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Downstream Flood Forecasting During Dam Breaching

Bharat Raj Pandey, Dr. Ram Krishna Regmi, Dr. Ram Chandra Tiwari
Tribhuvan University (TU), Institute Of Engineering (IoE), Pulchowk Campus
Deputy Head Of The Department Of Civil Engineering, TU, IoE
Lecturer, Department Of Civil Engineering, TU, IoE

Abstract
The heavy thunderstorm as equivalent to PMF (Probable Maximum Flood) was occurred in 2012 at 12MW Jhimruk small hydropower project located in Pyuthan District, Lumbini Province no 5 of Nepal. Three radial gates were operated to evacuate the reservoir level in dam. About 10kM far downstream of the existing Jhimruk, the flood level was marked and the peak flow was validated for the flood event.
The land and properties of downstream communities were damaged by the flood during operation of the gates. There was no early warning system installed beforehand to protect the life and property of the downstream communities. After this event the early warning system is installed and the mechanism to predict the flood damage during the operation of gates was established.
The article intends to discuss the various methods for solving unsteady surge wave problems of by the use of Dam Break equations. The HEC-RAS 2D flow equations were implemented to a wide rectangular channel by correlating the kinematic surge wave propagation to downstream with respect to time and space. The travel time of the peaks has been evaluated from upstream to downstream for the implication of early warning flood forecasting.
Keywords: *Dam Breaching, Peak Flow, Flood Attenuation, HEC-RAS Unsteady Flow Equation*

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I. Introduction

Background of 12MW Jhimruk impounding dam

The Jhimruk Hydropower Plant is in under operation since 1994 with an installed capacity of 12MW. The power plant has 3 Horizontal Francis turbines of each 4MW capacities. The Powerhouse of this project is semi-underground and is located on the bank of Madi River. The Jhimruk Power Plant is a basin transfer Project wherein water from Jhimruk River is discharged to Madi River after the generation of Power.

The power generated by the plant is transmitted to Nepal Electricity Authority (NEA) via 132kV National Transmission line to 41kM further away to a substation at Lamahi. It also supplies power to local through rural electrification 11kV transmission system to local consumers in Pyuthan, Rolpa and Arghakhachi Districts in addition to NEA consumers. The project is funded by NORAD through UMN and Government of Nepal.

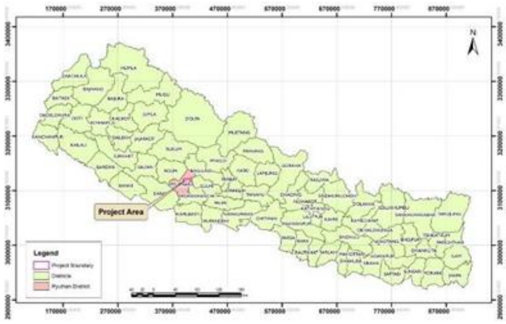

Figure 1.1 Location showing the Dam Axis of the 12MW Jhimruk impounding dam

Figure 1.2 Existing 12MW Jhimruk Daily Pondage Hydropower Project Downstream

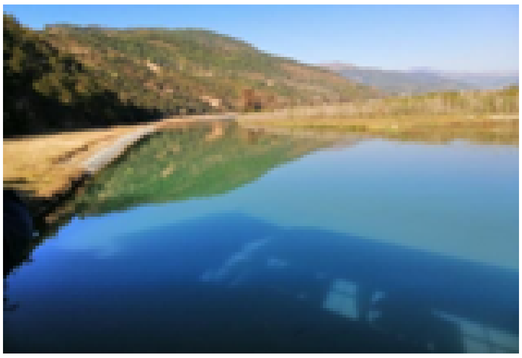

Figure 1.3 Existing 12MW Jhimruk Daily Pondage Hydropower Project Upstream Area

Figure 1.4 Existing 12MW Jhimruk Daily Pondage Hydropower Project Reservoir Floodgates

2 MATERIALS AND METHODS

2.1 Theoretical Foundation

The equations which are derived in the following paragraphs have justified to the HEC-RAS 2D Saint-Venant equations for the use of Dam Breaching cases. The Flood gate operation of 12MW Jhimruk Project in the event of PMF Flood is considered similar to the Dam breaching event.

