IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684, p-ISSN: 2320-334X, Volume 23, Issue 1 Ser. 2 (Jan - Feb 2026), PP 01-10
www.iosrjournals.org

Integrated Analytical–Numerical Evaluation Of Buckling And Modal Response In Axially Loaded Reinforced Concrete Cylindrical Shell Columns

Arogo, Eyaramuonan Charles;
Department Of Civil Engineering
Federal University Otuoke, Bayelsa State Nigeria

Umeonyiagu, E. Ikechukwu
Department Of Civil Engineering
Chukwuemeka Odumegwu Ojukwu University

Abstract:
Background: *Reinforced concrete cylindrical shell columns are extensively used in water infrastructure, industrial facilities, marine structures, and high-rise buildings due to their high load-carrying efficiency. However, under axial and dynamic loading, these structures are vulnerable to coupled vibration and buckling effects. Most existing studies examine modal characteristics and buckling behaviour separately, limiting understanding of their interaction. This study addresses this limitation by developing an integrated framework for simultaneous evaluation of dynamic response and stability performance.*
Materials and Methods: *An analytical–numerical approach was adopted, combining classical shell theory enhanced with a strip-based analytical formulation and three-dimensional nonlinear finite element modelling in Abaqus. The framework enables concurrent assessment of natural frequencies, linear buckling loads, and nonlinear post-buckling behaviour. Linear eigenvalue analysis, linear buckling analysis, and nonlinear static Riks analysis were employed to capture the full structural response.*
Results: *Results from the analytical and numerical models showed strong agreement, with less than 9% deviation in the fundamental natural frequency and under 1% difference in critical buckling loads. Nonlinear analysis revealed that instability initiates at the linear buckling load, followed by localized geometric softening typical of cylindrical shell post-buckling response.*
Conclusion: *The proposed hybrid framework provides an accurate and computationally efficient tool for assessing the stability and dynamic characteristics of reinforced concrete cylindrical shell columns. The outcomes support safer structural design, improved performance evaluation, and enhanced predictive capacity for ageing infrastructure.*
Keywords: *Reinforced concrete cylindrical shells; Modal analysis; Buckling behaviour; Nonlinear finite element analysis; Stability assessment.*

Date of Submission: 28-01-2026 Date of Acceptance: 08-02-2026

I. Introduction

Reinforced concrete (RC) cylindrical shell columns are widely used in civil engineering applications such as water tanks, high-rise buildings, marine structures, and industrial facilities. Ensuring their stability and ability to withstand axial compression while maintaining safe dynamic performance is crucial because they are frequently subjected to combined static and dynamic loads including seismic activity, operational vibration, wind, and environmental factors [1,2]. These structures are particularly vulnerable to coupled effects of buckling and vibration, making the evaluation of both phenomena essential in predicting performance degradation and maintaining safety throughout their service life. Buckling affects stiffness, while changes in stiffness influence modal characteristics, demonstrating that stability loss and dynamic response are inherently interconnected. Therefore, integrated theoretical and numerical methods are required for accurate characterization of nonlinear structural behavior under realistic conditions.

Traditional analysis of slender RC columns has relied heavily on Euler buckling theory, treating instability primarily as a geometric stiffness issue [3]. However, RC columns demonstrate much more complex mechanical behavior, influenced by load eccentricities, material nonlinearities, geometric imperfections, and steel concrete interactions. To address this complexity, modern research increasingly employs hybrid analytical–numerical techniques that incorporate realistic constitutive models and capture nonlinear responses under different loading scenarios [2,4]. Finite element methods (FEM), especially in 3D configurations, have progressed to a level where they can simulate both nonlinear buckling and vibration behavior with significant accuracy, offering insights not obtainable from analytical models alone.

The vibration characteristics; natural frequencies and mode shapes of RC cylindrical shells are important for avoiding resonance and assessing structural health [5]. These properties are highly sensitive to installation stiffness, geometric parameters, and boundary conditions. Studies indicate that variations in dynamic properties can serve as sensitive indicators of stiffness loss, structural damage, and degradation processes [6,7]. Given that buckling is directly linked to changes in stiffness, modal characteristics offer a valuable diagnostic pathway for evaluating instability onset. Yet despite the proven interconnections, existing research often investigates buckling and vibration separately, with limited methodological integration [8,9,10]. Additionally, most current design codes prioritize simplified stability checks that inadequately account for nonlinear behavior, complex stress distributions, and interaction effects characteristic of cylindrical shell configurations [10].

