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Vibration Study Of Sandwich Structure Using Equivalent Finite Element Model

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Abstract:
The sandwich structure is a type of structure widely used in many fields, especially in modern engineering and industry, due to its lightweight yet high stiffness and durability. The study of the mechanical behavior of sandwich structures has attracted the attention of many researchers. However, the heterogeneous and complex characteristics of the core make it difficult and costly to construct simulation models and perform direct simulations. This study proposes the use of an equivalent homogeneous model to replace the 3D sandwich panel structure with a 2D panel with comparable mechanical properties, thereby constructing a simpler finite element model that still accurately reflects the dynamic response. The dynamic response analysis results show that the equivalent model has higher accuracy than the 3D model. This is an important basis for applying the homogeneous model in studying the dynamic response of systems consisting of multiple sandwich panel structures.
***Key Word**: Sandwich structures; Equivalent model; Finite element; Vibration; Numerical simulation.*

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I. Introduction

Sandwich panels are a type of lightweight structure with a high stiffness-to-weight ratio, commonly used in fields such as aerospace, automobiles, marine, and construction. One of the important properties of this type of structure is its ability to reduce vibration and resist impact, which plays a key role in protecting the structure and increasing the working life of the system. Therefore, studying the vibration behavior of sandwich panels is essential in the design and evaluation of structures.

In recent years, there have been many studies on the natural vibration analysis of sandwich panels using different types of cores, such as foam cores, honeycomb cores, or truss cores. The methods used include analytical methods, energy methods, and finite element methods (FEM). For example, studies [1]–[9] have focused on the bending vibration analysis of foam core and honeycomb sandwich panels, with different boundary conditions and geometries. These studies mostly modeled the sandwich structure as a laminated panel or used equivalent models to simplify the problem. However, compared to the popular core types mentioned above, research related to corrugated core sandwich panels is still relatively limited. The corrugated core structure offers many advantages, such as high load-bearing capacity in one direction, good energy absorption capacity, and ease of fabrication, but causes complexity in modeling due to anisotropic properties and nonlinear geometry. Lok and Cheng [10] were one of the first research groups to analyze the free vibration of a truss sandwich panel, in which both the face and core are made of aluminum alloy. In this study, the sandwich panel was modeled as an equivalent homogeneous plate using the energy method to determine the natural frequencies. The results showed a high compatibility between the theoretical model and the numerical simulation using 2D and 3D finite elements. In a subsequent study, Lok and Cheng [11] continued to investigate the dynamic behavior of a four-sided, uniformly loaded aluminum truss sandwich panel. The results were also compared with the equivalent model to check the accuracy of the approach. More recently, Lou et al. [12] focused on the natural vibration analysis of stainless steel beams and sandwich panels with pyramidal truss cores. The truss core in this study was replaced by an equivalent homogeneous material layer, and the vibration frequencies were calculated under different boundary conditions, including single supports and clamps.

In summary, although there have been some efforts in studying the natural vibrations of sandwich panels with atypical cores such as truss cores or pyramid cores, in-depth studies on sandwich panels using corrugated cores, especially with homogenization modeling and finite element analysis, are still relatively rare. Therefore, the development of accurate and efficient models for vibration analysis of this type of structure is a research direction of high practical and academic significance.

II. Materials And Methods

Research materials

Corrugated cardboard is among the most commonly employed packaging materials for the fabrication of boxes and interlayer pads used in the transportation of goods. Its manufacturing process induces a pronounced anisotropy, resulting in three principal material directions: the machine direction (MD), the cross direction (CD), and the through-thickness or Z-direction (ZD) (Figure 1). The in-plane properties of cardboard can be determined quite easily through tensile tests. However, due to the small thickness, the determination of the out-of-plane properties (perpendicular to the plane) is more difficult. According to Stenberg [13], the Young's modulus of elasticity in the direction perpendicular to the plane (ZD direction) is about 200 times lower than that in the main fiber direction (MD). In addition, Stenberg et al. [14] found that the in-plane deformation is negligible when the sample is compressed in the thickness direction, resulting in Poisson's ratios ν_{xz} and ν_{yz} being close to zero. The research object in this study is corrugated core cardboard type B. The geometrical dimensions of the cardboard are shown in Figure 2. The properties of the paper layers are shown in Table 1.