The catastrophic PMF events in the 12MW Jhimruk Impounding Reservoir as aforementioned can be solved as the principle of a negative wave, steep at the beginning, will deform itself much faster than in the case for a positive wave. A true front does not establish itself and notion of a wave height, Δh, makes less(physical)sense than for a positive wave.

The negative wave can however be considered as being an unsteady flow of gradually varied flow. The equation of Saint-Venant equation of continuity equation and the dynamic equation of **"Gradually Varied Unsteady Flow (GVUF)"** shall be used to solve the negative wave propagation and assumed that the frictional forces are practically compensated by the gravity forces; for a rectangular channel, these yields:

$$\frac{\partial h}{\partial t} + \frac{\partial (Vh)}{\partial x} = 0; \text{.................}(1)$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial h}{\partial x} = 0 \text{......} (2)$$

The above equations represent now a simple wave, when Δh and V are only dependent on the flow depth, h, these can be written as: (see Jaeger,1954, p.364)

$$\frac{\partial (Vh)}{\partial x} = \frac{\partial (Vh)}{\partial h}\frac{\partial h}{\partial x};\ \frac{\partial V}{\partial x} = \frac{\partial V}{\partial h}\frac{\partial h}{\partial x}$$

Also: $\frac{\partial V}{\partial t} = \frac{\partial V}{\partial h}\frac{\partial h}{\partial t}$

Consequently, the expression of $\frac{\partial h}{\partial t}$ in the above Saint-Venant continuity equation and dynamic equation can be eliminated, which gives $g - h\left(\frac{\partial V}{\partial h}\right)^2 = 0$; and by taking the roots of the two members: $\frac{\partial V}{\partial h} = \pm\sqrt{\frac{g}{h}}$

which up on integration, renders: $(h) = \pm 2\sqrt{gh} + constant of\ integration$ (3)

The upper sign, $(+)$, corresponds to an elementary negative wave; but it could also be positive; which propagates in the direction of the initial flow, the lower sign $(-)$ corresponds to a wave propagating into the opposite direction.

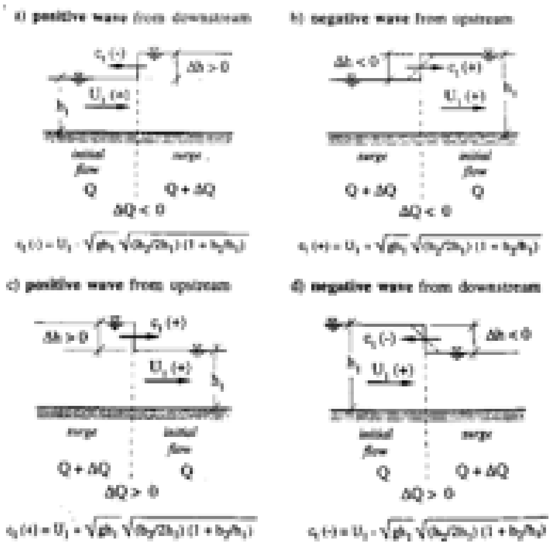

Figure 2-1 Rapidly Varied Flow due to sudden variation on discharge

For the evaluation of the integration constant, (see Favre,1935, p.29) one takes $V = V_1$ and $h = h_1$ in this part of the channel, which has as yet not been attained by the wave, thus one gets:

$V(h) = V_1 \pm 2\sqrt{gh} \mp 2\sqrt{gh_1}$ ….(4)

using the celerity of the small wave is given by:

$C_t(h) = V \pm \sqrt{gh} = V_1 \pm 3\sqrt{gh} \mp 2\sqrt{gh_1}$ (5)

This celerity is the velocity of propagation of wave a small amplitude.

