IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 21, Issue 3 Ser. 2 (May. – June. 2024), PP 39-47
www.iosrjournals.org

Validation Of A Mathematical Model For Semi-Elliptical Fins: A Combined Experimental And Finite Element Analysis Approach

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Abstract:
The study of heat transfer in fins is of utmost importance at a technological level and determining the effectiveness of the design can be expensive due to the need for experimentation. This article comparatively presents the results obtained from the analysis of a fin subjected to convection by different methods, namely physical experimental, mathematical modeling and simulation by Ansys® Software. The results reflect a very acceptable coherence between them which determines that the simulations made with the help of the computer (finite element methods) are a faithful reflection of the physical experiment and the theory that supports the phenomenon. It is concluded that simulations are a very good option for estimating the behavior of a heat transfer phenomenon, thereby avoiding the complicity of the mathematical model and the cost of the experimental model.
Background*: Experimental and theoretical methods applied to real phenomena that can be very complicated or expensive, these can be replaced by computer simulation methods, Ansys, a software for analysis on PC, is a good option. In this software you do not have to have advanced knowledge of the subject to carry out a simulation, only the boundary conditions are enough (in the case of heat transfer phenomena). The simulation is a fast and efficient calculation and can be repeated for different models or conditions.*
Materials and Methods*:* The cylindrical fin shown in the previous figure will be considered. The cross-sectional area is $A_c = \pi R^2$, where R is the radius of the glaive and the perimeter is $P = 2\pi R$. Both A_c and R are uniform, that is, they do not vary along the fin in the x direction (fin length).
Results*: The empirical relationships used in this article to determine convective coefficients are very close to each other and are verified from the experimental results, since using these convective coefficients in simulation and theory, curves are graphed very close to each other.*
Conclusion: *The methods used yield results with errors no greater than 2% compared to the theoretical methods and no greater than 7% with respect to the experimental ones. The largest errors recorded in the bars (between the experimental with Ansys or theory) were due to the fact that the screw that held the wooden plug (insulated end) to the fin was transferring heat to the outside. This is observed because the slope of the graph in the last points of the graph do not have a value of zero (as the theory would predict due to the insulation) and since the cross section in the copper was smaller than in the aluminum, this negative slope is accentuated even more since in proportion it affects the copper more (better conductor) than aluminum.*
Key Word*: Semiaslade Fin, Mathematical Modeling, Physical Experiment, Simulation.*

Date of Submission: 24-05-2024 Date of Acceptance: 04-06-2024

I. Introduction

A fin is an aggregate of material that is placed on a surface that transfers heat. This addition allows for increased heat transfer to the medium surrounding the surface. Some examples of fins are those used to cool electronic components and in automobile radiators. Analyzing, the fins increase the transfer area which helps dissipate heat more quickly. The drawback of a fin is observed in that the added area does not transfer heat like the original surface, since as is known, to transmit heat from one medium to another there must be a temperature difference between said mediums (the greater the difference the more heat is dissipated), and in a fin there is a distribution of temperatures along it with the highest temperature near the original surface and the lowest at the end. This fact makes a fin interesting for study since in general it is desired to optimize the material used to transfer heat to the maximum.

Generally a fin is thin in some direction, which results in the fact that there are no important temperature variations in that direction, then assuming conduction along the fin as if it were one-dimensional, thus simplifying the analysis. In this work, in addition to the comparison between the results of the methods that determine the temperature distribution, the results of different existing algorithms that obtain the convection heat transfer coefficient and the results were compared, based on experimental data. yielded by a method proposed in this article to obtain the real convective coefficient from the experimental temperature distribution, the formal relations for heat transfer in a needle fin and the use of least squares.

