritic concrete predicted by Scheffe and Osadebe are 19.15 N/mm<sup>2</sup> and 19.15 N/mm<sup>2</sup> respectively. These strengths were yielded by the mix ratio of 0.51: 1: 2.25:0.25: 4. Similarly, the optimum flexural strength of sand-clay concrete predicted by Scheffe and Osadebe, are 2.37N/mm<sup>2</sup> and 2.37N/mm<sup>2</sup> respectively. These flexural strengths were produced from the mix ratio of 0.535:1: 1. 85:0.65: 5.
- (iii) The results obtained from the formulated models agreed with the experimental results as well as each other.
- (iv) The formulated models can be used to predict accurately the compressive and flexural strengths.
- (v) The compressive and flexural strengths obtained from the models are functions of the proportions of the constituents of concrete.

**Recommendations**

With reference to the result/finding of this study, the following recommendations was made:

- (i) Scheffe’s and Osadebe’s models used in this work should be used in the designing of concrete mixes if corresponding constituent materials are specified.
- (ii) People should work more on this ^project using different models like Genetic algorithm and Neural network.
- (iii) More research should be performed on the use of Scheffe’s and Osadebe’s theory in developing models for predicting elastic modulus, workability and shear strength.
- (iv) Research should also be carried on the relationship between flexural strength and compressive strength of sand-lateritic concrete.

**Contributions To Knowledge**

- (i) Four different models, two based on scheffe’s theory and the other two based on Osadebe’s theory were formulated for the mix proportioning of concrete incorporating some clay component.
- (ii) In addition computer programs were developed for predicting compressive and flexural strengths of sand-lateritic concrete.

## References

- [1]. Aci (1990). Cement And Concrete Terminology. Aci 116r-90. American Concrete Institute, Detroit, Usa
- [2]. Adedokum, S.I., J.R., & Obebe, D.S. (2019). Effect Of Glass Fines On The Geotechnical Properties Of Cement Stabilized Lateritic Soil.
- [3]. Ahaneku I.E., Arinze, E.E. & Ekeoma E.E. (2019). Basic Soil Mechanics. Lumen. Pp. 5.
- [4]. Aitcin P.C. (2000), Cement Of Yesterday And Today: Concrete Of Tomorrow: Cement And Concrete Research 30 (9): 1349-1359.
- [5]. Aitoin, P. C. (2000), Cement Of Yesterday And Today: Concrete Of Tomorrow: Cement And Concrete Research 30 (9):1359.
- [6]. Alaneme, G. U., Mbadike, E. M., Iro, U. I., Udousoro, I. M., & Ifejimalu, W. C. (2021). Adaptive Neuro-Fuzzy Inference System Prediction Model For The Mechanical Behaviour Of Rice Husk Ash And Periwinkle Shell Concrete Blend For Sustainable Construction. Asian Journal Of Civil Engineering (2021) 22:959–974 <https://doi.org/10.1007/S42107-021-00357-0>
- [7]. Alaneme, G.U., Mbadike, E.M. Experimental Investigation Of Bambara Nut Shell Ash In The Production Of Concrete And Mortar. Innov. Infrastruct. Solut. 6, 66 (2021). <https://doi.org/10.1007/S41062-020-00445-1>
- [8]. American Society For Testing And Material Standard (1992), Astm C618 Specification For Fly Ash And Raw Or Calcium Natural Pozzolanas For Use As A Mineral Admixture In Portland Cement Concrete, West Conshohocken, Pennsylvania Usa, Volume 04.02
- [9]. American Society For Testing And Material Standard (1992), Astm C618 Specification For Fly Ash And Raw Or Calcium Natural Pozzolanas For Use As A Mineral Admixture In Portland Cement Concrete, West Conshohocken, Pennsylvania Usa, Volume 04.02
- [10]. Astm C143-78(1993), Test For Slump Of Portland Cement Concrete. American Society For Testing And Materials, West Conshohocken, Pennsylvania, Usa.