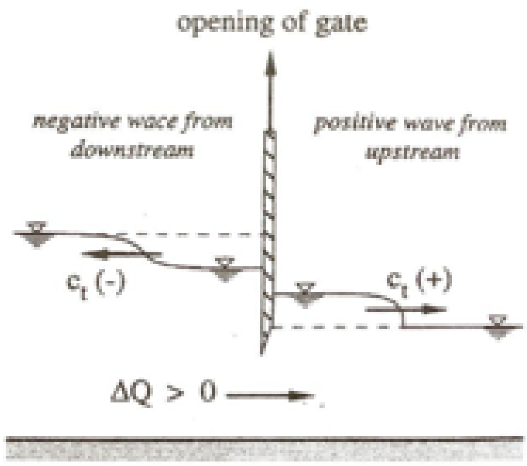

Figure 2-2 Illustrations of four different types of Rapidly varied flow during the gate operation cases

If one considers a negative wave from upstream the equation is written as:

$C_t(h) = V + \sqrt{gh} = V_1 + 3\sqrt{gh} - 2\sqrt{gh_1}$ ……………………….(6)

By putting the references; creation of wave; at time $t = 0$, the position of the wave is given by $x = C_t t$; thus, one gets: $x(h) = ((V_1 + 3\sqrt{gh} - 2\sqrt{gh_1})t$ ....(7).

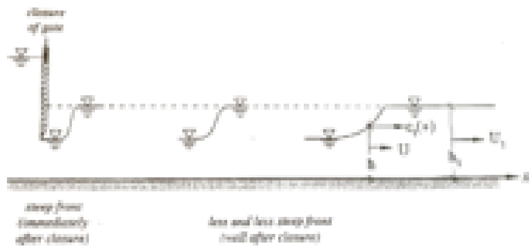

Figure 2-3 Negative wave from upstream during the gate closure

Being an expression of the water surface of the wave, comparing on between these eqs. one finds a relation of velocity, in terms of x and t,

thus, $V(h) = \frac{V_1}{3} + \frac{2}{3}\frac{x}{t} - \frac{2}{3}\sqrt{gh_1}$......... (8)

If one considers now a negative wave from downstream this relation, above eq. is written as:

$V(h) = V_1 - 3\sqrt{gh} + 2\sqrt{gh_1}$............. (9)

Now the equation no.9 is a problem of a" Dam Break" in hydraulic structures which thus can be addressed, considering it to be a negative from downstream propagating on a frictional less, horizontal and a rectangular channel (see Streeter,1971, p.681). This is the solution to the problem of a simple wave of Saint-Venant. "Thus, the above Saint-Venant for the use of dam break equation that could be solved by using the method of characteristics, explicit and implicit method of analysis. "

From the above we can summarize few points as follows:

i) Before the failure of the gate (or dam) the water depth upstream of the dam is $h = h_1$ and there is no water on the other side, $h = 0$

ii) After the sudden failure point of the gate or the breaching opening of the dam, an expression for the velocity at any section is obtained using the above equation and assuming that $V_1 = 0$, or:

$V(h) = 0 - 2\sqrt{gh} + 2\sqrt{gh_1}$ ...(10)

iii) The Celerity is given by:

$C_t(h) = -3\sqrt{gh} + 2\sqrt{gh_1}$ ....(11)

which renders in the shown fig below:

For $h = 0$: $C_t = +2\sqrt{gh_1}$

For $h = h_1$: $C_t = -2\sqrt{gh_1}$

iv) The shape of the water surface is given by:

$x(h) = C_t t = (0 - 3\sqrt{gh} + 2\sqrt{gh_1})t$ ...(12)

