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Finite Element Analysis of a Pressure Vessel Subjected to Uniform Internal Pressure

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Abstract
Pressure vessels are widely used in the oil and gas industry to hold and transport liquids and gases at pressures substantially different from atmospheric pressure. Such vessels are usually subjected to internal pressures due to the vapour pressure of the liquid or storage conditions of the gas being held. The design of pressure vessels requires detailed sizing calculations and material selection to ensure that they can withstand the stresses developed due to liquid containment, and thus prevent possible loss of containment.
In this study, finite element analysis will be performed on a fixed storage tank designed to hold liquefied petroleum gas (LPG). In doing this, we shall create a geometrical model of the LPG tank and mesh it. It will then be subjected to uniform internal pressure with the vessel support rigidly fixed to avoid translation. Mesh convergence shall be performed to establish an optimum mesh size that guarantees repeatability of the variable of interest.
We shall evaluate the stress and displacement distributions on the tank using different materials and wall thicknesses and thus, predict an appropriate material selection and wall thickness for the pressure vessel. The results of the finite element analysis shall be verified by comparison with analytical calculation results.

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I. Problem Description

The storage of dangerous fluids, especially gases, is a challenging problem in the industry, as it bears huge impact on the safety of personnel, properties, and the environment.

There have been incidences of loss of containment of dangerous fluids leading to accidents and fatalities. The main causes of these accidents were lack of proper storage facilities due to cost constraints or errors in design. This underpins the importance of proper design at optimum cost, to avoid catastrophic failures.

The proposed work is intended to analyse an LPG storage tank with a capacity of 23,000 litres and internal design pressure of 1.8 MPa. By varying the construction material and wall thickness, we strive to achieve an optimum design for the pressure vessel which will produce minimal stresses and deformations.

II. Objective

The objective of our study is to analyse an LPG storage tank using FEA, verify the results with analytical design calculation, and then extend the analysis to other materials of construction and wall thickness to strive to provide some perspective on optimal material and size selection.

III. Literature Review

Introduction

A pressure vessel is a container designed to hold gases or liquids at a pressure substantially different from the ambient pressure. In most cases, the internal pressure is higher than the external.

The method for construction of a typical pressure vessel, and the choice of material is governed by the vessel size, contents, working pressure, and weight considerations. The number of vessels required for a typical plant may also play a key role.

In the United States, Pressure vessels are usually built in accordance with the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Code (BPVC) [1]. Other Countries have similar codes or adopt one of the more popular ones.

As part of the ASME BPVC, an authorized inspector is required to sign off on every newly constructed vessel [1]. Such vessels will bear a nameplate with details about the vessel such as maximum allowable working pressure, maximum temperature, name of manufacturer, date of manufacture, registration number and the ASME stamp (U-stamp) [1]. This allows for traceability and officially renders such vessel an ASME Code vessel.

Historical Context

Historically, early attempts to design a tank capable of holding pressures up to 10,000 psi led to the construction of a 6-inch diameter tank in 1919. The tank was spirally wound with two layers of high tensile strength steel wire to prevent sidewall rupture, and the end caps were longitudinally reinforced with lengthwise high-tensile rods [2].

With subsequent technological advancements, the need for high pressure and temperature vessels for petroleum refineries and chemical plants gave rise to vessels joined with welding instead of rivets. In recognition of this success, the BPVC included welding as an acceptable means of construction in the 1930s [2].

Applications

Pressure vessels are used in a variety of applications. Some of the common uses include:

1. Compressed air receivers
2. Boilers and domestic hot water storage tanks
3. Diving cylinders and compression chambers
4. Distillation towers
5. Autoclaves
6. Nuclear reactor vessels
7. Storage vessels for high pressure permanent gases and liquified gases such as ammonia, chlorine, and LPG (propane, butane).

Pressure Vessel Shapes

Most pressure vessels are either spherical or cylindrical in shape as shown in Fig. 1.1. Cylindrical vessels typically have end caps also known as heads. The end caps are frequently either hemispherical, torispherical, or dished shape.

Historically, more complicated shapes have been much harder to analyse for safe operation and are usually very difficult to construct [1].

(a) (b)

Fig 1.1: Spherical (a) and Cylindrical (b) Pressure Vessels.

Theoretically, a spherical pressure vessel has approximately twice the strength of a cylindrical pressure vessel with the same wall thickness and is the ideal shape to hold internal pressure. However, a spherical shape is difficult to manufacture, and therefore more expensive, hence, most pressure vessels are cylindrical [3].

There are two main types of cylindrical pressure vessels: thin-walled and thick-walled cylindrical pressure vessels. If the diameter of the pressure vessel is at least 10 – 20 times greater than the wall thickness, then the cylinder may be treated as a thin one [4], otherwise, it is thick-walled.

Construction Materials

Most pressure vessels are made of steel due to availability and cost. To manufacture a cylindrical or spherical pressure vessel, rolled sheet plates and possibly forged parts would have to be welded together. The overall quality of the pressure vessel would depend on the quality of construction materials.