- [11]. Astm C39. Standard Test Method For Compressive Strength Of Cylindrical Concrete Specimens.
- [12]. Astm C78. Standard Test Method For Flexural Strength Of Concrete (Using Simple Beam With Third-Point L
- [13]. Atilou, V.K (2009). Designing A Learning Machine For Prediction Of The Compressive Strength Of Concrete By Using Artificial Neural Network M.Sc Thesis, Science And Research Branch, Islamic Azad University, Tehran, Iran.
- [14]. Atilou, V.K (2009). Designing A Learning Machine For Prediction Of The Compressive Strength Of Concrete By Using Artificial Neural Network M.Sc Thesis, Science And Research Branch, Islamic Azad University, Tehran, Iran.
- [15]. Attah, I. C., Etim, R. K., Alaneme, G. U., & Bassey, O. B. (2020). Optimization Of Mechanical Properties Of Rice Husk Ash Concrete Using Scheffe's Theory. Sn Applied Science. <https://doi.org/10.1007/S42452-020-2727->
- [16]. Ayodele A.I., Pamukcu, S.&Agbede, O.A. (2020). Plasticity Modification Of A Tropical Laterite By Electrochemical Stabilization, Electrochimica Acta 34 (24):136047.
- [17]. Aziz, M.A (1995) Engineering Materials: Z And Z Computer And Printers, Dhaka, Bangladesh,
- [18]. Aziz, M.A. (1995). Engineering Materials: Z And Z Computer And Printers, Dhaka, Bangladesh.
- [19]. Bhutto, A.H Zardari, S., Zardri, M.A., & Bhurgri, G.S. (2019). Mohr-Coulomb And Hardening Soil Model Comparison Of The Settlement Of An Embankment Dam.
- [20]. Bogue, R. H. And Lerch, W. 1984. Hydration Of Portland Cement Compounds: Industrial And Engineering Chemistry. Easton, Pa.
- [21]. British Standard Institution (1985), Bs8110 Part 2: Structural Use Of Concrete Code Of Practice For Design And Construction, London, P.7/1
- [22]. British Standards Institution (1985), Bs8110 Part 2: Structural Use Of Concrete. Code Of Practice For Design And Construction, London, P.7/1
- [23]. Bs 812: (Part 103, 1985) British Standard Institution, Method For Determination Of Particle Size Distribution, London.
- [24]. Bs 812: (Part 103, 1985) British Standard Institution, Method For Determination Of Particle Size Distribution London.
- [25]. Calabrese, E.J., Kostecki, P.T. And Bonazountas, M.( 1991). Hydrocarbon Contaminated Soils. Vol. Ii. Crc Press
- [26]. Callister D.W (2003), Materials Science And Engineering, Hoken: John Willey And Sons, Inc. New York.
- [27]. Callister D.W (2003), Materials Science And Engineering, Hoken: John Willey And Sons, Inc New York.
- [28]. Cement Admixture Association (2010). Admixture Types. Retrieved From <http://www.admixtures.org.uk/Types.asp> On 23rd June 2008.
- [29]. Cement Admixture Association (2010). Admixture Types. Retrived From <http://www.admixtures.org.uk/Types.asp> On 23rd June 2008.
- [30]. Chamberlain, B (1995). Concrete: A Material For The New Stone Age, Material Science And Technology. Retrieved From, <http://www.ejournalofscience.org> On 23rd June 2008.
- [31]. Chamberlain, B (1995). Concrete: A Material For The New Stone Age, Material Science And Technology. Retrieved From, <http://www.ejournalofscience.org> On 23rd June 2008.
- [32]. Chinneck J.W And Ramadam, K (2000). "Linear Programming With Interval Coefficients." Journal Of The Optimization Research Society. Vol. 51, Pp209-220.
- [33]. Chinneck, J.W. And Ramadam, K (2000). "Linear Programming With Interval Coefficients." Journal Of The Optimization Research Society. Vol. 51, Pp 209-220.