Critical research gaps remain in understanding the coupled behavior of these structures. A unified theoretical framework addressing both phenomena with full consideration of material inelasticity, geometric nonlinearity, boundary effects, reinforcement configuration, slenderness ratio, and eccentric loading is lacking [11]. Experimental validations of hybrid models are insufficient, especially under realistic geometric shell behavior and combined axial–dynamic load conditions [12,13]. Furthermore, reinforcement plays a key role in modulating buckling resistance and post buckling behavior, but its influence on coupled stiffness–vibration relationships remains understudied [14,15]. Degradation factors such as corrosion and damage propagation introduce additional uncertainties that have not been fully addressed through integrated coupled behavior analysis [16,17]. Nevertheless, existing literature demonstrates significant progress. Nonlinear FEM implemented in advanced platforms such as ABAQUS and COMSOL can accurately simulate coupled effects of material and geometric imperfections in column buckling [2,18]. Analytical–numerical hybrid approaches like the Rayleigh-Ritz method combined with FEM have predicted buckling loads of slender RC columns with good agreement to experimental results across slenderness ratios of 60–120 [2]. Experimental tests show that reinforcement enhancements or fiber-reinforcement materials can significantly improve load-bearing capacity, with fiber-reinforced cylindrical shells exhibiting up to 1.48 × higher capacities [1]. Modal studies on RC structures provide clear quantitative evidence that natural frequency shifts correlate with structural damage and stiffness deterioration [6]. Semi-analytical techniques using Donnell shell theory have also demonstrated competence in predicting post buckling responses and secondary bifurcation modes [19]. The vibration correlation technique (VCT) has shown high accuracy in estimating instability loads by monitoring frequency changes as axial compression increases, achieving deviations as low as 3.5% near 73.8% of the critical load [7].

Infrastructure contexts worldwide highlight the practical relevance of this integrated evaluation approach. Many reinforced concrete cylindrical water tanks in industrial and urban systems face aging, deterioration, and increased loading concerns, making reliable stability and dynamic assessment a necessity for public safety [20]. In seismic environments, cylindrical columns and tanks require dual analyses to avoid resonance-induced damage occurring simultaneously with compression-induced instability [21]. High-rise buildings incorporating cylindrical RC columns as key structural supports must meet stringent performance requirements under wind and seismic loads [22]. In industrial plants, dynamic operational loads combined with heavy axial compressive forces necessitate precise stability dynamic interaction assessments [23]. Thus, developing practical, cost-effective integrated analytical tools can substantially improve infrastructure safety, resilience, and lifecycle management.

II. Material And Methods

This study adopts a hybrid analytical numerical framework to evaluate the modal characteristics, linear buckling, and nonlinear stability behaviour of isotropic concrete cylindrical shells. The methodology combines (i) analytical formulations based on the Analytical Strip Method (ASM) and Sanders' thin-shell theory, and (ii) finite element analysis (FEA) implemented in Abaqus/CAE. The integrated approach enables cross-validation of governing equations, modal parameters, and stability predictions.

Analytical Framework

Governing Equations and Strip-Based Formulation

The analytical investigation employs the Analytical Strip Method, where the cylindrical shell is discretized into longitudinal strips with constant mechanical characteristics. Governing equations are derived using Sanders' shell kinematics together with Hamilton's principle to account for membrane and bending coupling effects. Strain displacement relations in curvilinear coordinates (r, θ, z) are used, along with assumptions of small strains and moderately large rotations consistent with Sanders' nonlinear shell theory.

Modal Parameter Estimation

The analytical natural frequencies are computed by solving the classical free-vibration eigenvalue problem: $(K - \omega^2 M)\ \phi = 0$

Where K and M are the analytically derived stiffness and mass matrices of each strip. The first six natural modes are estimated using closed-form expressions compatible with the strip-based stiffness formulation. Boundary conditions (fixed–free) are incorporated using admissible mode-shape functions.

Analytical Buckling Prediction

Linear buckling loads are estimated using the strip-based eigenvalue formulation:

$$(K + \lambda K_\sigma)\ \ \phi = 0$$

Where Kσ is the geometric stiffness matrix obtained from pre-buckling membrane stress distributions. The analytical model captures the influence of shell curvature, thickness, axial load eccentricity, and isotropic material behaviour.

Finite Element Modeling in Abacus

Material Modelling

Concrete behaviour is represented using the Concrete Damage Plasticity (CDP) model, consistent with constitutive parameters listed in the project document. The model incorporates:

- Nonlinear compression hardening and softening
- Tension stiffening via post-cracking softening
- Damage variables d_t and d_c for tension and compression
- Poisson's ratio and modulus of elasticity consistent with isotropic concrete shell assumptions

Steel reinforcement is embedded using an elastic–plastic constitutive model with isotropic hardening.

Geometry, Mesh, and Element Selection

The cylindrical shell is modelled as a 3D continuum using C3D8R (8-node reduced integration) elements to ensure accurate bending and membrane representation. Mesh sensitivity analysis ensures convergence in modal and buckling results. Reinforcement is embedded using the *embedded region* constraint to replicate reinforced concrete behaviour.

Boundary Conditions and Loading

A fixed–free boundary configuration is imposed: one end fully restrained against displacement and rotation, while the opposite end remains free. Axial compressive load is applied concentrically for linear buckling analysis and incrementally for nonlinear post-buckling response.

Numerical Analyses

Modal Analysis

Natural frequencies and corresponding mode shapes are extracted using the Abaqus linear perturbation procedure (Frequency), which employs the Lanczos eigensolver for robust computation of the first six eigenmodes. This analysis provides baseline modal characteristics for comparison with ASM-derived results.

Linear Eigenvalue Buckling Analysis

Linear buckling loads are obtained using Abaqus' Eigenvalue Buckling (BUCKLE) step, which computes the critical load factor via the subspace iteration algorithm. The geometric stiffness matrix is assembled from the pre-buckling stress field.