The most important characteristics to be considered when selecting a material of construction are:
1. Mechanical properties
2. Corrosion resistance
3. Ease of fabrication – forming, welding, casting etc.
4. Availability in standard sizes – plates, sections, tubes etc.
5. Cost

For pressure vessels built to an approved code, the materials used must be selected from Code-approved material specifications. The ASME BPVC, for instance, provides a large catalogue of acceptable materials, which is readily available [5]. A sample table from the BPVC showing maximum allowable stress values for ferrous materials is provided below.

Table 1A
Section I; Section III, Classes 2 and 3;* Section VIII, Division 1; and Section XII
Maximum Allowable Stress Values, *S*, for Ferrous Materials
(*See Maximum Temperature Limits for Restrictions on Class)

Line No.	Min. Tensile Strength, MPa	Min. Yield Strength, MPa	Applicability and Max. Temperature Limits (NP = Not Permitted) (SPT = Supports Only) I	III	VIII-1	XII	External Pressure Chart No.	Notes
1	275	140	NP	NP	343	NP	CS–6	...
2	275	140	NP	NP	343	NP	CS–6	...
3	310	155	NP	343 (Cl. 3 only)	482	343	CS–6	G10, G22, T10
4	310	165	NP	149 (Cl. 3 only)	NP	NP	CS–1	W12
5	310	165	NP	149 (Cl. 3 only)	343	343	CS–1	...
6	310	165	482	371	482	343	CS–1	G10, T2
7	310	165	NP	371	NP	NP	CS–1	S6, W10, W12
8	310	170	NP	NP	482	343	CS–1	G10, T2
9	325	180	538	NP	NP	NP	CS–1	G4, G10, S1, T2, W13
10	325	180	538	NP	538	343	CS–1	G3, G10, G24, S1, T2, W6
11	325	180	NP	NP	482	343	CS–1	G10, T2
12	325	180	538	NP	538	343	CS–1	G10, S1, T2
13	325	180	NP	NP	538	343	CS–1	G24, T2, W6
14	325	180	NP	NP	538	343	CS–1	G10, T2
15	325	180	NP	NP	538	343	CS–1	G24, T2, W6
16	330	205	482	NP	NP	NP	CS–2	G3, G10, S1, T2
17	330	205	482	149 (Cl. 3 only)	NP	NP	CS–2	G10, S1, T2, W12, W13
18	330	205	NP	NP	482	343	CS–2	G24, T2, W6
19	330	205	399	NP	NP	NP	CS–2	G2, G10, S10, T2, W15
20	330	205	482	149 (Cl. 3 only)	NP	NP	CS–2	G10, S1, T2

Pressure Vessel Design

Pressure vessels are held together against the internal pressure by tensile forces within the walls of the container. The normal (tensile) stress in the walls of the container is proportional to the internal pressure and radius of the vessel and inversely proportional to the wall thickness.

Based on the foregoing analysis, pressure vessels are designed to have a wall thickness proportional to the radius and the internal pressure of the tank and inversely proportional to the maximum allowable normal stress of the material used in the walls of the container. The exact formula for calculation of the wall thickness depends on the vessel shape [5].

In designing a pressure vessel, the minimum mass requirement is a key consideration. The exact formula for determining the minimum mass varies with the vessel shape but depends on the density, ρ, and maximum allowable stress σ of the material in addition to the pressure P and volume V of the vessel [5].

An additional consideration in the design of pressure vessels is the deflection of the material of the vessel due to loading. Deflections must be minimized as much as possible to avoid any plastic deformations. The deflection in a pressure vessel depends on the internal pressure, radius, height, wall thickness and the material's modulus of elasticity and Poisson's ratio [6].

Stress in Thin-Walled Pressure Vessels

In accordance with the ASME Boiler and Pressure Vessel Code (BPVC) [5], the stresses in a spherical thin-walled pressure vessel are given by:

$$\sigma_\theta = \sigma_{long} = \frac{P(R + 0.2t)}{2tE}$$

Equation 1

Similarly, the stresses in a thin-walled cylindrical pressure vessel (see Fig. 1.2) are given by:

$$\sigma_\theta = \frac{P(R + 0.6t)}{tE}$$

Equation 2

$$\sigma_{long} = \frac{P(R - 0.4t)}{2tE}$$

Equation 3

Where:
is hoop stress, or stress in the circumferential direction.
is stress in the longitudinal direction.
P is internal gauge pressure.
R is the inner radius of the cylinder.
t is thickness of the cylinder wall.
E is the joint efficiency

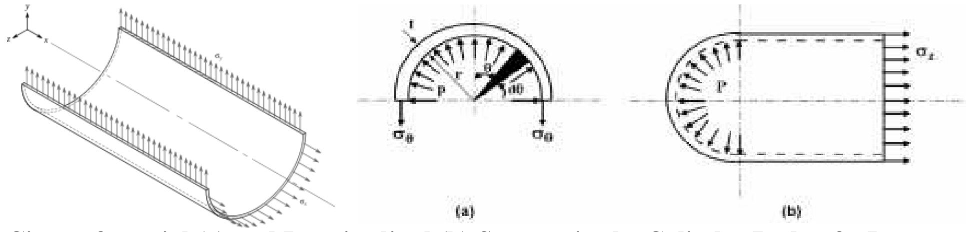

Fig 1.2: Circumferential (a) and Longitudinal (b) Stresses in the Cylinder Body of a Pressure Vessel.