Subsequently, one obtains for the section at the gate or failure opening of dam breaching point, $x = 0$ for all time instants t, flow depth as being $h_{x0} = \frac{4}{9}h_1$ This flow depth remains thus independent of time. The water surface profile for all time instants pivots around this point of $h_{x0} = \frac{4}{9}h_1$

v) The velocity, V in this section, $x = 0$ is obtained from the above equation. Or

$V_{x0} = -2\sqrt{g}\left(\frac{2}{3}\sqrt{h_1} - \sqrt{h_1}\right) = +\frac{2}{3}\sqrt{gh_1}$

Showing that it is also independent of time.

vi) The unit discharge, q, can readily be computed:

$q = hV = \frac{8}{27}h_1\sqrt{gh_1}$ being constant with time.

vii) In reality, the shape of the water surface, should be modified since due to the friction on the bed a positive wave from upstream will establish itself in the downstream reach in the channel.

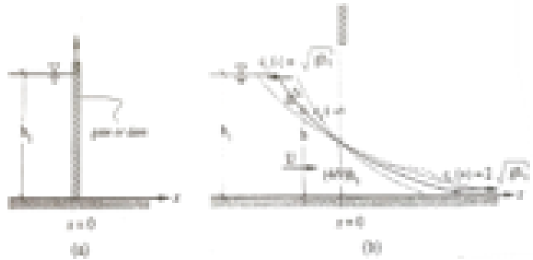

Figure 2-4 Dam Break Problem of negative wave from downstream during gate opening (can be illustrate similar to as dam breaching)

2.2 Comparison of the Negative surges in Rapidly Varied Unsteady Flow (RVUF)

Negative surges are not stable in form, because the upper portions of the wave travel faster than the lower portions (Art 19-1), Ven Te Chow page 566. If the initial profile to the surge is assumed to have a steep front, it will soon flatten out as the surge moves through the channel. If the height of the surge is moderate or small compared with the depth of flow, the equations derived for a positive surge can be applies to determine approximately the propagation of the negative surge. If the height of the surge is relatively large, a more elaborate analysis is necessary as follows:

The wave velocity of the surge actually varies from point to point. For example, V_w is the wave velocity at a point on the surface of the wave where the depth is h and the velocity of flow through the section is V. During a time, interval dt, the change in depth is dh. The value of dh is positive for an increase of h and the negative for a decrease of h.by the momentum principle, the corresponding change in hydrostatic pressure should be equal to the force required to the change the momentum of the vertical element between h and $h + dh$. Considering a unit width of the channel and assuming $\beta_1 = \beta_2 = 1$,

$$\frac{w}{2} h^2 - \frac{w}{2}(h + dh)^2 = \frac{w}{g}(h + \frac{1}{2}dh)(V + V_w)dV \quad \ldots\ldots\ldots\ldots\ldots\ (13)$$

Simplifying the above equation and neglecting the differential terms of higher order,

$$dh = -\frac{V \pm V_w}{g}\, dV \quad (14)$$

The whole wave front can be assumed to be made up of a large number of very small wave placed one on top of the other. The velocity of small wave at the point under consideration may be expressed as

$$V_w = \sqrt{gh} - V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (15)$$

Similarly, the velocity at the wave crest is

$$V_{w2} = \sqrt{gh_2} - V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (16)$$

And, at the wave trough,

$$V_{w1} = \sqrt{gh_1} - V_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (17)$$

When the surge is not too high, a straight-line relation between V_{w} and V_{w_2} may be assumed. Thus, the mean velocity of the wave may be considered to be

$$V_w = \frac{V_{w1} + V_{w2}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (18)$$

Now, eliminating V_w

$$\frac{dh}{\sqrt{h}} = -\frac{dV}{\sqrt{g}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (19)$$