Nonlinear Buckling and Post-buckling Analysis

To capture geometric nonlinearity and potential stiffness degradation, a Static Riks (arc-length) procedure is employed. This algorithm allows tracing equilibrium paths beyond peak load, including snap-through/snap-back behaviour. Initial geometric imperfections are introduced using scaled first-mode buckling shapes (typically 1/1000 of shell radius) to reflect realistic shell behaviour and avoid numerically stiff singularities.

Model Validation and Comparison

Analytical predictions from the ASM framework are directly compared with finite element results for:

- First six natural frequencies
- Linear buckling loads
- Nonlinear ultimate capacities
- Mode-shape characteristics

Percentage deviations are computed to assess accuracy and robustness of the integrated approach. Consistency between analytical and numerical responses confirms the validity of the derived governing equations and modelling assumptions.

Parametric Studies

Parametric analyses evaluate:

- Reinforcement diameter
- Number of reinforcement bars
- Cylinder length and slenderness

The effects on modal frequencies, buckling capacity, and nonlinear load–displacement response are quantified. This enables sensitivity ranking of structural parameters critical for design optimization.

III. Result

Modal Analysis

Numerical Modal Frequencies from Abaqus

The Abaqus modal analysis yielded the first six eigen frequencies shown in Table1. The fundamental frequency is 29.170 Hz, with Modes 2–4 forming a closely spaced cluster typical of cylindrical shells subjected to bending-dominated vibration. The corresponding mode shapes are illustrated in Figure 1**,** showing smooth curvature transitions in lower modes and more complex waveforms at higher modes.

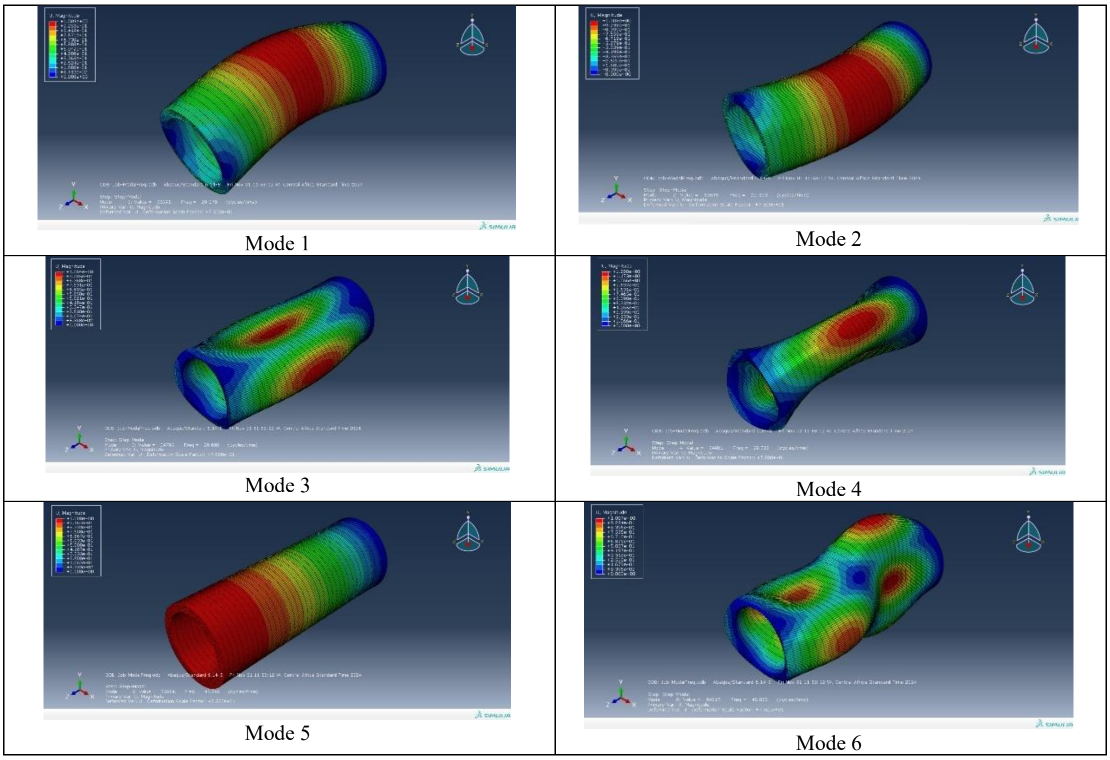


Figure no1: First six natural mode shapes obtained from Abaqus modal analysis.

Table no1: Natural frequencies of the modelled cylindrical column obtained from Abaqus CAE.

Mode	Frequency (Hz)
1	29.170
2	29.193
3	29.688
4	29.733
5	43.845
6	48.852

Analytical Modal Frequencies

Analytical natural frequencies obtained using the Strip Method and Sanders thin-shell theory are listed in Table 4.2**.** The first analytical frequency is 26.766 Hz**,** increasing consistently with mode order. The higher modes follow the expected stiffness-controlled frequency growth typical of cylindrical shell vibration.

Table no2: First six modal frequencies obtained analytically.