Minimum Mass of Pressure Vessels
For a spherical pressure vessel, the minimum mass required is:

$$M = \frac{3}{2} PV \frac{\rho}{S}$$

Equation 4
For a cylindrical vessel with hemispherical ends, the minimum mass required is:

$$M = 2\pi R^2 (R + W) P \frac{\rho}{S}$$

Equation 5
For a cylindrical vessel with semi-elliptical ends, the minimum mass required is:

$$M = 6\pi R^3 P \frac{\rho}{S}$$

Equation 6
Where:
M is mass, (kg)
P is the pressure difference from ambient (the gauge pressure), (Pa)
V is volume, (m^3)
ρ is the density of the pressure vessel material, (kg/m^3)
S is the maximum working stress that material can tolerate. (Pa)
R is the Radius (m)
W is the middle cylinder width only, and the overall width is W + 2R (m)

Deflections in Pressure Vessels
For a cylindrical pressure vessel under uniform radial load (see Fig. 1.3), the radial and longitudinal deflections are given by:

$$\Delta R = \frac{PR^2}{\mathrm{E}t}$$

Equation 7

$$\Delta y = \frac{-PRvy}{\mathrm{E}t}$$

Equation 8
For a hemispherical end cap under uniform radial load, the radial and longitudinal deflections are given by:

$$\Delta R = \Delta y = \frac{PR^2(1 - v)}{2\mathrm{E}t}$$

Equation 9
Where:
P is the uniform internal pressure
R is the inner radius
v is the Poisson's ratio of the cylinder material
y is the cylinder height
t is thickness of the cylinder wall
E is the modulus of elasticity of the cylinder material

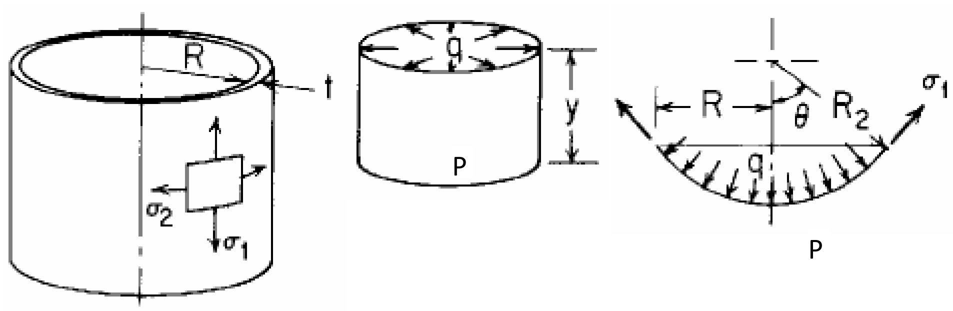

Fig 1.3: Deflections in the Cylinder Body and Spherical End Cap of a Pressure Vessel.

Use Of Finite Element Analysis In Design

Integrated finite element method is commonly used in the design and development of products across Engineering specializations notably the aeronautical, biomechanical, and automotive industries [7].

The most common FEM packages in the industry include ANSYS, ABAQUS, COMSOL etc. Most modern FEM packages include working environments for thermal, electromagnetic, fluid, and structural analysis.

In structural analysis, FEM allows detailed visualization of areas of stress concentration, bending and twisting as well as areas of stiffness and strength. It also indicates the distribution of stresses and displacements, thus allowing for material selection and cost optimization [7].

FEM softwares provide the platform and a wide range of simulation options for modelling and analysis of complex systems. The desired level of accuracy and associated computational time required to address most engineering applications can also be managed simultaneously [7].

FEM allows conceptual designs to be modelled, tested, refined, and optimized at a minimal cost, before a final design is selected for manufacturing [7].

Analytical Model

Vessel Specifications

The fixed storage tank in this study is designed to hold Liquefied Petroleum Gas (LPG). A thin-walled cylindrical pressure vessel is considered with the following specifications:

Equipment Specification	
Equipment Name	LPG Storage Tank
Function	Storage and dispensing of LPG
Material	Steel (SA-240 Gr.316L)
Type	Cylinder with Hemispherical Ends
Design Data	
Temperature	40°C
Pressure	1.8 MPa
Dimensions	
Capacity	23 m^3
Internal Diameter	2 m
Height	8 m
Wall Thickness	20 mm
Joint Efficiency E	1

A cylindrical vessel type was used for the LPG product storage tank for ease of fabrication. The end caps of the cylindrical tank are hemispherical shaped with a conical base support. The material used for the storage tank is stainless steel because of its high resistance to acidity and corrosion.

Stress Analysis

The following properties for stainless steel SA-240 Gr. 316L are obtained from the ASME Pressure Vessel Code [5]:

$S = 1.15 \times 10^8$ Pa

$E = 2 \times 10^{11}$ Pa

$\nu = 0.3$

Using the vessel radius and internal pressure, we can determine the minimum required cylinder wall thickness from equation (2) as follows:

$$t = \frac{PR}{SE - 0.6P}$$

$$t = \frac{1.8 \times 10^6 \times 1}{1.15 \times 10^8 \times 1 - 0.6 \times 1.8 \times 10^6}$$

$$t = 0.0173\ m = 17.3\ mm$$

Considering corrosion allowance of 2.7 mm, minimum wall thickness of 20 mm is selected.