II. Development

Theoretical Analysis

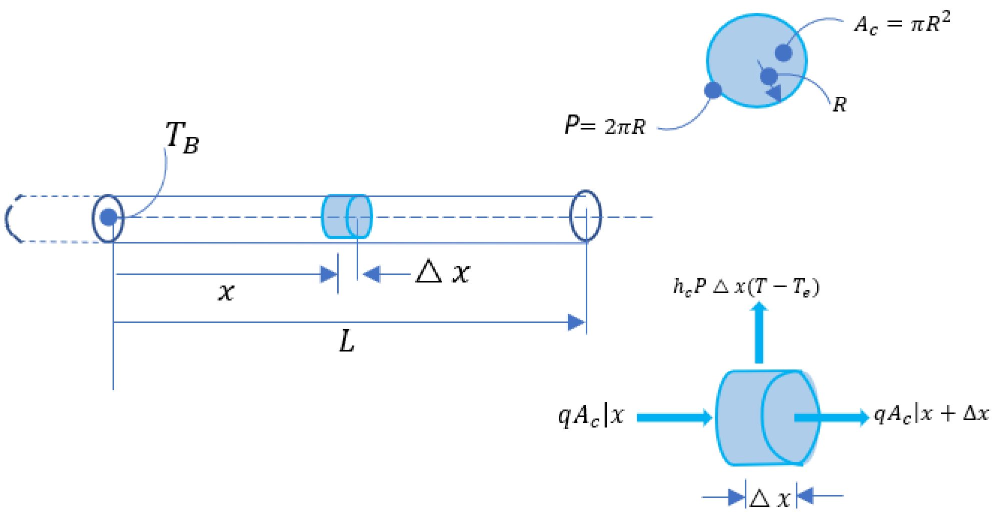

Figure 1: Cylindrical bar Obtained from reference

Obtaining the differential equation and boundary conditions

The cylindrical fin shown in the previous figure will be considered. The cross-sectional area is $A_c = \pi R^2$, where R is the radius of the glaive and the perimeter is $P = 2\pi R$. Both A_c and R are uniform, that is, they do not vary along the fin in the x direction (fin length). The principle of conservation of energy is applied to an element of the fin between the points x and $x + \Delta x$ (see previous Figure 1). Heat can enter and leave the element by conduction along the fin and can also be lost by convection (h_c) from the surface of the element at temperature T to the surrounding fluid at temperature T_e. The area of the element is $P * \Delta x$; therefore,

$$qA_c|_x - qA_c|_{x+\Delta x} - h_c P \Delta x (T - T_e) = 0$$

Dividing by Δx and doing $\Delta x \rightarrow 0$ (differential element) we obtain

$$-\frac{d}{dx}(qA_c) - h_c P(T - T_e) = 0 \quad (0)$$

For a cylindrical fin, Ac is independent of x; using Fourier's law

$$q = -k\frac{dT}{dx}$$ where k is the thermal conductivity.

keeping k constant, we obtain

$$kA_c\frac{d^2T}{dx^2} - h_c P(T - T_e) = 0 \quad (1)$$

which is a second-order ordinary differential equation for $T = T(x)$. Assuming conduction along the fin as a one-dimensional process causes convective heat loss from the sides of the fin to appear in the differential equation. Now it is necessary to know the boundary conditions for equation (1) Since we want to study the behavior of the fin itself and we have the temperature condition at the end (see experiment)

$$T|_{x=0} = T_B \tag{2}$$

At the other extreme, the fin loses heat due to Newton's law of cooling

$$-A_c k \frac{dT}{dx}\bigg|_{x=L} = A_c h_c (T|_{x=L} - T_e) \tag{3a}$$

where the convection heat transfer coefficient is, in general, different from that of the lateral faces of the fin because the geometry is different. However, since the area of the end AC is small and in our experiment it was isolated with a piece of wood, it is concluded that

$$\frac{dT}{dx}\bigg|_{x=L} \cong 0 \tag{3b}$$

Furthermore, this boundary condition is easier to use than equation (3a).