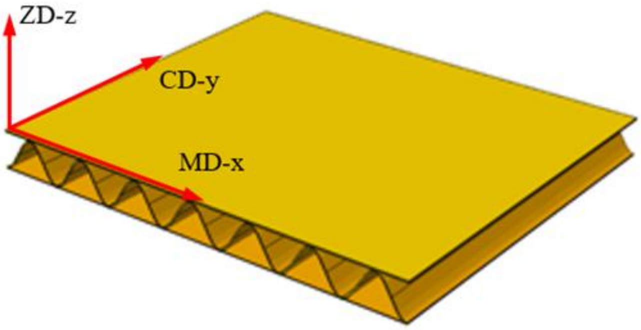

Figure 1. *Corrugated core cardboard structure*

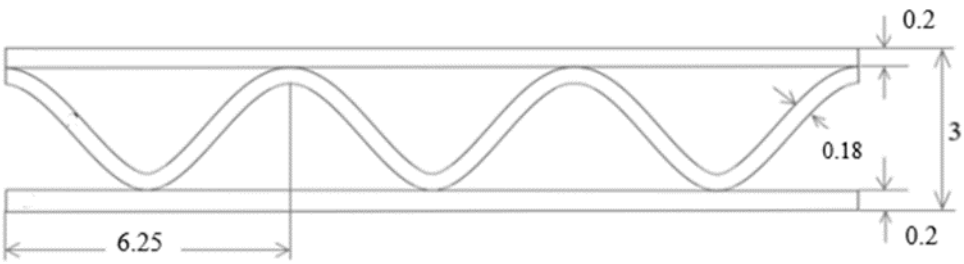

Figure 2. *Geometric dimensions of corrugated cardboard type B*

Table 1. *Weight and thickness of paper layers*

Layer	Weight (g/m^2)	Thickness (mm)
Skins	150	0.2
Flute	115	0.18

Paper is an anisotropic material that follows the IPE behavior model [15]. Accordingly, an inverse identification procedure is performed to calibrate the parameters of the constitutive model, based on minimizing the discrepancy between experimental observations and numerical simulations. The optimization relies on a scalar least-squares objective function, as defined in Equation (1). The minimization of this quadratic difference was carried out using the mode optimization software FrontierTM combines the finite element software ABAQUS. The workflow of the identification process is schematized in the context of the optimization software mode FrontierTM. There are currently many different standard optimization algorithms as Simplex, MOGA-II, Levenberg-Marquardt…ect or advanced optimization algorithms as MOSA, NGSA-II…ect, can be used depending on the complexity of the problem to be solved. In this study, we used the genetic optimization algorithm MOGA-II to minimize the objective function in equation (1). Uniaxial tensile tests were conducted on each constituent of the corrugated cardboard (skins and flute) in three principal orientations—machine direction (MD), cross direction (CD), and 45°—using a constant crosshead speed of 10 mm/min. All tests were performed under standardized environmental conditions of 23 °C and 50% relative humidity. Perform the reverse identification process to obtain the material parameter values as shown in Figure 3 and Tables 2 and 3.

$$F_{obj} = \frac{1}{N}\sum_{i=1}^{N}\left(F_{num}\left(t_i\right) - F_{\exp}\left(t_i\right)\right)^2 \quad (1)$$

Where N is the number of data set, t_i denotes the time of the corresponding experimental point i and F_{num} and F_{exp} are the indenter forces numerically computed and experimentally measured, respectively