Integrating this equation from h_2 to h and from V_2 to V, and solving for V,

$$V = V_2 + 2\sqrt{gh_2} - 2\sqrt{gh} \quad \ldots\ldots\ldots\ (20)$$

$$V_w = 3\sqrt{gh} - 2\sqrt{gh_2} - V_2 \quad \ldots\ldots\ (21)$$

Thus, the wave velocity at the rough of the wave is

$$V_{w1} = 3\sqrt{gh_1} - 2\sqrt{gh_2} - V_2 \ldots\ (22)$$

(Dam Break Equation via RVUF)

Hence the Rapidly Varied Unsteady flow negative surge dam break equation and derivation of the Gradually Varied Unsteady Flow Saint-Venant equation from equation (9) prove to be concurrent in dam breaching equation. Therefore, these can be interchangeably use in Flood wave propagation during the case of dam breaching rapidly varied unsteady flow equations.

The hydrograph generated during the operation of the 12MW Jhimruk Hydropower flood gates is compatible similar to the hydrograph generated in the dam breaching event.

2.3 Implementation of HEC-RAS 2D Flow Model to solve and validate the above Dam Breaching Equations

The flood gate opening modality of 12MW Jhimruk Hydropower project has been set up to a computer model for different flood scenario of various gate opening for 700m3/sec, 900m3/sec and 1000m3/sec respectively to evacuate the flood from the upstream pondage.

Also, this would validate the downstream flood propagation by 2D HEC-RAS (Hydrological Engineering Centre's River Analysis System) which is a mathematical solver based upon the finite volume method for solving full dynamic flood wave equations. This has been widely used for water related issues in different field of River Engineering, Hydropower with the implementation of Hydraulic Structure in the model itself.

2.3.1 Computation of Downstream Peak Flow by the use of 2D HEC-RAS Modelling

Continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

Momentum equation

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

Local acceleration term — Convective acceleration term — Pressure force term — Gravity force term — Friction force term

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

Kinematic Wave

Diffusion Wave

Dynamic Wave

A HEC-RAS model was run for computing the water level at the downstream of the barrage of Jhimruk hydropower project. For this purpose, the model was prepared and then run for a discharge of 750, 800 and 850 cumecs. The 2D model is shown in Figure below.

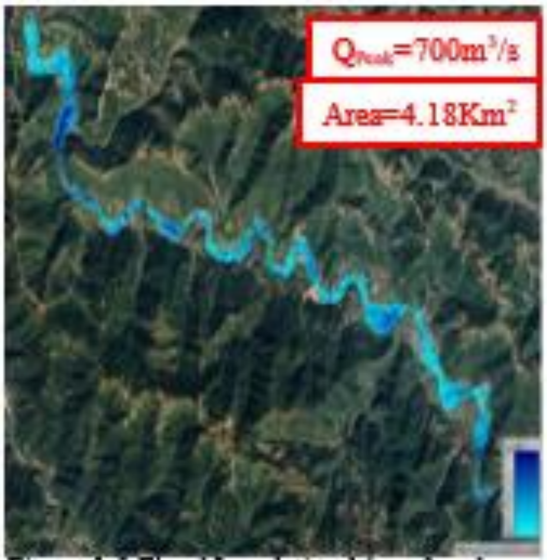

Figure 2-5 Flood Inundation Map after dam breaching for inflow of 700 cumecs

Figure 2-6 Flood Inundation Map after dam breaching for inflow of 900 cumecs

Figure 2-7 Flood Inundation Map after dam breaching for inflow of 1000 cumecs

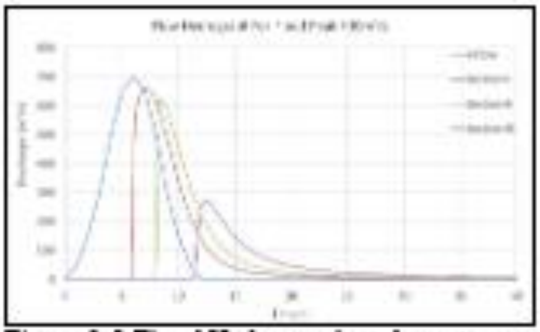

Figure 2-8 Flood Hydrograph at downstream of dam for flood inflow of 700 cumecs

Table 2-1 Discharge at downstream of the dam for inflow of 700 cumecs

Peak Values (m3/s)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
700	659.799	625.735	274.957
Time to reach Peak (h)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
6	7.166667	8.5	12.5