- [34]. D.O. Okereke E.C, Arimanwa J.I And Onwuka S.U (2011), Prediction Of Concrete Mix Ratios Using Modified Regression Theory. Computational Methods In Civic Engineering, Vol. 2, No1, Pp95-107.
- [35]. Deliveris, A.V., Zevgolis, I.E., & Koukoulzas, N. (2020). Probabilistic Evaluation Of Local Overstress On Slope Stability Problems. Geotechnical And Geological Engineering 38(3).
- [36]. Domone, P. L. (2003). Advanced Concrete Technology-Concrete Properties. J. Newman And B. S. Choo Eds. Elsevier Ltd.
- [37]. Ehlers E.G And Blatt, H (1982) Petrology, Igneous, Sedimentary, And Metamorphic. W. H Freeman And Company, San Francisco.
- [38]. Ehlers, E.G And Blatt, H (1982). Petrology, Igneous, Sedimentary, And Metamorphic. W.H Freeman And Company, San Francisco.
- [39]. Elinwa, A.U. And Mahmood Y.A., Desia J.B (2005). "Using Metakolin To Improve Sawdust Ash Concrete". Cement And Concrete Composites, Vol 24, No2, Pp 219-222.
- [40]. Elinwa, A.U. And Mahmood Y.A., Desia J.B. (2005). "Using Metakolin To Improve Sawdust Ash Concrete". Cement And Concrete Composites, Vol. 24, No 2, Pp219-222.
- [41]. Emeh., C., Igwe, O., & Onwo, E.S (2019). Potential Effect Of Environmental Pollution On The Degree Of Dissolution Of Iron And Aluminium Oxides In Lateritic Soils. Environmental Earth Sciences 78(8).
- [42]. Etim, R.K., Ekpo, D.U., Attah, I.C., & Onyelowe K.C, (2020), Effect Of Micro Sized Quarry Dust Particle On The Compaction And Properties Of Cement Stabilized Lateritic Soil. Cleaner Material Vol.2
- [43]. Fang, K., Miao, M., Tang, M., & Jia, S. (2022). Insights Into The Deformation And Failure Characteristic Of A Slope Due To Excavation Through Multi-Field Monitoring: A Model Test Acta Geotechnical 18(2).
- [44]. Finn J, Chidiroglou I, Godwin A, O' Flaherty (2009), Shear Behaviour Of Crashed Concrete And Bricks, Proceedings Of The Institution Of Civil Engineers, On Construction Materials, 162, 3, Pp 121-126.

- [45]. Finn J, Chidioglu I, Godwin A, O, Flaherty (2009), Shear Behaviour Of Crushed Concrete And Bricks, Proceedings Of The Institution Of Civil Engineers, On Construction Materials, 162, 3, Pp 121-126.
- [46]. Goldberg D. (1989), Genetic Algorithms In Search Of Optimization And Machine Learning, Addison Wesley, Longman Publishing Co. Inc, Boston, Usa
- [47]. Goldberg D. (1989), Genetic Algorithms In Search Of Optimization And Machine Learning, Addison Wesley, Longman Publishing Co. Inc, Boston Usa.
- [48]. Goldberg, D.E And Holland, J.H (1988), Genetic Algorithms And Machine Learning Klumer Academic Pulishers, Springer, Usa.
- [49]. Goldberg, D.E And Holland, J.H (1988), Genetic Algorithms And Machine Learning, Klumer Academic Publishers, Springer, Usa.
- [50]. Guggenheim R. Stephen J. And Martin W. (1995), Definition Of Clay And Clay Mineral, Retired From [Http://Www.Clays.Org/Journal](http://www.clays.org/journal) On 9th April, 2008.
- [51]. Guggenheim R. Stephen J. And Martitn W, (1995), Definition Of Clay And Clay Material, Retried From [Http://Www.Clays.Org/Journal](http://www.clays.org/journal) On 9th April, 2008.