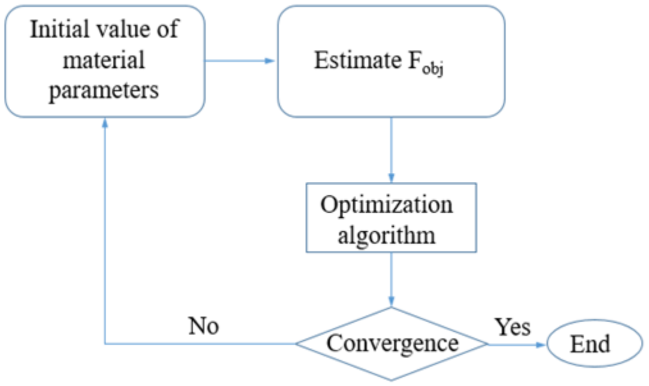

Figure 3. *Reverse engineering of paper material parameters*

Table 2. *Elastic properties of paper*

Layer	E_x (MPa)	E_y (MPa)	ν_{xy}	G_{xy} (MPa)	E_0
Skins	2357.6	762.23	0.084	1027.2	90.12
Flute	1171.1	612.05	0.070	303.15	80.03

Table 3. *Plastic properties of paper*

Layer	n	A	B	C	D	ε_0
Skins	3.708	1.0	2.106	2.116	1.451	0.44e-3
Flute	3.017	1.0	2.708	2.116	1.582	0.9e-3

Equivalent finite element model

The developed homogenization models are applied in this study [6],[9],[16]–[20] to study the vibration behavior of corrugated core cardboard panels. Accordingly, a 3D structure of the sandwich panel is converted into a 2D panel (Figure 4) through the calculation of equivalent stiffnesses. Consider a corrugated core sandwich panel as shown in Figure 1. To construct an equivalent homogenization model, extract a representative volume element (VER) as shown in Figure 5.

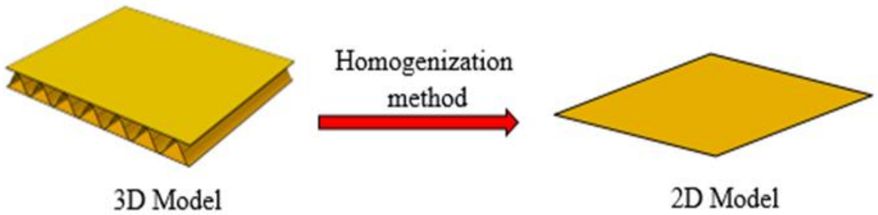

Figure 4. *Homogenization method*

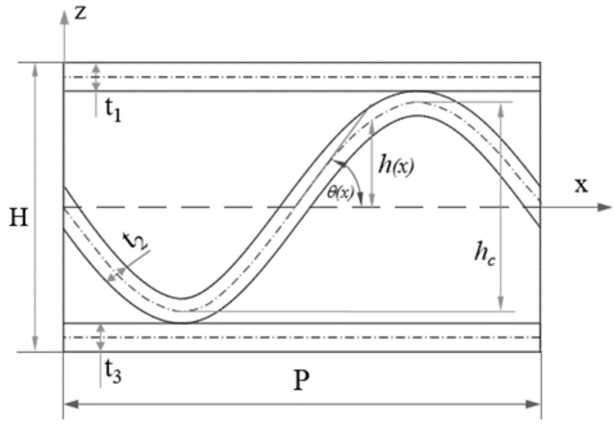

Figure 5. *Representative elements of corrugated core sandwich panels*

The geometric shape of the corrugated core is determined by formulas (2) and (3).

$$h(x) = \frac{h_c}{2}\sin(\frac{2\pi x}{P}) ; \qquad \theta(x) = \arctan(\frac{dh(x)}{dx}) \tag{2}$$

According to the theory of multilayer plate, A_{ij} represents the stiffness of the membrane, B_{ij} represents the terms of coupling between membrane and bending-torsion, D_{ij} represents the stiffness of bending and torsion, and F_{ij} represents the stiffness of transverse shears.