The above calculation refers to the thickness required for the longitudinal joints. This is higher than the thickness required for the circumferential joints and the hemispherical end caps, hence, it is adopted as the thickness of the vessel.

Using the above, the stress experienced by the pressure vessel in the circumferential direction is obtained using equation (2) as follows:

$$\sigma_\theta = \frac{P(R + 0.6t)}{tE}$$

$$\sigma_\theta = \frac{1.8 \times 10^6(1 + 0.6(0.02))}{0.02 \times 1}$$

$$\sigma_\theta = 9.1 \times 10^7\ Pa$$

The stress in the longitudinal direction is obtained from equation (3) as follows:

$$\sigma_{long} = \frac{P(R - 0.4t)}{2tE}$$

$$\sigma_{long} = \frac{1.8 \times 10^6(1 - 0.4(0.02))}{2 \times 0.02 \times 1}$$

$$\sigma_{long} = 4.46 \times 10^7\ Pa$$

Deflection Analysis

The radial deflection in the cylindrical part of the pressure vessel is obtained from equation (7) as follows:

$$\Delta R = \frac{PR^2}{\mathrm{E}t}$$

$$\Delta R = \frac{1.8 \times 10^6(1)^2}{2 \times 10^{11}(0.02)}$$

$$\Delta R = 4.5\ \times 10^{-4}\ m$$

The longitudinal deflection in the cylindrical part of the pressure vessel is obtained from equation (8) as follows:

$$\Delta y = \frac{-PR\nu y}{\mathrm{E}t}$$

$$\Delta y = \frac{-1.8 \times 10^6(1)(0.3)(6)}{2 \times 10^{11}(0.02)}$$

$$\Delta y = -8.1 \times 10^{-4}\ m$$

The radial and longitudinal deflections in the hemispherical end part of the pressure vessel are obtained from equation (9) as follows:

$$\Delta R = \Delta y = \frac{PR^2(1 - \nu)}{2Et}$$

$$\Delta R = \Delta y = \frac{1.8 \times 10^6(1)^2(1 - 0.3)}{(2)2 \times 10^{11}(0.02)}$$

$$\Delta R = \Delta y = 1.575 \times 10^{-4}\ m$$

Maximum longitudinal deflection (shrinkage) on vessel occurs on the cylindrical portion and is given by:

$$U_{long} = 8.1 \times 10^{-4}\ m$$

Maximum radial deflection (expansion) on vessel occurs on the cylindrical portion and is given by:

$$U_\theta = 4.5\ \times 10^{-4}\ m$$

We shall compare these stresses and deflections with those obtained from the FE model.

IV. Finite Element Model

In this study, we shall create a geometrical model of the LPG tank using ABAQUS and mesh it. It shall then be subjected to uniform internal pressure. The base of the tank support shall be fully fixed (encastre boundary condition) to prevent any form of translation or rotation.

We shall evaluate the stresses and displacement distributions on the tank shell using different materials and wall thicknesses.

Geometry

ABAQUS geometry editing software "Sketcher" is used to design the shell of the LPG Tank. The tank consists of a cylindrical section of 2 m diameter and 6 m long, with two hemispherical end caps, supported by a conical base 1.5 m from the bottom section of the cylindrical part.

The initial vessel wall thickness is 20 mm. Thereafter, other wall thicknesses are used to compare the performance of the vessel under uniform internal pressure.

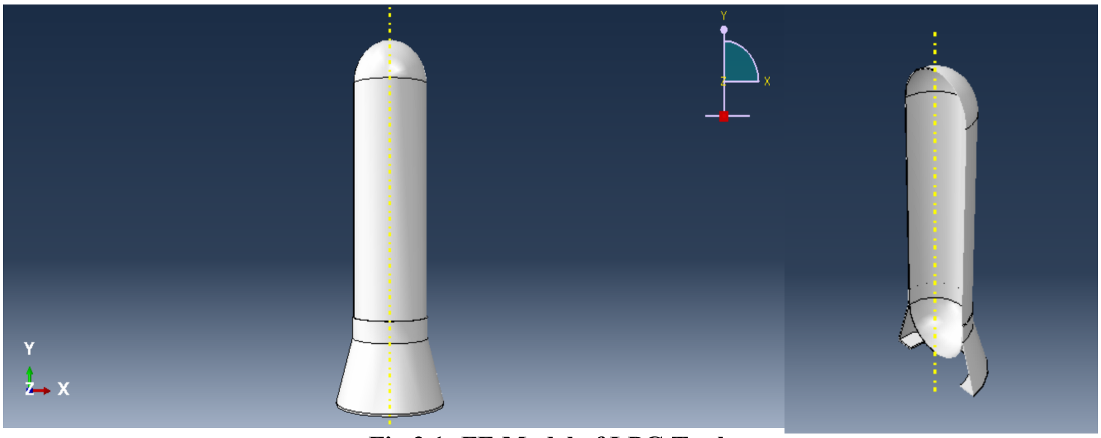

Fig 3.1: FE Model of LPG Tank

Material Properties

The initial model is built using stainless steel SA-240 Gr. F316L. Thereafter, other materials are used to compare the performance of the vessel under uniform internal pressure.