Temperature Distribution

To make the algebraic manipulation simpler, we take $\theta = T - T_e$ and $\beta^2 = \frac{h_c P}{k A_c}$ where

H_c = convective coefficient
P =Fin perimeter
k =Thermal conductivity
A_c =Transverse area to the heat flow in the fin

then equation (1) becomes

$$\frac{d^2\theta}{dx^2} - \beta^2\theta = 0 \tag{4}$$

For constant β, equation (4) has the solution

$$\theta = C_1 e^{\beta x} + C_2 e^{-\beta x}$$

or well

$$\theta = B_1 senh(\beta x) + B_1\, cosh(\beta x)$$

The second form turns out to be more convenient; therefore, we have

$$T - T_e = B_1 senh(\beta x) + B_2\, cosh(\beta x) \tag{5}$$

Using the two boundary equations [(2) and (3b)] two algebraic equations are obtained for the unknown constants B_1 and B_2

$$T_B - T_e = B_1 senh(0) + B_2\, cosh(0);\quad B_2 = T_B - T_e$$
$$\frac{dT}{dx}\bigg|_{x=L} = \beta = B_1 senh(\beta L) + B_2\, cosh(\beta L);\quad B_1 = -B_2\, tanh(\beta L)$$

Substituting B_1 and B_2 in equation (5) and reordering we obtain the temperature distribution along the fin. This equation is what will be called the theoretical equation.

$$\frac{T - T_e}{T_B - T_e} = \frac{cosh[\beta(L - x)]}{cosh[\beta L]} \text{ where } \beta = \sqrt{\frac{h_c P}{k A_c}} \tag{6}$$

Determination of convection coefficients

There are numerous empirical relationships in the literature for determining average free convection heat transfer coefficients. The following functional represents this phenomenon for different circumstances

$$\overline{Nu_f} = C \qquad (Gr_f Gr_f)^m \tag{7}$$

where the subscript f indicates that the properties in the dimensionless groups are evaluated at the film temperature and furthermore, given that the Grashof (Gr_f) and Prandtl (Gr_f) numbers have a characteristic dimension, this will vary depending on the problem to be studied: for a vertical plate is the height of the plate L and for our case, that is, a horizontal cylinder, it is considered as d (diameter). The mathematical definition of these numbers is shown below.

And since the average Nusselt Number ($\overline{Nu}$) has the following mathematical definition, having the other variables and the Nusselt Number (by equation 11) we can solve for the average convective coefficient h_c

$$\overline{Nu}_d = \frac{\overline{h_c} d}{k} \tag{8}$$

The Grashof number can be physically interpreted as a dimensionless group that represents the ratio of buoyancy forces to viscous forces in the free convection flow system and is defined as

$$Gr = \frac{g\beta(T_\omega - T_\infty)d^3}{\nu^2} \tag{9}$$

where
β = the inverse of the absolute temperature $[K]$ at which the film is
T_ω = surface temperature
T_∞ = temperature in the convective medium
ν = Kinematic viscosity (of the air in our experiment) ν
d = diameter
g = gravity

The Prandtl number is a dimensionless parameter that relates the relative thicknesses of the hydrodynamic and thermal boundary layers. The kinematic viscosity of a fluid carries information about the speed at which momentum can diffuse through the fluid due to molecular motion. In short, it is the link between the velocity field and the temperature field and is mathematically defined as:

$$Pr = \frac{c_p \mu}{k} \tag{10}$$

where

μ = Dynamic viscosity (of the air in our experiment)
k = thermal conductivity (of the air in our experiment)
c_p =Specific heat at constant pressure (of air in our experiment)

Once the necessary parameters have been defined to find the convective coefficient, we also have the mathematical definition of the Nusselt number (alternative to (8)) for horizontal cylinders found in the bibliography [2].

$$Nu_d = 0.60 + 0.387\left\{\left[\frac{Gr_d Pr}{\left[1+\frac{0.559}{Pr^{\frac{9}{16}}}\right]}\right]^{\frac{1}{6}}\right\} \tag{11}$$

Furthermore, as an alternative relationship to the previously explained study, there is another simplified formula for horizontal cylinders. This formula will be used to obtain the value through another method.