$$[A] = \int_{-h/2}^{h/2} Q_{ij}dz \Rightarrow A_{ij} = \sum_{k=1}^{3} Q_{ij}t_k = Q_{ij}^{(1)}t_1 + Q_{ij}^{(2)}(\theta(x))t_2^{'} + Q_{ij}^{(3)}t_3 \tag{3}$$

$$[B] = \int_{-h/2}^{h/2} zQ_{ij}dz \Rightarrow B_{ij} = \sum_{k=1}^{3} Q_{ij}z_k t_k = Q_{ij}^{(1)}z_1t_1 + Q_{ij}^{(2)}(\theta(x))z_2t_2^{'} + Q_{ij}^{(3)}z_3t_3 \tag{4}$$

$$[D] = \int_{-h/2}^{h/2} z^2Q_{ij}dz \Rightarrow D_{ij} = \sum_{k=1}^{3} Q_{ij}(z_k^2t_k + \frac{1}{12}t_k^3) = Q_{ij}^{(1)}(z_1^2t_1 + \frac{1}{12}t_1^3) + Q_{ij}^{(2)}(z_2^2t_2^{'} + \frac{1}{12}t_2^{'3}) + Q_{ij}^{(3)}(z_3^2t_3 + \frac{1}{12}t_3^3) \tag{5}$$

$$F_{ij} = \frac{5}{6}(C_{ij}^{(1)}t_1 + C_{ij}^{(2)}(\theta(x))\,t_2^{'} + C_{ij}^{(3)}t_3) \tag{6}$$

$$\text{with } t_2^{'} = \frac{t_2}{\cos\theta(x)}$$

The membrane force, bending, and torsional moments are obtained by integrating the stresses over the area to obtain the overall stiffness matrix. This homogenization model is implemented in ABAQUS using the UGENS subroutine.

$$A_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)}w_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}(\theta(x))w_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)}w_k \tag{7}$$

$$B_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)}z_kw_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}(\theta(x))z_kw_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)}z_kw_k \tag{8}$$

$$D_{ij}(x) = \frac{t_1}{2}\sum_{k=1}^{3} Q_{pij}^{(1)}z_k^2w_k + \frac{t_2^{'}}{2}\sum_{k=1}^{3} Q_{pij}^{(2)}(\theta(x))z_k^2w_k + \frac{t_3}{2}\sum_{k=1}^{3} Q_{pij}^{(3)}z_k^2w_k \tag{9}$$

III. Results And Discussion

The cardboard sheet is subjected to impact in the CD direction

An impact experiment in the CD direction for a corrugated cardboard board was carried out. The boundary conditions are shown in Figure 6. A mass of 0.5 kg is placed on the rigid board above. An acceleration with a trapezoidal amplitude A = 0.5g acts vertically for a very short time (0.012s). The deformations of the plate after the vertical impact obtained from the 3D model and the H-2D model are shown in Figure 7. Figure 8 shows the excitation acceleration amplitudes as well as the responses obtained in the two models. The 3D model simulation required 229 seconds of CPU time, but the simulation with the H-2D model required only 49 seconds (a difference of 4.67 times). The difference between the response acceleration amplitudes was about 0.498% (Table 4).