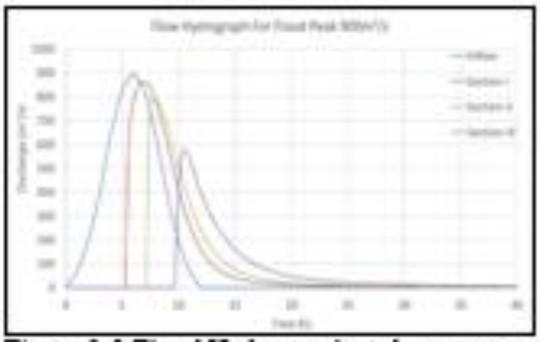

Figure 2-9 Flood Hydrograph at downstream of dam for flood inflow of 900 cumecs

Table 2-2 Discharge at the downstream of dam for inflow of 900 cumecs

Peak Values (m3/s)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
900	864.35	794.84	581.34
Time to reach Peak (h)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
6	6.8333	7.8333	10.5

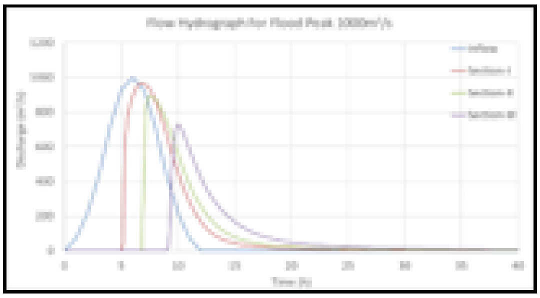

Figure 2-10 Flood Hydrograph at downstream of dam for flood inflow of 1000 cumecs.

Table 2-3 Discharge at downstream of dam for inflow of 1000 cumecs.

Peak Values (m3/s)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
1000	965.378	892.4131	728.386
Time to reach Peak (h)			
Inflow	**Sec-I**	**Sec-II**	**Sec-III**
6	6.833333	7.5	10

3 FINDINGS OF ILLUSTRATIVE EXAMPLE

Implication of 2D HEC-RAS Equation models in the natural rivers cross-section helps to evaluate flood area through a flood plain. The verification of the numerical solution as described in the theoretical basis of the dam break equation was performed in comparison of the result of 2D HEC-RAS model for the downstream flood peak.

The validation of these results with respect to the water level through observed data considering the natural river channel cross-section from the filed measurement was also carried out to compare these analytical results. It was found to be the 3% of water level variation higher in 2D HEC-RAS than the field observation to the similar successive events during the flood attenuation.

The results have also been tested as aforementioned with different discretization methods (Method of Characteristic, Explicit and Implicit Method) for solving the partial differential Unsteady Flow Equation as well as Dam Break Equation and found to be 5% discrepancy higher flood level with the measured water level in actual field observation.

3.1 Conclusions

- The dam breaching case is a rapidly varied problem however when the flood waves diffuse it would behave like in the case of gradually varied unsteady flow situation.
- It is concluded that the gradually varied unsteady flow Saint-Venant equation is compatible to use in the situation of rapidly varied unsteady flow dam break equation.
- Because of the gradually varied unsteady flow dam break equation and the rapidly varied equation proves to be concurrent, hence these equations are compatible to use interchangeably in place of each other for predicting the downstream peak flow in the event of dam breaching.
- The numerical model of 2D HEC-RAS simulation is proved to be most realistically accurate for the validation of the downstream flood wave propagation in comparisons of the field observation in the natural river channel during the operation of the gates as in the case of dam breaching event of the reservoir pondage.

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