$$h = 1.32\left(\frac{\Delta T}{d}\right)^{1/4} \tag{12}$$

where

ΔT is the temperature difference between the surface and that of the fluid
d is the diameter of the fin

Experiment
Characteristics

In the experiment, two cylindrical fins made of different materials were analyzed on a test bench: aluminum (diameter 1”) and copper diameter (0.5”). The fins are recessed at one end and set into an acrylic frame at the other end. The embedded end is located inside a tank where there is water vapor in such a way that using a thermometer that is inside the tank the temperature is measured at one end, the other end has as a boundary condition that it does not transfer heat since it was placed a wooden plug screwed to the fin at the end. In addition, along the fin there are 9 holes placed at different distances, in which the temperature can be sensed with the help of a thermocouple that reflects the temperature with the help of a multimeter that works as a transducer and digital encoder. In order to compare methods for determining convective coefficients, it was decided to place a semi-insulating medium (cotton for healing) around the entire length of the fin with an approximate thickness of 1” radial.

Procedure

Once the cotton was placed, it was decided to open the valve that allows the steam to pass to the chamber in which the end of the fin is located. The temperature was stabilized by maintaining a constant pressure with the help of the reading of a dial manometer and we waited until the phenomenon was no longer transient (repeated measurements were made over time until the temperature values did not change) for approximately 2 hr. In a permanent state, the measurements corresponding to each point along the metal fin were taken, the distances at which the test holes are located and the diameters of each of the metal rods (fins) were measured and finally the the temperature on the surface of the cotton (at the ends) in order to obtain the average convective coefficients.

Used materials

- Aluminum bar with a circular cross section with a diameter of 1” and a length of 88 cm.
- Copper with a circular cross section with a diameter of 1/2” and a length of 88 cm.
- Thermocouple zone
- Steam service
- Dial temperature and pressure gauges
- Cotton
- Scotch tape
- Measuring tape

The following figures reflect the test bench and the fins

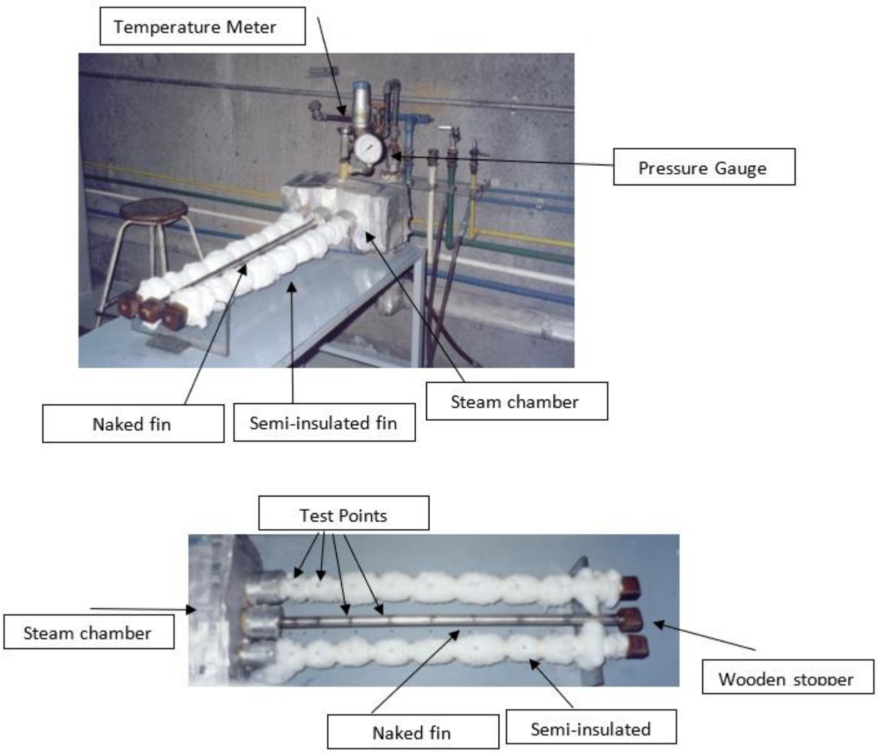

Figure 1: Test bench and fins

Simulation

The fin was modeled as a 3-dimensional solid partial cylinder (see figure). At one end there was a constant temperature condition and at the other an insulated end. The curved wall is where the convective load was applied and the flat side faces were considered insulated (heat flux = 0), since as the fin has the same radial temperature distribution, there is no heat transfer to the other body of the fin. A partial cylinder was decided since, as the academic version was used, it only allows the use of a small number of nodes that, having used the entire cylinder, would have been exceeded.