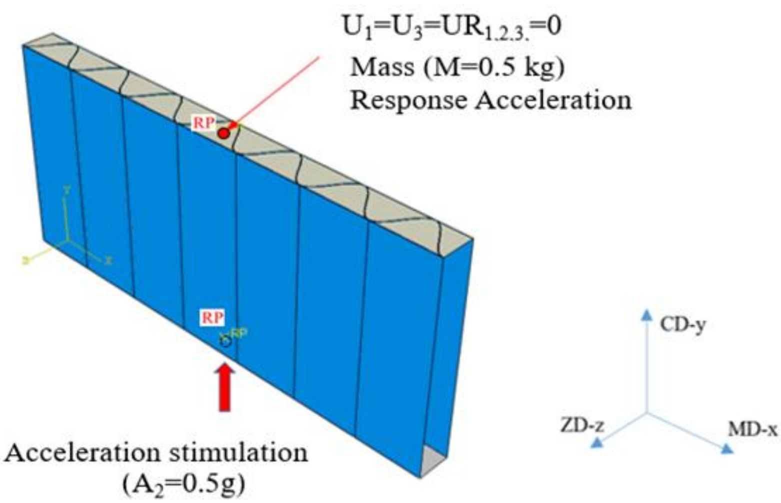

Figure 6. *Sandwich panel subjected to vertical impact in the CD direction*

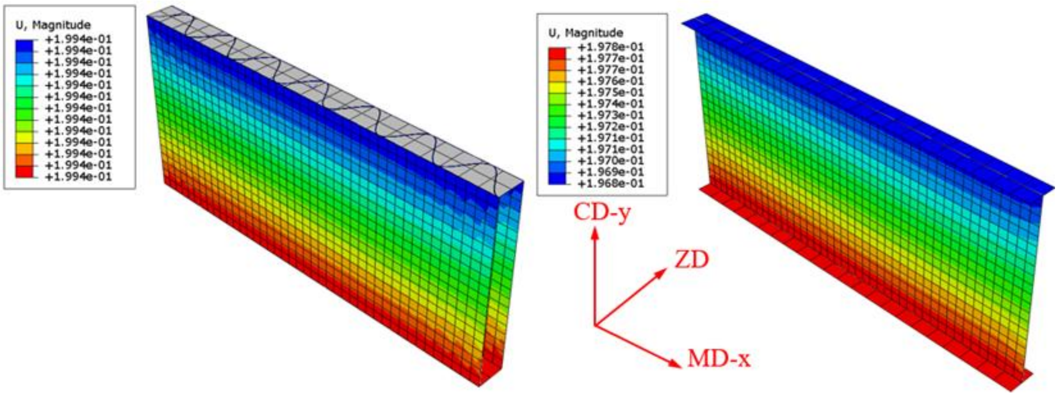

Figure 7. *Deformation of the plate after the vertical collision of a 3D model and 2D model*

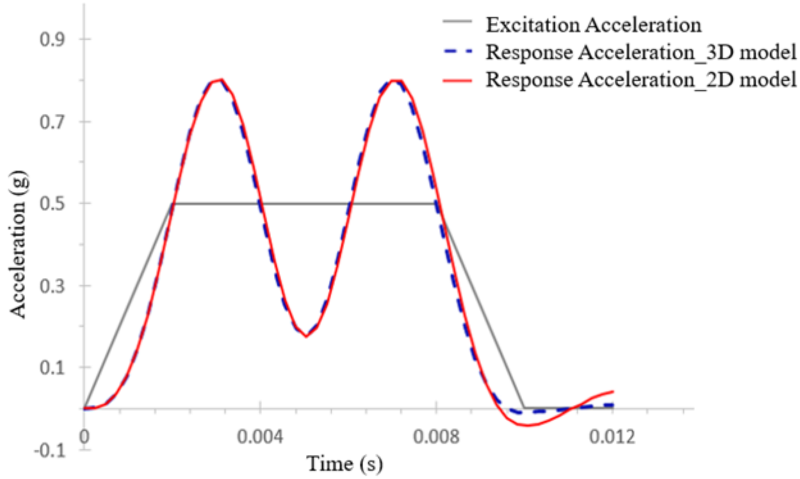

Figure 8. *Excitation, acceleration, and response of two models*

Table 4. *Comparison of the acceleration responses of the two models*

	Response acceleration (g)	CPU time (s)
3D model	0.802	229
2D model	0.798	49
Error	0.498 %	4.67 times

Vibration of a cardboard sheet under bending

Consider a cantilever beam, with dimensions of 100 mm in length and 24.8 mm in width, subjected to a periodic vertical acceleration (direction ZD) with amplitude A = 1g and frequency f = 0.8 Hz. The feedback acceleration at a point in the middle of the beam is selected for study (Figure 9).

