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Backwater in Circular Channels with Zero Slope

S. David Graber, P.E. [1]

[1] *Consulting Engineer, Stoughton, Massachusetts, USA*

Abstract:
The author previously published a paper that addressed spatially increasing flow in collection conduits [1]. Analyses were given for circular and rectangular channels flowing with free-surface and full-channel flow. That paper referred to, but did not present, generalized solutions obtained by the author for horizontal, circular channels with a free or submerged discharge with constant or spatially-increasing flow. The author has used those solutions for the design of skimmer channels at wastewater treatment plants. Those solutions are presented here. Examples from the author's practice are presented.
Keywords: *Dimensional analysis; Hydraulic design; Numerical Analysis; Open channel flow; Pipe flow.*

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I. Introduction

The author previously published a paper that addressed spatially increasing flow in collection conduits [1]. Analyses were given for circular and rectangular channels flowing with free-surface and full-channel flow. That paper referred to, but did not present, generalized solutions obtained by the author for horizontal, circular channels with a free or submerged discharge with constant or spatially-increasing flow. The author has used those solutions for the design of skimmer channels at wastewater treatment plants. Those solutions are presented here.

Skimming pipes typically consist of a horizontal circular pipe with a slot cut symmetrically about the vertical axis of the pipe. The slot typically comprises about 60 degrees of arc. The edges of the slot serve as weirs over which scum and/or surface grease flows into the pipe when the pipe is rotated (see Fig. 1). The skimming pipes are commonly connected in series (and operated individually) when parallel rectangular basins are employed, and the downstream-most skimmer discharges freely into a scum or grease sump or box. Pipes typically range from 8 to 18 inches (20 to 45 cm) in diameter in 2-inch (5 cm) increments. The water head on the weir when the pipe is rotated is usually on the order of 1/2 to 3/4 inch (1.3 to 1.9 cm) so that the high velocity of flow over the weir will efficiently remove floating scum and grease while minimizing water volume.

The Keifer-Chu method for constant flow [2] is not applicable to channels of zero slope (although it is for negative slope) because, as noted by Chow [3, p. 267], for zero slope the varied-flow functions become meaningless. Hager's [4] explicit approach for circular cross sections with constant flow also applies only for nonzero slopes.

The importance of proper hydraulic design is made clear by the report of problems associated with the Massachusetts Water Resources Authority's newly-modified Deer Island Wastewater Treatment Plant [5]: "The [secondary clarifier scum] system was found to be inadequate for removing scum from the clarifier furthest away from each scum drain box. The scum tube was found to be hydraulically overloaded, because it relied on 60 feet of scum tube to move collected scum from the far clarifier to the scum box….[Control system] changes…did not fully correct the underlying hydraulic problem."

II. Free Discharge

The equation for free discharge is of the following functional form [1, Equation (38b)]:

$$\frac{y_o}{\sigma} = \Phi\left(\frac{fL}{\sigma}, \frac{y_c}{\sigma}\right) \qquad (1)$$