III. Results

Theoretical Analysis

Since the theoretical temperature distribution is given by the following form and the only unknown variable is h_c, different methods (mentioned in the theoretical development) were tested to determine it.

$$\frac{T-T_e}{T_B-T_e} = \frac{cosh[\beta(L-x)]}{cosh[\beta L]} \quad \text{where} \quad \beta = \sqrt{\frac{h_c P}{kA_c}}$$

Applying the Churchill and Chu equation for wide ranges of $GrPr$ r in horizontal cylinders and the simplified method, the following data are obtained:

Aluminum Facts

Temperature at x = 0 on the cotton [ºC]	33
Temperature at x = 88 cm on cotton [ºC]	26
Average temperatures on cotton [ºC]	29.5
Temperature difference between average and ambient (Tω-T∞) [ºC]	10.5
Beta β[K^{-1}]	0.00331
g (gravity) [m/s^2]	9.78
ν = Kinematic viscosity (of the air in our experiment) [m^2/s]	15.68
d = diameter (includes cotton) [m]	0.0762
Grashof	611615
μ = Dynamic viscosity (of the air in our experiment) [kg/m s *10^5]	1.983
k= thermal conductivity (of the air in our experiment) [W/mºC]	0.02624
C_p =Specific heat at constant pressure (of air in our experiment) [kJ/kgºC]	1.0057
Prandtl	0.708
Grashof per PrandtlGr*Pr	4.33*10^5
Nuselt Nu using (11)	11.52
Solving for hc of (8) [W/m^2K]	3.97
Hc using (12) [W/m^2K]	4.52
Average hc's [W/m^2K]	4.24

Copper Facts

Temperature at x = 0 on the cotton [ºC]	35
Temperature at x = 88 cm on cotton [ºC]	25
Average temperatures on cotton [ºC]	30
Temperature difference between average and ambient (Tω-T∞) [ºC]	11
Beta β[K^{-1}]	0.003298
g (gravity) [m/s^2]	9.78
ν = Kinematic viscosity (of the air in our experiment) [m^2/s]	15.68
d = diameter (includes cotton) [m]	0.0762
Grashof	369575
μ = Dynamic viscosity (of the air in our experiment) [kg/m s *10^5]	1.983
k= thermal conductivity (of the air in our experiment) [W/mºC]	0.02624
C_p =Specific heat at constant pressure (of air in our experiment) [kJ/kgºC]	1.0057
Prandtl	0.708
Grashof per PrandtlGr*Pr	2.62*105
Nuselt Nu using (11)	10.04
Solving for hc of (8) [W/m^2K]	4.15
Hc using (12) [W/m^2K]	4.58
Average hc's [W/m^2K]	4.36

Aluminum Facts

Furthermore, an approximation of h_c was made using the graph of the real temperature distribution and the theoretical form of the distribution in a fin like the one in the experiment. With the help of an electronic spreadsheet, the squared differences between the theoretical result and the real distribution were calculated, then the objective was to minimize the squared differences from the modification of h_c, which gave a result that was also very close to the others. convective coefficients.

Copper = 4.44 W/m^{2}K
Aluminum = 4.28 W/m^{2}K

By making a numerical average between the convective coefficients, the value was obtained that was used both in the theoretical calculation and in the simulation by Ansys as data.

Simulation by Ansys

In the simulation by Ansys, data was obtained that is represented in the General Results section and the graph delivered by Ansys when giving a Nodal solution on the graph is shown here.