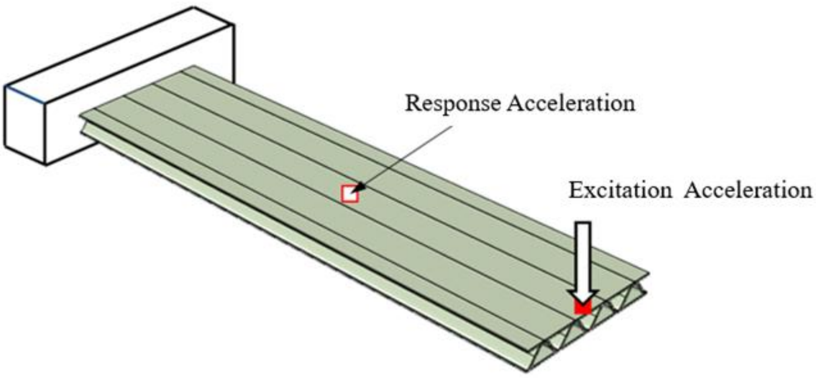

Figure 9. *Experimental model simulation of response acceleration measurement.*

In case the beam length is in the MD direction

The excitation cyclic acceleration with amplitude A=1g is applied to the free end of the beam. The response acceleration obtained at the beam midpoint for the 3D and H-2D models, respectively, is shown in Figure 10. Accordingly, the correlation between the two models is noticeable after the transient phase at the beginning of the simulation. The difference between the response acceleration is 10.2%, as shown in Table 5, and the CPU time is 12.5 times larger for the 3D model.

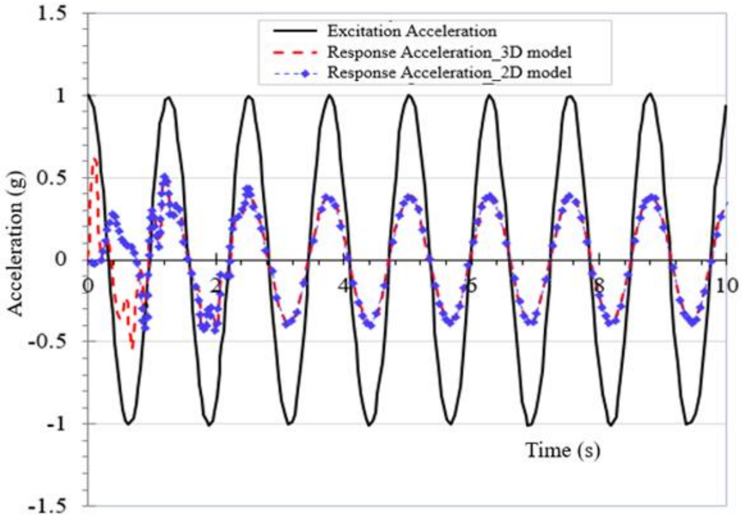

Figure 10. *Excited acceleration on the beam in MD direction and response of the two models*

Table 5. *Response acceleration and calculation time in the MD direction between the two models*

Model	Response acceleration (g)	CPU time (s)
3D model	0.380	932
2D model	0.341	74.6
Error	10.2%	12.5 (times)

In case the beam length is in the CD direction

Similar simulations are performed on the beam with a length in the CD direction. The results obtained are shown in Figure 11. The difference in response acceleration amplitude between the two models is 2%. The CPU time is 11.2 times greater for the 3D model (Table 6).