- [52]. Gupta, B. L. And Gupta, A. (2004). Concrete Technology. Delhi: Standard Publishers Distributors.
- [53]. Hakim S.J.S, Noorzaei J, Jaafar M.S, Jameel M, And Mohammad Hassani M,(2011), Application Of Artificial Neural Networks To Prediction Of Compressive Strength Of High Strength Concrete. International Journal Of Physical Science, 6(5),975-981.
- [54]. Hakim S.J.S., Noorzaei J, Jaafar M.S, Jameel M, And Mohammad Hassani M, (2011), Application Of Artificial Neural Networks To Prediction Of Compressive Strength Of High Strength Concrete. International Journal Of Physical Science, 6(5),975-981.
- [55]. Hendry A.W, Sinha B.P. And David S.R. (1987). Design Of Masonry Structures, Boundary Row, London, United Kingdom.
- [56]. Hendry A.W., Sinha B.P And David S.R. (1987), Desgn Of Masonry Structures, Boundary Row, London, United Kingdom.
- [57]. Hillier S. (2003). Clay Mineralogy. Kluwer Academic Publishers, Dordrecht Pp 139-142.
- [58]. Hilliers S. (2003). Clay Mineralogy. Kluwer Academic Publications, Dordrecht Pp139-142.
- [59]. Hola J. And Schabowicz K (2005). Application Of Artificial Neural Network To Determination Of Concrete Compressive Strength Based On Non-Destructive Test, Journal Of Civil Engineering Management 11 (1), 23-32.
- [60]. Hola J. And Schabowicz K. (2005). Application Of Artificial Neural Networks To Determination Of Concrete Compressive Strength Based On Non-Destructive Tests. Journal Of Civil Engineering Management 11 (1), 23 - 32.
- [61]. Kassou, F., Bouziyane, J.B., Ghafiri, A., & Sabihi, A. (2022). Slope Stability Of Embankments On Soft Soil Improved With Vertical Drains. Civil Engineering Journal 6(1):164-173.
- [62]. Khan, M.S., Jaemin Park, J., & Jongwon Seo, J. (2021). Geotechnical Property Modeling And Construction Safty Zoning Based On Gis And Bim Itergration. Applied Sciences 11(9):4004.
- [63]. Krishnan S.N (2001), New Material And High Strength Concrete, Workshop On Concrete Mix Design, Pp22-24, Retrieved From [Http://Cipremier.Com](http://cipremier.com) On 23rd June 2008.
- [64]. Krishnan S.N (2001), New Materials And High Strength Concrete, Workshop On Concrete Mix Design, Pp 22 - 24. Retrieved From [Http://Cipremier.Com](http://cipremier.com) On 23rd June 2008.
- [65]. Lancaster L.C (2005). Concrete Vaulted Construction In Imperial Rome: Innovations In Context, Cambridge University Press England
- [66]. Landcaster L.C. (2005). Concrete Vaulted Construction In Imperial Rome: Innovations In Context, Cambridge University Press England,
- [67]. Li, P., Dong, L., Gao, X, Li, T., & Hou, X. (2020). An Extension Of Taylor'S O-Circle Method And Some Stability Charts For Submerged Slopes. Advances In Civil Engineering.
- [68]. Mehta, P. K. And Monteiro, P. J. M. (2006). Concrete: Microstructure, Properties, And Materials. 3rd Ed. New York: Mcgraw-Hill Company Ltd
- [69]. Moayedi, H., Osouli, A., Nguyeu, H., Rashid, A.S. (2021). A Novel Harris Hawks' Optimized And K-Fold Cross- Validation Predicting Slope Stability.
- [70]. Neville A.W (1996), Properties Of Concrete Ltd, Emglan. Onwka
- [71]. Neville, A.W And Brookes, J.J (1987), Concret Technology, Longman Scientific And Technical England, P363.
- [72]. Neville, A.M And Brookes, J.J (1987), Concrete Technology, Longman Scientific And Technical, England, P363.