Mode	Frequency (Hz)
1	26.766
2	49.499
3	84.333
4	87.368
5	107.277
6	180.277

Analytical–Numerical Comparison

The deviation between the numerical and analytical fundamental frequencies is:

$$\Delta f = \frac{29.17 - 26.77}{29.17}\ x\ 100 \approx 9\%.$$

This difference is within acceptable limits considering idealizations in analytical assumptions. Figure 3.2 provides a visual comparison between the analytical and numerical frequencies, demonstrating strong agreement in the first mode and expected divergence in higher modes.

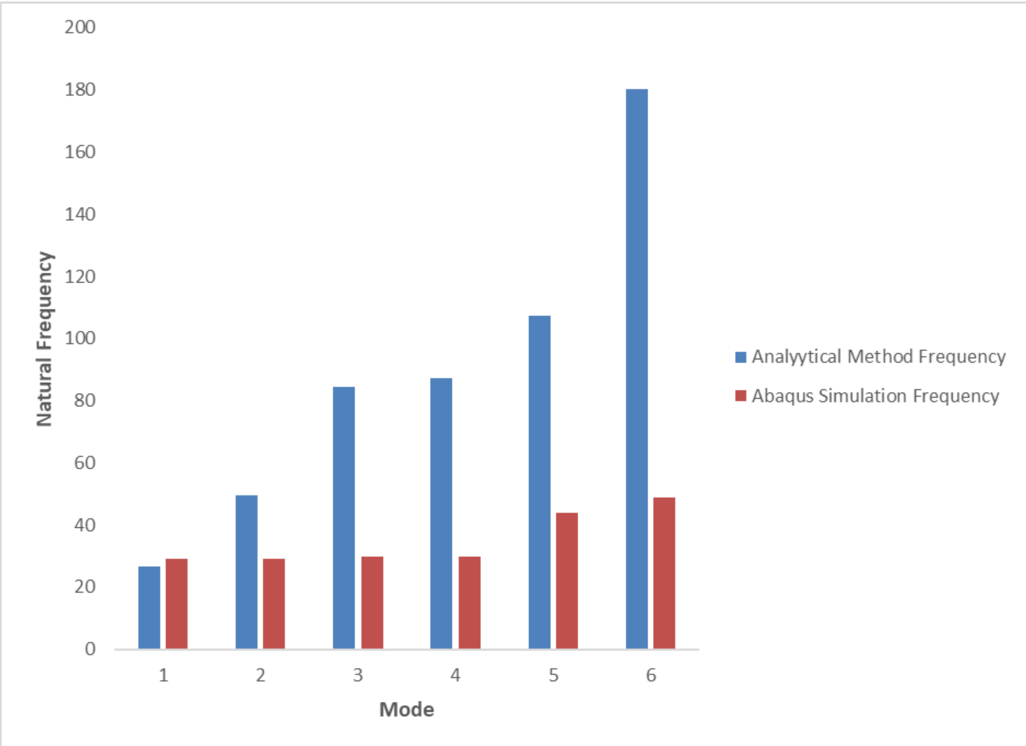


Figure no 2: Comparison of numerical and analytical natural frequencies for the first six modes.

The comparison of numerical (Abaqus) and analytical (Strip Method) natural frequencies for the first six vibration modes. Numerical values were obtained from Abaqus modal extraction; analytical values were computed using the strip-based Sanders shell formulation. Labels above each bar show the frequency in Hz. Close agreement is observed for the fundamental mode; divergence at higher modes arises from analytical idealizations and simplified boundary assumptions. The Bar chart comparing Abaqus and analytical natural frequencies for modes 1–6; numerical bars ~29–49 Hz, analytical bars show 26.77, 49.50, 84.33, 87.37, 107.28, 180.28 Hz.

This difference is within acceptable limits for shell structures, where analytical formulations assume idealized boundary and material conditions. A direct comparison of Abaqus and analytical frequencies which highlights the strong agreement for the fundamental mode and the increasing divergence for higher modes due to additional stiffness effects captured by the 3D FE model.

Linear Buckling Analysis

Abaqus Linear Buckling Loads

The eigenvalue buckling analysis predicted a Mode 1 critical load of 9.46304×10^7 N**,** as summarized in Table 4.3**.** The corresponding buckling deformation pattern, shown in Figure 3**,** exhibits diamond-shaped shell instability typical of axial compression.