Table 6. *Comparison of response acceleration and CPU time in the CD direction between the two models*

Model	Response acceleration (g)	CPU time (s)
3D model	0.335	952
2D model	0.330	85
Error	2%	11.2 (times)

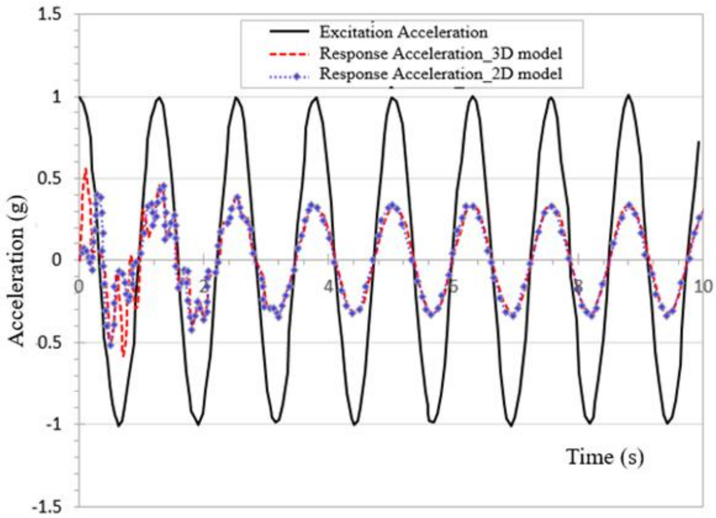

Figure 11. *Excited acceleration on the beam in the CD direction and responses of the two models*

IV. Conclusion

The paper proposes and presents an effective method for vibration analysis of corrugated core cardboard panels through equivalent finite element modeling. Compared with the full 3D model, this method gives results with acceptable accuracy while significantly reducing computational costs in terms of time and resources. With these advantages, the equivalent modeling method can be fully applied in the optimal design of sandwich structures as well as in simulations of complex systems where computational efficiency plays an important role.

References

[1] J. R. Vinson, “The Behavior Of Sandwich Structures Of Isotropic And Composite Materials,” Behav. Sandw. Struct. Isotropic Compos. Mater., 2018, Doi: 10.1201/9780203737101.

[2] Q. Liu And Y. Zhao, “Effect Of Soft Honeycomb Core On Flexural Vibration Of Sandwich Panel Using Low Order And High Order Shear Deformation Models,” J. Sandw. Struct. Mater., Vol. 9, No. 1, Pp. 95–108, 2007, Doi: 10.1177/1099636207070588.

[3] Y. Frostig And O. T. Thomsen, “High-Order Free Vibration Of Sandwich Panels With A Flexible Core,” Int. J. Solids Struct., Vol. 41, No. 5–6, Pp. 1697–1724, 2004, Doi: 10.1016/J.Ijsolstr.2003.09.051.

[4] L. J. Lee And Y. J. Fan, “Bending And Vibration Analysis Of Composite Sandwich Plates,” Comput. Struct., Vol. 60, No. 1, Pp. 103–112, 1996, Doi: 10.1016/0045-7949(95)00357-6.

[5] S. . Wang, T., Sokolinsky, V., Rajaram, S. And Nutt, “Consistent Higher-Order Free Vibration Analysis Of Sandwich Plates,” Compos. Struct., 2008.

[6] V. D. Luong Et Al., “Finite Element Simulation Of The Strength Of Corrugated Board Boxes Under Impact Dynamics,” In Lecture Notes In Mechanical Engineering, 2018, Vol. Partf3, Pp. 369–380. Doi: 10.1007/978-981-10-7149-2_25.

[7] A. Paternoster, S. Vanlanduit, J. Springael, And J. Braet, “Measurement And Analysis Of Vibration And Shock Levels For Truck Transport In Belgium With Respect To Packaged Beer During Transit,” Food Packag. Shelf Life, Vol. 15, No. January, Pp. 134–143, 2018, Doi: 10.1016/J.Fpsl.2017.12.007.