The following material properties obtained from the ASME Pressure Vessel Code [5] are used:

Material	Modulus of Elasticity (E)	Poisson's Ratio (ν)	Maximum Allowable Stress (S)
Stainless Steel SA-240 Gr. F316L	200 x 10^9 Pa	0.3	115 MPa
Glass Fibre Reinforced Plastic (GFRP)	36.233 x 10^9 Pa	0.1615	86 MPa
Cast Iron	161 x 10^9 Pa	0.29	138 MPa

Loads And Boundary Conditions

The boundary condition specified was full fixing of the base of the tank support. This was achieved by checking encastre (all three translational and rotational degrees of freedom).
At the base, for $y = 0$:

$$u = v = w = 0$$
$$M_x = M_y = M_z = 0$$

The internal walls of the pressure vessel were then subjected to uniform pressure of 1.8 MPa.

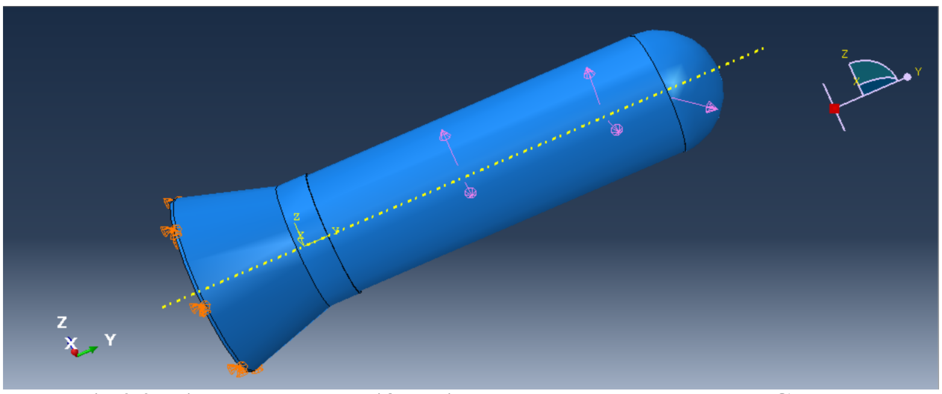

Fig 3.2: Fixed base and uniform internal pressure load on LPG Tank

Meshing

Since the vessel is subjected to uniform internal pressure, two-dimensional, 8-node quadrilateral elements with tri meshing on bounding faces were used. This produces a more accurate displacement field and does not have the problem of fitting the geometry. The approximate global seed size was set at 0.1. A total of 41,897 elements were generated in the initial model with an average aspect ratio of 3.57.

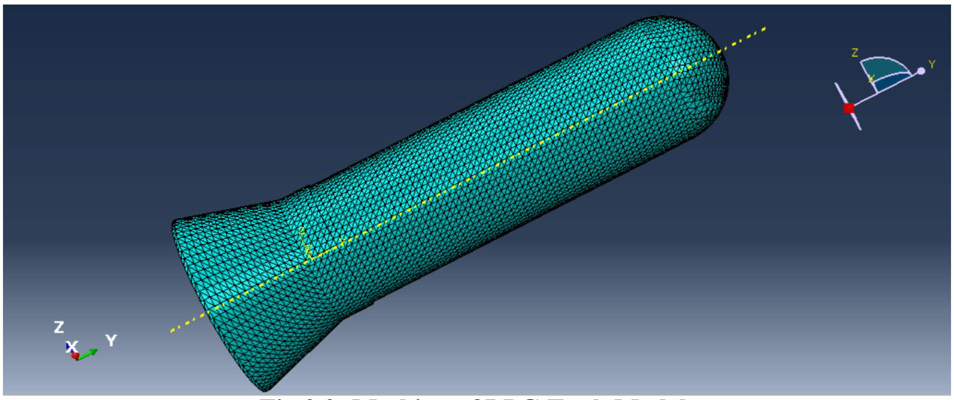

Fig 3.3: Meshing of LPG Tank Model

V. Results And Discussions

Finite Element Model Results

The following results were obtained from our initial finite element model, a 20mm thick stainless steel cylindrical pressure vessel:

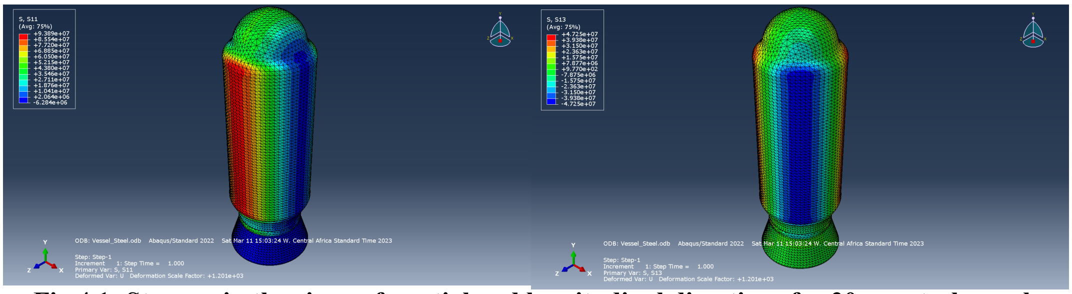

Fig 4.1: Stresses in the circumferential and longitudinal directions for 20mm steel vessel