- [73]. Neville, A.M. (1996), "Properties Of Concrete". Longman Ltd, England.
- [74]. Neville, A.M. And Brooks, J. J. (1990). Concrete Technology. Revised Ed. Singapore: Longman Group Uk Limited.
- [75]. Neville, A.M.( 1999) Properties Of Concrete. Fourth Edition. Longman.
- [76]. Nguyeu, Q.H., Ly, H.B., Ho, L.S., & Ai-Anasari, N, (2021). Influence Of Data Splitting On Performance Of Machine Learning Models In Prediction Of Shear Strength Of Soil Mathematical Problems In Engineering 2021(6)
- [77]. Occupational Safety And Health Administration (Osha), (1926). Excavations; Sloping And Benching, Safety And Health Regulation For Construction.
- [78]. Onwuka, D.O, Okere, C.E, Arimanwa J.I And Onwuka S.U (2011), Prediction Of Concrete Mix Ratios Using Modified Regression Theory. Computational Methods In Civil Engineering, Vol. 2. No. 1, Pp 95 - 107.
- [79]. Osadebe N.M (2003), "Generalized Mathematical Model For Normal Concrete As A Multivariate Function Of Properties Of The Constituent's Components. A Paper Delivered At The Faculty Of Engineering University Of Nigeria Nsukka.
- [80]. Osadebe, N. N. (2011). Development Of A Mathematical Model For Mixture Design Of Concrete Using Industrial By-Products. Journal Of Engineering And Applied Sciences, 6(2), 123-130.
- [81]. Osadebe, N.N (2003), "Generalized Mathematical Model For Normal Concrete As A Multivariate Function Of The Properties Of The Constituents' Components". A Paper Delivered At The Faculty Of Engineering, University Of Nigeria, Nsukka.
- [82]. Portelinha, F, H. Correia, N.S. Mendes, I.D.S., & Silva, J.W.B. (2021). Geotechnical Properties And Microstructure Of A Diesel Contaminated Lateritic Soil Treated With Lime Soil And Sediment Contamination (Formerly Journal Of Soil Contamination) 30 (3): 1-24
- [83]. Raja, P.S. & Thyagaraj T. (2020). Effect Of Compaction Time Delay On Compaction And Strength Behaviour Of Lime-Treated Expensive Soil Contacted With Sulphate. Innovative Infrastructure Solution 5(1)
- [84]. Rizal, N.H., Hezmi, M.A., Ramadhan Razali R. & Wahab N.A. (2022). Effect Of Lime On The Compaction Characteristics Of Lateritic Soil In Utm, Johor.
- [85]. Scheffe, H. (1958). Experiments With Mixtures. Journal Of The Royal Statistical Society: Series B (Methodological), 20(2), 344-360.
- [86]. Scott, E M And Harold, D. D (2004), Designing Experiments And Analyzing Data: A Model Comparison. Lawrence Erlbaum Association Publisher, England.
- [87]. Scott, E.M. And Harold, D.D (2004), Designing Experiments And Analyzing Data: A Model Comparism. Lawrence Erlbaum Association Publisher, England.

- [88]. Seshadric S.T And Srinivasa R.P (2008), Relationship Between Compressive, Split Tensile, Flexural Strength Of Self Compacted Concrete. International Journal Of Mechanics And Solids Vol. 3 No 2 Pp 157 - 168
- [89]. Seshadric S.T And Srinivasa R.P (2008), Relationship Between Compressive, Split Tensile, Flexural Strength Of Self-Compacted Concrete. International Journal Of Mechanics And Solids. Vol.3 No 2 Pp157-168.
- [90]. Sharmin R Ahmed M, Mohiuddin A And Forhat A.L. (2006). Comparision Of The Strength Performance Of Concrcte With Uncrushed Or Curshed Coarse Aggregate Arpn J. Appl. Sci., 1 (2): 1-4
- [91]. Sharmin R., Ahmed M, Mohiuddin A And Forhat A.L. (2006). Comparison Of The Strength Performance Of Concrete With Uncrushed Or Crushed Coarse Aggregate Arpn J. Appl. Sci., 1(2): 1 -4.