[8] V. D. Luong, B. Abbes, F. Abbes, J. B. Nolot, And D. Erre, “Experimental Characterisation And Finite Element Modelling Of Paperboard For The Design Of Paperboard Packaging,” In Iop Conference Series: Materials Science And Engineering, 2019, Vol. 540, No. 1. Doi: 10.1088/1757-899x/540/1/012014.

[9] V. D. L. Lien Tien Dao, Pham Tuong Minh Duong, “Analysis Of Stress And Strain In Sandwich Structures Using An Equivalent Finite Element Model,” Int. J. Eng. Technol. Innov., 2024, Doi: Https://Doi.Org/10.46604/Ijeti.2024.

[10] T. S. Lok And Q. H. Cheng, “Free Vibration Of Clamped Orthotropic Sandwich Panel,” J. Sound Vib., Vol. 229, No. 2, Pp. 311–327, 2000, Doi: 10.1006/Jsvi.1999.2485.

[11] T. S. Lok And Q. H. Cheng, “Bending And Forced Vibration Response Of A Clamped Orthotropic Thick Plate And Sandwich Panel,” J. Sound Vib., Vol. 245, No. 1, Pp. 63–78, 2001, Doi: 10.1006/Jsvi.2000.3543.

[12] J. Lou, L. Ma, And L. Z. Wu, “Free Vibration Analysis Of Simply Supported Sandwich Beams With Lattice Truss Core,” Mater. Sci. Eng. B, Vol. 177, No. 19, Pp. 1712–1716, 2012, Doi: 10.1016/J.Mseb.2012.02.003.

[13] N. Stenberg, “A Model For The Through-Thickness Elastic-Plastic Behaviour Of Paper,” Int. J. Solids Struct., Vol. 40, No. 26, Pp. 7483–7498, 2003, Doi: 10.1016/J.Ijsolstr.2003.09.003.

[14] N. Stenberg, C. Fellers, And S. Östlund, “Plasticity In The Thickness Direction Of Paperboard Under Combined Shear And Normal Loading,” J. Eng. Mater. Technol. Trans. Asme, Vol. 123, No. 2, Pp. 184–190, 2001, Doi: 10.1115/1.1352747.

[15] P. Mäkelä And S. Östlund, “Orthotropic Elastic-Plastic Material Model For Paper Materials,” Int. J. Solids Struct., Vol. 40, No. 21, Pp. 5599–5620, 2003, Doi: 10.1016/S0020-7683(03)00318-4.

[16] V. D. Luong Et Al., Finite Element Simulation Of The Strength Of Corrugated Board Boxes Under Impact Dynamics, Vol. Partf3. 2018. Doi: 10.1007/978-981-10-7149-2_25.

[17] V. D. Luong, A. S. Bonnin, F. Abbès, J. B. Nolot, D. Erre, And B. Abbès, “Finite Element And Experimental Investigation On The Effect Of Repetitive Shock In Corrugated Cardboard Packaging,” J. Appl. Comput. Mech., Vol. 7, No. 2, Pp. 820–830, 2021, Doi: 10.22055/Jacm.2020.35968.2771.

[18] L. V. Dung, D. L. Tien, And D. P. Tuong Minh, “Numerical Simulation For The Honeycomb Core Sandwich Panels In Bending By Homogenization Method,” Int. J. Mech., Vol. 15, Pp. 88–94, 2021, Doi: 10.46300/9104.2021.15.9.

[19] V. D. L. Hoang Manh Nguyen, Lien Tien Dao, “Study The Mechanical Behavior Of Corrugated Box Using The Finite Element Method,” J. Mech. Civ. Eng., Vol. 21, No. 2, Pp. 42–49, 2024.

[20] Dao Lien Tien And Viet Dung Luong, “Inverse Identification Method Of Plasticity Parameters Of Anisotropic Material,” J. Serbian Soc. Comput. Mech., Vol. 18, No. 2, Pp. 106–119, 2024.