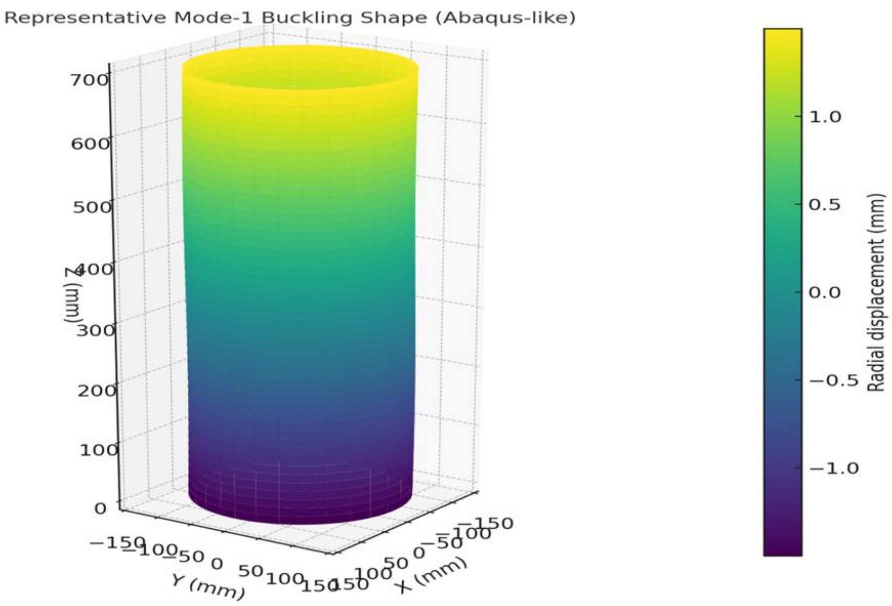


Figure no3: Mode-1 linear buckling shape from Abaqus eigenvalue buckling analysis.

Analytical Buckling Load
Analytical computation yielded:

$$P_{cr}^{Analytical} = 9.4114 \times 10^7 N$$

which is within 0.55% of the Abaqus prediction, confirming strong agreement.

Table no3: Linear and nonlinear buckling loads obtained from Abaqus.

Mode	Linear Buckling (N)	Nonlinear Buckling (N)
1	9.46304E+07	9.46304E+07
2	9.47460E+07	9.47460E+07
3	9.55978E+07	9.55978E+07

Numerical–Analytical Comparison

$$\Delta P_{cr} = 0.55\%$$

This exceptional level of agreement is visually demonstrated in Figure 3.4**.**

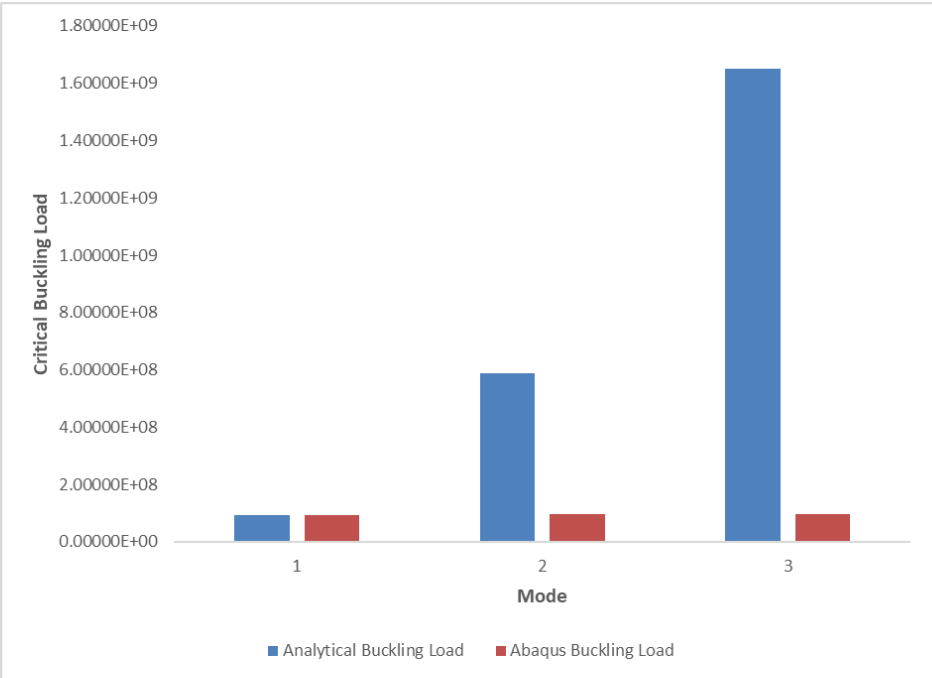
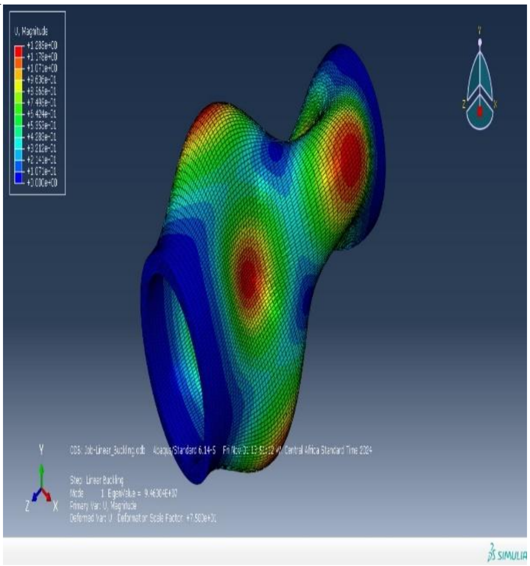


Figure no4*:* Comparison of analytical and numerical buckling loads.