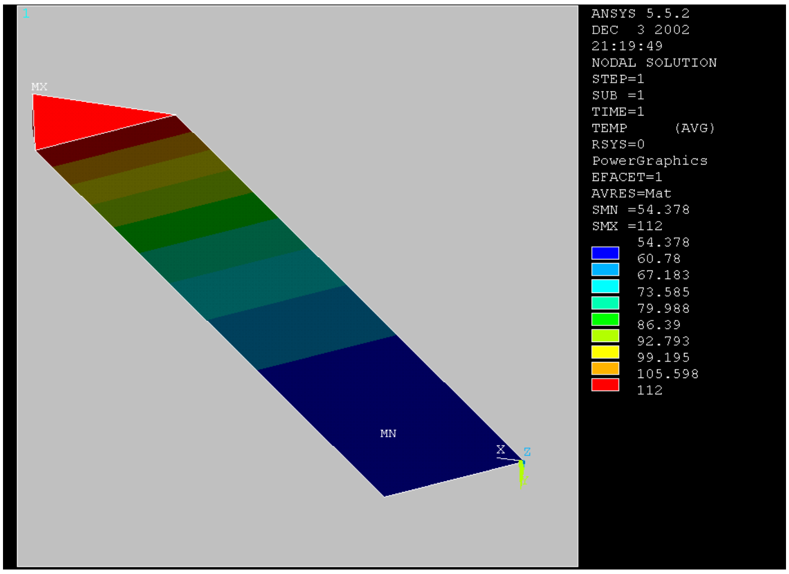

Figure 2: Nodal solution

IV. General Results

The following table and graph summarize the results of the calculations made for Aluminum

Table Num. 1: Calculations made for Aluminum.

Length (m)	Temp real (ºC)	Temp theory (ºC)	Ansys Temperature (ºC)	Deviation % Ansys theory	Deviation % Ansysr-real	Deviation % theory-real %
0	112	112.000	112	0.00	0.00	0.00
0.16	93	90.732	91.463	0.81	1.65	2.44
0.191	89	87.358	87.107	0.29	2.13	1.85
0.237	84	82.744	82.156	0.71	2.20	1.49
0.309	78	76.400	75.275	1.47	3.49	2.05
0.395	71	70.092	68.291	2.57	3.82	1.28
0.485	66	64.817	63.163	2.55	4.30	1.79
0.578	61	60.644	58.92	2.84	3.41	0.58
0.676	56	57.524	56.078	2.51	0.14	2.72
0.777	54	55.579	54.576	1.80	1.07	2.92
0.888	52	54.853	54.378	0.87	4.57	5.49
			Average	1.49	2.43	2.06