The maximum stress in the circumferential direction is obtained as:

$$\sigma_{\theta} = 9.389 \times 10^{7}\ Pa$$

The maximum stress in the longitudinal direction is obtained as:

$$\sigma_{long} = 4.725 \times 10^{7}\ Pa$$

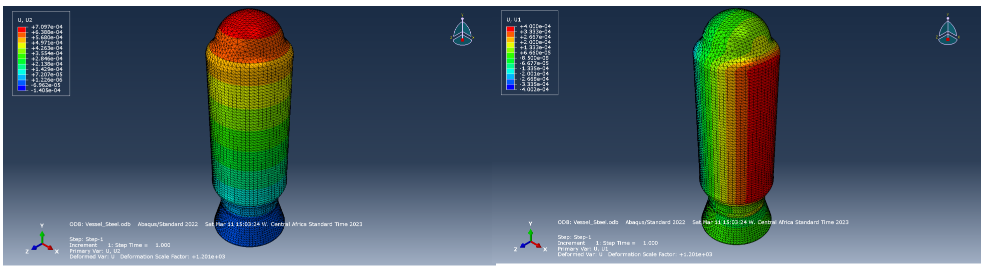

Fig 4.2: Deflections in the radial and longitudinal directions for 20mm steel vessel

The maximum longitudinal deflection on vessel is obtained as:

$$U_{long} = 7.097 \times 10^{-4}\ m$$

The maximum radial deflection on vessel is obtained as:

$$U_{\theta} = 4 \times 10^{-4}\ m$$

The above results are verified by comparison with the analytical results as shown in the table below:

PARAMETER	ANALYTICAL RESULT	FINITE ELEMENT RESULT
Maximum circumferential stress (MPa)	91.00	93.89
Maximum longitudinal stress (MPa)	44.60	47.25
Maximum radial deflection (mm)	0.81	0.71
Maximum longitudinal deflection (mm)	0.45	0.40

Mesh Convergence Analysis

Mesh convergence is carried out on the FE model to verify the accuracy of the meshing used for analysis.

The variable of interest used for the mesh convergence analysis is the radial deflection on the pressure vessel. The element size was varied from 0.5 to 0.085 with the number of elements increasing from 2,296 to 58,213.

Global Seed Size	Number of Elements	Radial Deflection (10^{-4}m)
0.500	2,296	3.637
0.300	5,094	5.054
0.250	7,004	6.362
0.200	10,756	6.814
0.150	19,299	7.054
0.125	27,068	7.090
0.100	41,897	7.097
0.095	46,472	7.092
0.090	52,027	7.087
0.085	58,213	7.098

A plot of the radial deflection with number of elements is provided in the figure below:

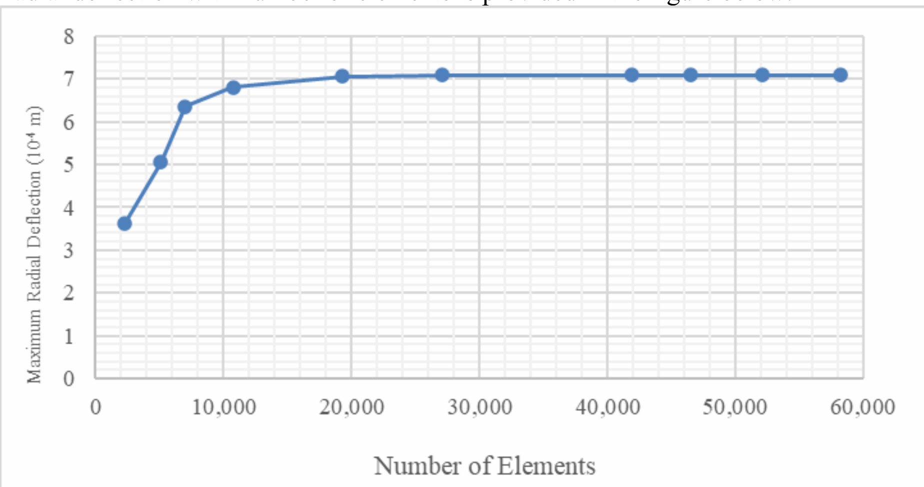

Fig 4.3: Mesh convergence for 20mm steel vessel

From the analysis above, we observe that mesh convergence occurs at around 20,000 elements. Beyond this, the desired improvement in accuracy does not justify the increased computational time and memory usage.

Sensitivity On Wall Thickness

Sensitivity was carried out on the response of the pressure vessel using stainless steel with wall thicknesses of 16 mm and 24 mm respectively, in addition to the initial model of 20 mm.