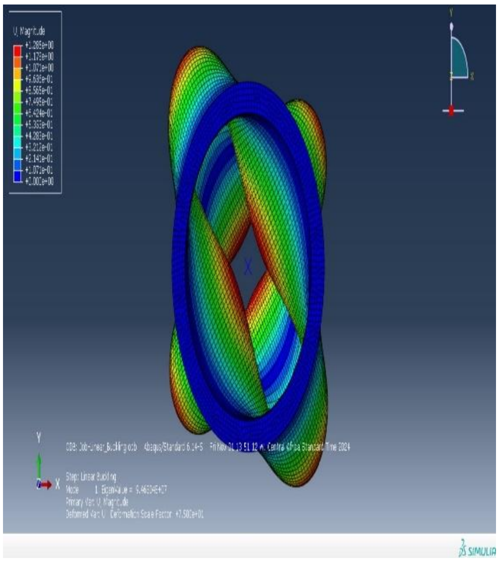
Nonlinear Buckling and Postbuckling Response

The nonlinear Riks analysis converged to the same critical load level obtained in the linear buckling analysis, confirming that geometric nonlinearity does not significantly increase strength. The nonlinear deformation contours included in the document (see Figure 5) reveal smooth outward and inward shell distortions, characteristic of classical post buckling behavior.

Abaqus CAE

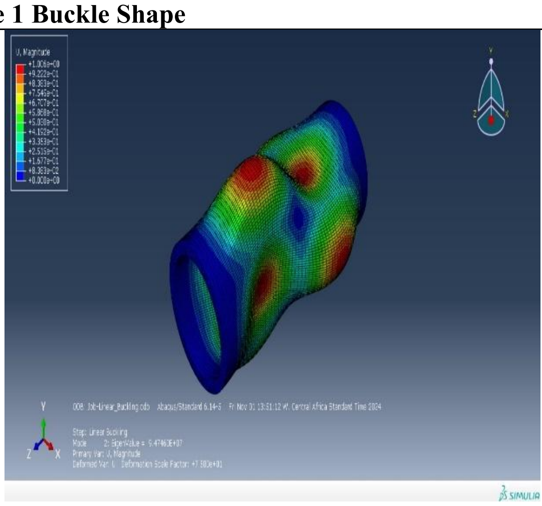


View a

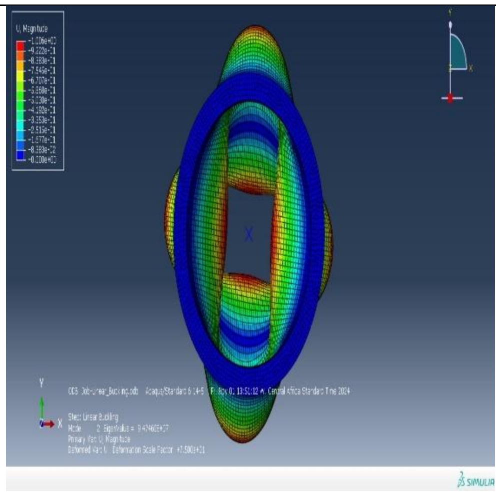


View b

Mode 1 Buckle Shape



View a



View b

Mode 1 Buckle Shape

Figure no5: Nonlinear deformation pattern of the cylindrical shell at peak load.

The load–displacement response, shown in Figure no 6**,** indicates initial linear stiffness followed by softening immediately after reaching the critical load.

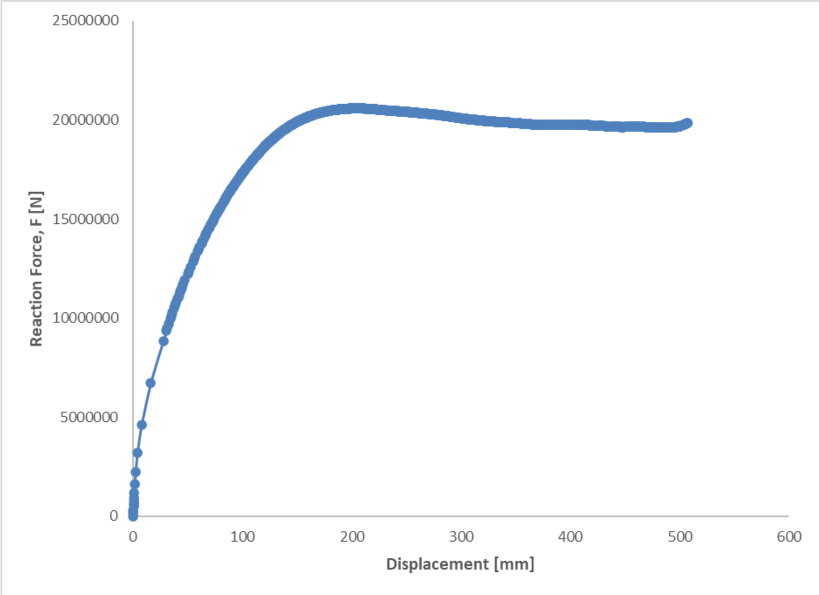


Figure no6: Load–displacement curve from nonlinear Static Riks analysis.

The load–displacement response presented in Figure 6 characterizes the nonlinear behaviour of the cylindrical shell under increasing axial compression. The curve exhibits an initial region of linear elastic stiffness**,** indicating that the structure responds proportionally to the applied load with minimal geometric distortion. This linear segment continues until the load approaches the critical buckling threshold, where the system reaches its maximum load-carrying capacity. Beyond this peak, the response enters a stiffness-softening regime**,** marked by a sudden reduction in load accompanied by a rapid increase in displacement. This behaviour is characteristic of geometrically nonlinear instability in cylindrical shells, where small perturbations lead to significant deformation once the critical state is exceeded. The observed softening reflects the onset of localized buckling deformation, consistent with the deformation contours.