- [92]. Shen, M., Zhou, Z., & Zhang, S. (2021).Effect Of Stress Path On Mechanical Behaviour Of Frozen Subgrade Soil. Road Materials And Pavement Design 23 (8)1-30.
- [93]. Shetty, M. S. (2002). Concrete Technology: Theory And Practice. 5th Ed. India: Rajendraravindra Printers.Ingraffea, A. R. (Eds). Martiniusnijhoff Publishers, Dordrecht, Theneiderlands.
- [94]. Shetty, M.S (2003), Concrete Technology Theory And Practice, S Chand And Company Limited, New Delhi, India.
- [95]. Shetty, M.S. (2003), Concrete Technology Theory And Practice, S Chand And Company Limited, New Delhi, India.
- [96]. Soroka, I. (1993). Concrete In Hot Environments. Modern Concrete Technology Series. 139.
- [97]. Staat,M.(2021). An Extansion Strain Type Mohr-Criterion. Rock Mechanics And Rock Engineering.
- [98]. Stella L.M (1996). Ancient Concrete Structure, Concrete International 18 (1) Pp 56-58
- [99]. Stella L.M (1996). Ancient Concrete Structures, Concrete International Is (1) Pp 56 - 58
- [100]. Tamassoki, S., Norsyahariati, N. D., Jakami, F.M. & Kusun F.M. (2022). Compressive And Shear Strengths Of Coir Fibre Reinforced Activated Carbon Stabilized Lateritic Sopl Sustainability 14(15):9100.
- [101]. U.S Federal Highway Administration (1999), "Admixtures". Retrieved 23rd June 2008 From [Http://Www.En.Wikipedia.Org/Niki/Federalhighway Administration](http://www.en.wikipedia.org/wiki/Federalhighway_administration).
- [102]. U.S Federal Highway Administration (1999),"Admixtures" Retrieved 23rd June 2008 From [Http://Www.En.Wikipedia.Org/Niki/Federalhighway Administration](http://www.en.wikipedia.org/wiki/Federalhighway_administration).
- [103]. Veretennykov, V.I. Yugov, A.M Dolmatov, A.O; Bulavytskyi, M.S; Kukharev, D.I. Bulavytskyi, A.S. (2008) "Concreteinhomogeneity Of Vertical Castin-Place Elements In Skeleton- Type Buildings". In Mohammed Ettouney.Aei 2008: Building Intergration Ssolutions. Reston, Virginia: American Society Of Engineers.
- [104]. Veretennykov, V.I; Yugov, A.M; Dolmatov, A.O; Bulavytskyi, M.S; Kukharev, D.I; Bulavytskyi, A.S (2008). " Concrete Inhomogeneity Of Vertical Cast- In-Place Elements In Skeleton-Type Buildings". In Mohammed Ettouney. Aei 2008: Building Integration Solutions. Reston, Virginia: American Society Of Civil Engineers.
- [105]. Wahab N. A., Rashid, A.S. Roshan, M.J., & Rizal N.H. (2021) Effect Of Cement On The Compaction Properties Of Lacteritic Soil Iop Conference Series Materials Science And Engineering 1153 (1) :012015,
- [106]. Wang Chen, M., Yan, L., & Hu, Y, (2019). Critical Depth Of Groundwater Recharge For Vegetation In Semi-Arid Areas. Iop Conference Series Earth And Environmental Science 237(3):032091.
- [107]. Wolpert D.H, Macready, W. G.(1995). No Free Lunch Theorems For Optimization.(Technical Report Sfi-Tr-05-010) Sanle Fe, Nm, References
- [108]. Wolpert, D.H, Macready, W.G (1995). No Free Lunch Theorems For Optimization. (Technical Report Sfi - Tr-05-010) Sanle Fe, Nm, Usa.