The following results were obtained:

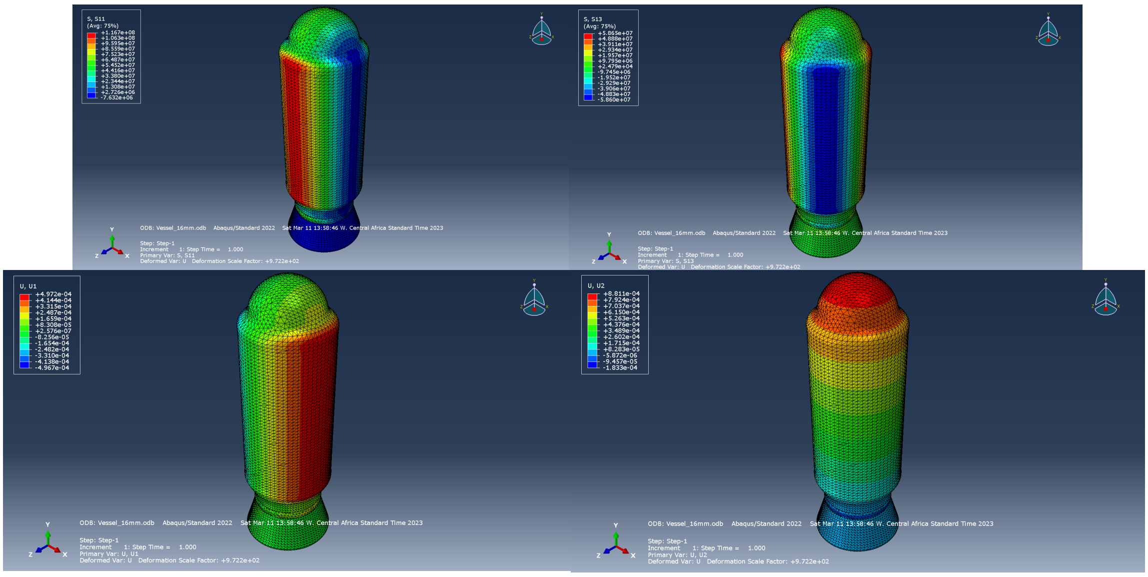

Fig 4.4: Stresses and deflections for 16mm steel vessel

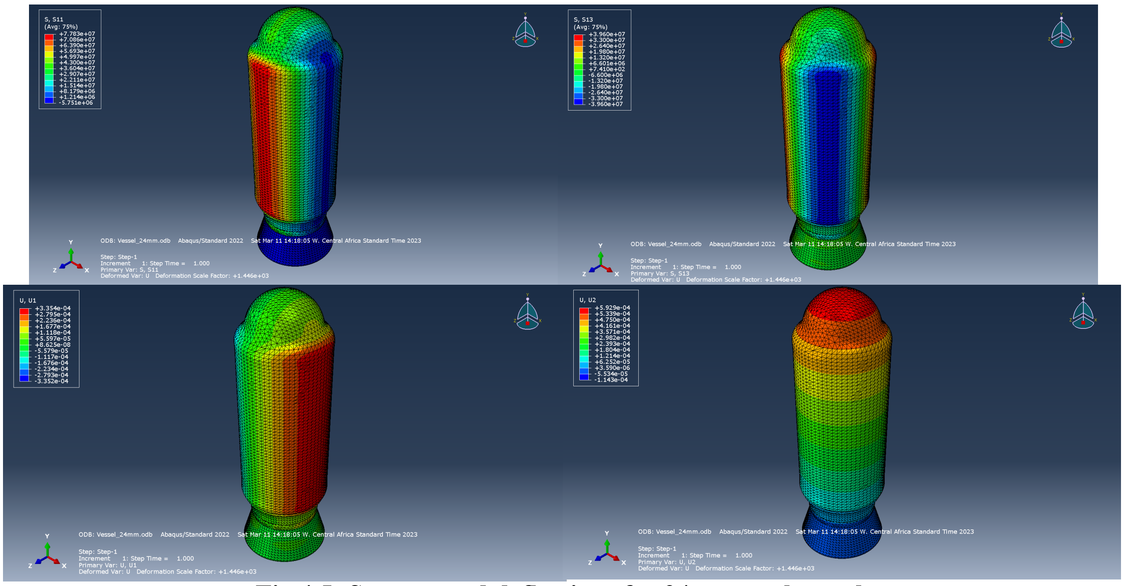

Fig 4.5: Stresses and deflections for 24mm steel vessel

Comparison of the results obtained for different wall thicknesses is shown in the table below:

PARAMETER	20 mm	16 mm	24 mm
Maximum circumferential stress (MPa)	93.89	116.7	77.83
Maximum longitudinal stress (MPa)	47.25	58.65	39.60
Maximum radial deflection (mm)	0.7097	0.8811	0.5929
Maximum longitudinal deflection (mm)	0.4000	0.4972	0.3354

The maximum allowable working stress for the stainless steel material as obtained from the ASME BPVC, S = 1.15 x 10^8 Pa [5].

We observe that the 16mm stainless steel vessel does not satisfy this requirement. The requirement is however satisfied by both the 20mm and 24mm variants. The 24mm option produces minimal stresses and deflections. However, the additional cost does not justify the expected additional benefits, hence, the 20mm option is the most technically and economically feasible option.

Sensitivity On Material Properties

Sensitivity was carried out on the response of the pressure vessel using 20mm glass fiber reinforced plastic (composite) and cast iron respectively, in addition to the initial stainless steel model.