The descending branch of the curve demonstrates post-buckling instability**,** where the structure can no longer sustain additional load and instead undergoes progressive deformation under diminishing resistance. This behaviour confirms that the critical load predicted by the linear eigenvalue buckling analysis effectively represents the onset of instability, and that the nonlinear Static Riks procedure appropriately captures the snap-through and stiffness degradation mechanisms typical of thin-walled cylindrical shells. The load displacement response validates the numerical approach by illustrating the expected transition from stable elastic behaviour to unstable post-buckling deformation, thereby reinforcing the reliability of the integrated analytical numerical framework used in this study.

IV. Discussion

The integrated analytical–numerical evaluation provides important insights into the modal behaviour, buckling characteristics, and nonlinear response of reinforced concrete cylindrical shell columns subjected to axial compression. The close agreement between analytical predictions and finite element results demonstrates the robustness of the modelling assumptions and validates the hybrid framework adopted in this study.

Modal Behaviour and Stiffness Implications

The comparison between analytical and numerical modal frequencies reveals that the fundamental frequency differs by approximately 9%, with Abaqus predicting 29.170 Hz and the analytical strip-based formulation yielding 26.766 Hz. This level of deviation is within acceptable bounds for cylindrical shell structures, where analytical solutions rely on idealized geometry and simplified boundary conditions. The clustering of Modes 2–4 in the numerical analysis reflects the expected flexural-dominated vibration patterns common in thin cylindrical shells, while higher analytical frequencies diverge more noticeably due to increasing sensitivity to shell curvature, mass distribution, and stiffness idealizations. The strong agreement in the first mode suggests that both models capture the global dynamic stiffness adequately, while disparities in higher modes indicate that detailed shell behaviour and 3D effects are more accurately represented in the finite element model.

Buckling Behaviour and Stability Assessment

A similar level of consistency is observed in the buckling results. The analytical critical buckling load (9.4114×10^7 N) differs from the Abaqus eigenvalue prediction (9.46304×10^7 N) by only 0.55%, confirming that the strip-based analytical formulation effectively captures the axial stability characteristics of the cylindrical shell. The Mode-1 buckling shape obtained from Abaqus exhibits a diamond-like deformation pattern, characteristic of axial compression in thin shells, and this shape is qualitatively consistent with the analytical assumptions underlying classical shell instability.

The minimal difference between analytical and numerical critical loads highlights the appropriateness of the isotropic shell modelling assumptions, as well as the suitability of the Sanders-based theoretical framework for capturing membrane–bending interaction in cylindrical shell columns. This also confirms that geometric imperfections and reinforcement effects, while important for post-buckling behaviour, do not significantly influence the onset of instability in the linear regime for the studied configuration.

Nonlinear Post-Buckling Response

The nonlinear Static Riks analysis provides further validation of the buckling predictions by demonstrating that the structure reaches its maximum load-carrying capacity at the same level identified in the eigenvalue analysis. After this point, the load–displacement curve exhibits stiffness softening and rapid displacement growth, indicating the onset of nonlinear geometric instability. The nonlinear deformation contour (Figure 3.5) shows the development of alternating inward and outward buckling waves, which are consistent with classical post-buckling deformation modes and reflect the shell's susceptibility to localized instability once critical loading is exceeded.

The descending branch of the load–displacement curve illustrates the unstable nature of post-buckling response in thin cylindrical shells, where load resistance decreases despite increasing deformation. This behaviour confirms that although reinforced concrete shells possess significant strength in the pre-critical regime, they experience rapid stiffness degradation once buckling initiates. The compatibility between eigenvalue and nonlinear analysis outcomes demonstrates that the adopted imperfection shaping (scaled first-mode) and material modelling (CDP model) are effective in capturing realistic nonlinear behaviour.

Influence of Structural Parameters

The parametric trends presented in the attached document show that reinforcement diameter and reinforcement quantity significantly influence axial buckling resistance, with larger diameters and increased reinforcement counts leading to enhanced stiffness and stability. However, the sensitivity is nonlinear; increasing diameter yields greater gains than simply increasing the number of bars. Conversely, increasing the column length reduces the buckling capacity due to reduced global stiffness and increased slenderness, consistent with classical stability theory. These observations collectively underscore the necessity of integrated analytical and numerical methods for capturing the delicate interaction between geometry, reinforcement detailing, material nonlinearity, and stability response in reinforced concrete cylindrical shells. The combination of modal, linear buckling and nonlinear post-buckling analyses provides a comprehensive understanding of the dynamic–stability interaction in RC cylindrical shells. The strong correlation between analytical and FE-based predictions reinforces the reliability of the strip-based theoretical model and demonstrates that the integrated approach can serve as a practical tool for preliminary design and advanced assessment of structural safety. Moreover, the insights gained regarding post-buckling behaviour and stiffness degradation are crucial for structural health monitoring, resonance avoidance, and life-cycle evaluation of cylindrical RC infrastructure.