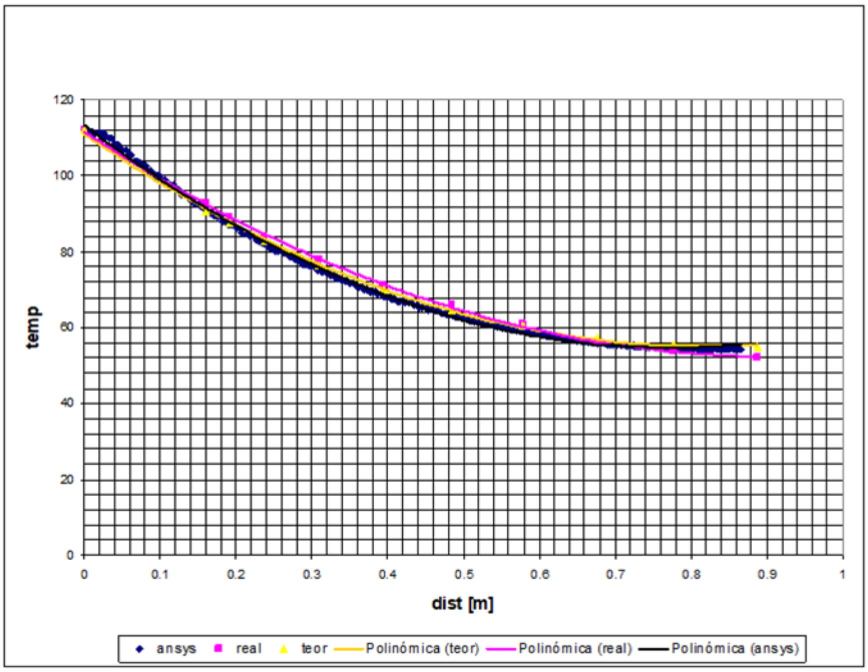

Figure 3: Graphic comparison of methods for aluminum fin

The following table and graph summarize the results of the calculations made for Copper

Table Num. 1: Calculations made for Copper.

Length (m)	Temp real (ºC)	Temp theory (ºC)	Ansys Temperature (ºC)	Deviation % Ansys theory	Deviation % Ansysr-real	Deviation % theory-real %
0	112	112.000	112	0.00	0.00	0.00
0.16	94	89.586	89.998	0.46	4.45	4.93
0.19	91	86.162	86.567	0.47	5.12	5.61
0.238	88	81.131	81.319	0.23	8.22	8.47
0.31	82	74.536	74.059	0.64	10.72	10.01
0.395	75	68.074	67.347	1.08	11.36	10.17
0.485	68	62.618	62.03	0.95	9.62	8.60
0.578	56	58.313	57.688	1.08	2.93	3.97
0.678	51	55.046	54.333	1.31	6.13	7.35
0.78	48	53.049	52.428	1.18	8.45	9.52
0.89	45	52.323	51.934	0.75	13.35	14.00
			Average	0.74	7.30	7.51

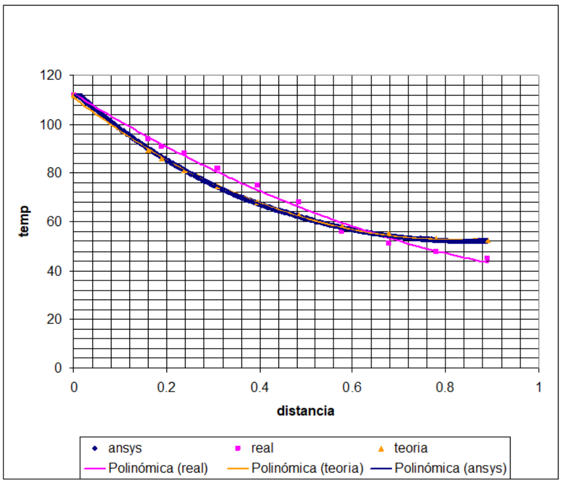

Figure 3: Graphic comparison of methods for Copper fin.

Note: For the graphs all the data from the nodes provided by Ansys were used, but for the tables only the temperatures corresponding to those distances are shown.

V. Conclusion

The empirical relationships used in this article to determine convective coefficients are very close to each other and are verified from the experimental results, since using these convective coefficients in simulation and theory, curves are graphed very close to each other.

The methods used yield results with errors no greater than 2% compared to the theoretical methods and no greater than 7% with respect to the experimental ones.

The largest errors recorded in the bars (between the experimental with Ansys or theory) were due to the fact that the screw that held the wooden plug (insulated end) to the fin was transferring heat to the outside. This is observed because the slope of the graph in the last points of the graph do not have a value of zero (as the theory would predict due to the insulation) and since the cross section in the copper was smaller than in the aluminum, this negative slope is accentuated even more since in proportion it affects the copper more (better conductor) than aluminum.

Experimental and theoretical methods applied to real phenomena that can be very complicated or expensive, these can be replaced by computer simulation methods, Ansys, a software for analysis on PC, is a good option. In this software you do not have to have advanced knowledge of the subject to carry out a simulation, only the boundary conditions are enough (in the case of heat transfer phenomena). The simulation is a fast and efficient calculation and can be repeated for different models or conditions.

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