The following results were obtained:

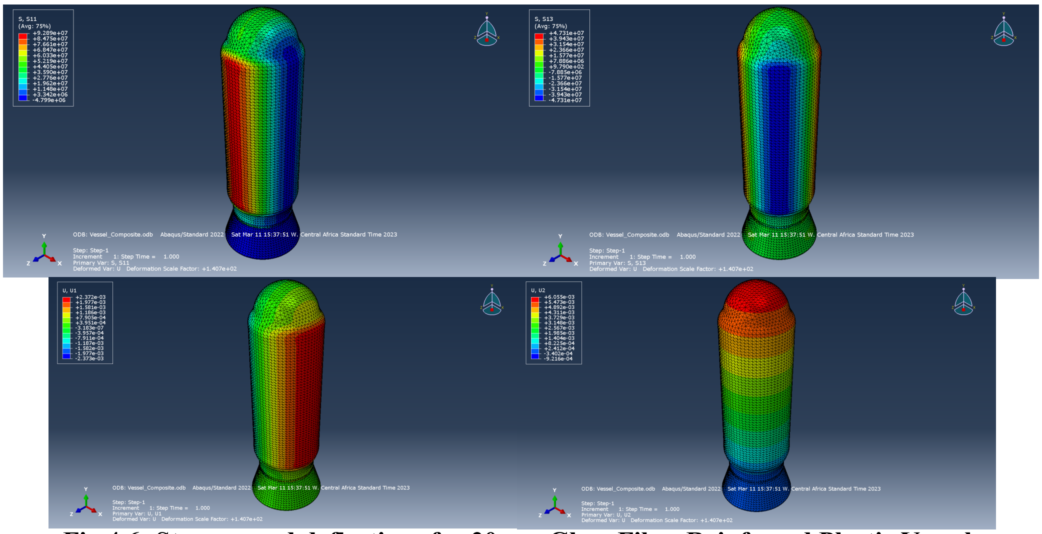

Fig 4.6: Stresses and deflections for 20mm Glass Fiber Reinforced Plastic Vessel

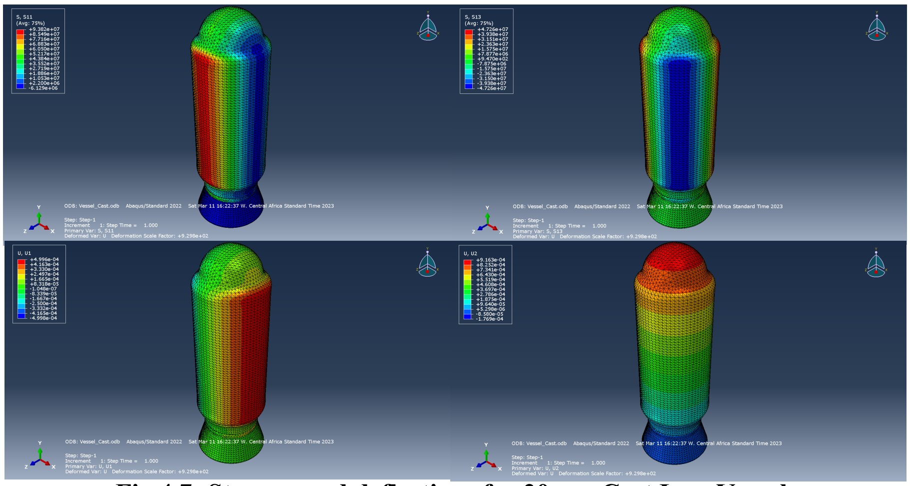

Fig 4.7: Stresses and deflections for 20mm Cast Iron Vessel

Comparison of the results obtained for different materials is shown in the table below:

PARAMETER	Stainless Steel	Glass Fiber Reinforced Plastic	Cast Iron
Maximum circumferential stress (MPa)	93.89	92.89	93.82
Maximum longitudinal stress (MPa)	47.25	47.31	47.26
Maximum radial deflection (mm)	0.7097	6.055	0.9163
Maximum longitudinal deflection (mm)	0.4000	2.372	0.4996

We observe that the cast iron variant is subjected to similar stress as the stainless steel vessel. However, the deflections are slightly higher.

The glass fiber reinforced plastic variant, on the other hand, also experiences similar maximum stress as the stainless steel variant. However, the deflections in this case are significantly higher compared to stainess steel.

VI. Conclusion

The aim of our study was to analyse an LPG storage tank using FEA, to verify the analytical design of the vessel, and then extend the analysis to other materials of construction and wall thickness to strive to provide some perspective on optimal material and size selection.

The FE Model for the LPG tank studied produced similar stresses and deformation as the analytical model, when subjected to the design pressure of 1.8 MPa. To ensure that mesh convergence has been achieved, mesh refinement was carried out. Convergence was achieved at around 20,000 elements.

When the wall thickness of the tank studied is reduced to 16mm, we observe that the stress on the walls of the vessel exceeds the maximum allowable working stress for the material of construction. Increasing the thickness, on the other hand, reduces the stress on the walls of the vessel. We note that this reduction in stress does not necessarily justify the additional cost of material.

In the final analysis, the use of Glass Fibre Reinforced Plastic (GFRP) and Cast Iron as construction materials for the vessel were evaluated. We note significant increase in deformation using GFRP compared to the initial model.

The FE analysis carried out justifies the material and wall thickness selection for the LPG tank and affirms the use of FEA in design which allows conceptual designs to be modelled, tested, refined, and optimized at a minimal cost, before a final design is selected for manufacturing.

The results of this FE analysis can further be validated by experimental results. This will involve using gauges to physically measure stresses and deflections on the LPG vessel during operation.