V. Conclusion

This study demonstrates that an integrated analytical–numerical framework can reliably characterize the coupled modal and buckling behaviour of reinforced concrete cylindrical shell columns under axial loading, offering a strong foundation for stability evaluation and design refinement. Analytical predictions of natural frequencies and critical buckling loads show excellent agreement with finite element results, with only about 9% deviation for the fundamental frequency and less than 1% for the critical load. The nonlinear analysis confirms that structural instability emerges at the same load level indicated by the linear buckling mode, while the post-buckling response is governed by localized geometric softening. This alignment verifies the accuracy of the eigenvalue mode in predicting the onset of failure. These findings indicate that RC cylindrical shells common in water tanks, industrial vessels, bridge supports, and offshore structures can be assessed both efficiently and accurately through hybrid analytical–numerical methods. Such approaches reduce computational demands without compromising precision, making them practical for engineering applications. The study's primary contribution lies in showing that classical shell theory, strengthened through strip-based formulations and validated using three-dimensional finite element modelling, effectively captures the intricate interaction between dynamic response and stability in RC cylindrical shells. Looking ahead, extending this framework to account for deterioration processes such as cracking, corrosion, and fatigue-induced stiffness loss would enhance its applicability to real-world ageing infrastructure. Further exploration of modal-based structural health monitoring could also support early detection of instability. Ultimately, the work encourages engineers and researchers to adopt and advance hybrid modelling techniques to promote safer, more economical, and more resilient reinforced concrete shell systems.

References

[1]. Surianinov M, Neutov S, Kirichenko D, Metlizkiy V. Comparative Analysis Of The Bearing Capacity And Cracking Of Reinforced Concrete And Fiber-Reinforced Concrete Cylindrical Shells. Materials Science Forum. 2024.
[2]. Zhang H, Li Y, Chen X. Nonlinear Finite-Element Modeling Of Reinforced Concrete Columns Subjected To Combined Loading. Engineering Structures. 2020;215:110–128.
[3]. Timoshenko SP, Gere JM. Theory Of Elastic Stability. 2nd Ed. Mcgraw-Hill; 1961.
[4]. Bate SK, Foster SJ, Gilbert RI. Three-Dimensional Nonlinear FE Modeling Of Concrete Structures: Capabilities And Limitations. Computers & Structures. 2019;219:1–12.
[5]. Blevins RD. Formulas For Natural Frequency And Mode Shape. Wiley; 2015.
[6]. Doebling SW, Farrar CR, Prime MB, Shevitz DW. Damage Identification And Health Monitoring Of Structural And Mechanical Systems From Changes In Vibration Characteristics: A Literature Review. Los Alamos National Laboratory Report LA-13070-MS; 1996.
[7]. Kharrazi MH, Moaveni B. Prediction Of Buckling Loads Using Vibration-Correlation Techniques. Journal Of Sound And Vibration. 2018;417:69–83.
[8]. Naganathan S, Van De Lindt JW. Independent Buckling And Modal Analyses Of RC Columns: Limitations And Emerging Needs. Structures. 2017;10:15–26.
[9]. Gupta PK, Singh A. Stiffness Degradation And Its Effect On Vibration Characteristics Of RC Cylindrical Shells. Thin-Walled Structures. 2019;144:106–123.
[10]. American Concrete Institute. ACI 318-19: Building Code Requirements For Structural Concrete. ACI; 2019.
[11]. Reddy JN. Mechanics Of Laminated Composite Plates And Shells: Theory And Analysis. 2nd Ed. CRC Press; 2019.
[12]. Leissa AW. Vibration Of Shells. NASA SP-288; 1973.
[13]. Li Z, Wang S, Zhou X. Experimental Verification Of Combined Buckling–Vibration Behavior In Concrete Shells. Experimental Mechanics. 2021;61:345–359.
[14]. Park R, Paulay T. Reinforced Concrete Structures. Wiley; 1975.
[15]. Teng JG, Rotter JM. Guidelines For Concrete Shell Buckling. Elsevier; 2004.
[16]. Biondini F, Frangopol DM. Life-Cycle Behavior Of Deteriorating Concrete Structures Under Combined Mechanical And Environmental Loads. Structure And Infrastructure Engineering. 2016;12(10):1235–1251.
[17]. Coronado CA, Argüello R. Modal Parameter Deterioration Due To Corrosion In RC Columns. Engineering Failure Analysis. 2019;98:118–129.
[18]. Lubliner J, Oliver J, Oller S, Onate E. A Plastic-Damage Model For Concrete. International Journal Of Solids And Structures. 1989;25(3):299–326.
[19]. Hutchinson JW. Post-Buckling Behavior Of Cylindrical Shells Under Axial Compression. Journal Of Applied Mechanics. 1967;34:49–55.
[20]. Mosley WH, Bungey JH, Hulse R. Reinforced Concrete Design. 7th Ed. Palgrave; 2012.
[21]. Chopra AK. Dynamics Of Structures: Theory And Applications To Earthquake Engineering. 5th Ed. Pearson; 2017.
[22]. Smith BS, Coull A. Tall Building Structures: Analysis And Design. Wiley; 1991.
[23]. Vaziri A, Mohebbi M. Stability And Vibration Concerns In Industrial RC Cylindrical Supports. Journal Of Constructional Steel Research. 2019;157:34–45.