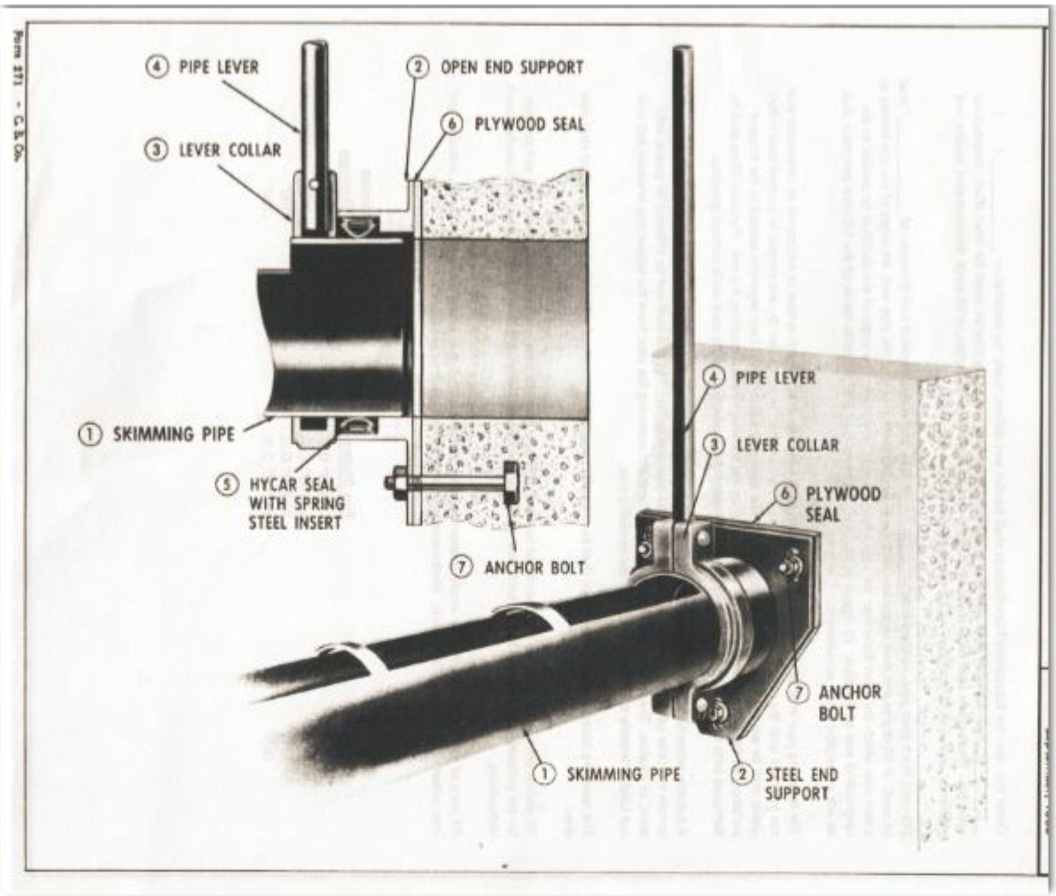

Fig. 1. Skimming Equipment (after Envirex, Waukesha, Wisconsin)

in which y = depth of flow, y_o = value of y at the upstream end of the channel; y_c = critical depth; σ = characteristic cross-sectional dimension, here taken to be the skimmer diameter d_o; f = Darcy-Weisbach friction factor; and L = channel length. We have specifically:

$$\frac{y_o}{d_o} = \Phi\left(\frac{fL}{d_o}, \frac{y_c}{d_o}\right) \quad \text{for} \quad \mathbf{F}_L = 1 \qquad (2)$$

A plot of y_o / d_o versus y_c / d_o with fL / d_o as a parametric variable is presented as Figure 2. The procedure that led to a generalized solution is as follows. For values of the central angle θ of the water surface from 0 to 180 degrees, successive values were tabulated using geometric elements formulae of $\cos(\theta / 2)$; y / d_o; D / d_o where D is hydraulic depth; A / d_o^2 where A is cross sectional area of flow; and $Z / d_o^{2.5} = Q_{crit} / (\sqrt{g} d_o^{2.5})$ in which Z is the critical-flow section factor, Q_{crit} is critical flow rate, and g is acceleration of gravity. Then, with no loss in generality, we assume $d_o = 1$ ft, and calculate y_c, $Q_{crit} = \left[Q_{crit} / (\sqrt{g} d_o^{2.5}) \right] \sqrt{32.2} \left(d_o \right)^{2.5}$, and Froude Number at the downstream end of the channel $\mathbf{F}_L = (Q_{crit} d_o^{2.5}) / \left[\left(A / d_o^2 \right) \sqrt{32.2} \left(D / d_o \right) \right]$. The latter was done as a check to assure $\mathbf{F}_L = 1$ for every value of θ, which it exactly was. Then, using a formulation given by Graber [1] for numerical solution of open channel conduits and the Newton-Raphson method (also described in [1]), the author computed, using suitably small intervals along the channel length (e.g., 100 ft channel length and 1 ft intervals) upstream from critical depth with given values of f and corresponding values of Manning's n, successive values of upstream depth y_o and corresponding y_o / d_o.

More recently, Shang *et al*. [6] derived explicit dimensionless relationships for critical depth in closed conduits of various cross sections. For circular sections, they provide a relationship which is accurate to within $\leq$0.182% over the range of y_c / d_o equal to 0.005 to 1. That relationship is given by:

$$\frac{y_c}{d_o} = \left(1 + 3.83\varepsilon_c^{-2.1454} - 3.2\varepsilon_c^{-2.1}\right)^{-0.115} \quad , \quad \varepsilon_c = \frac{Q^2}{g d_o^5} \qquad (3)$$

The author found that relationship to give results virtually identical to those shown on Fig. 2. By using Equation (3), some of the steps described in the preceding paragraph are eliminated (those prior to $Z / d_o^{2.5}$).

Although the author has not found a need to do so, Graber [1, p. 73] gives a simple device that may be used to extend the generalized solution for constant-flow to horizontal channels with submerged discharges ($\mathbf{F}_L < 1$).

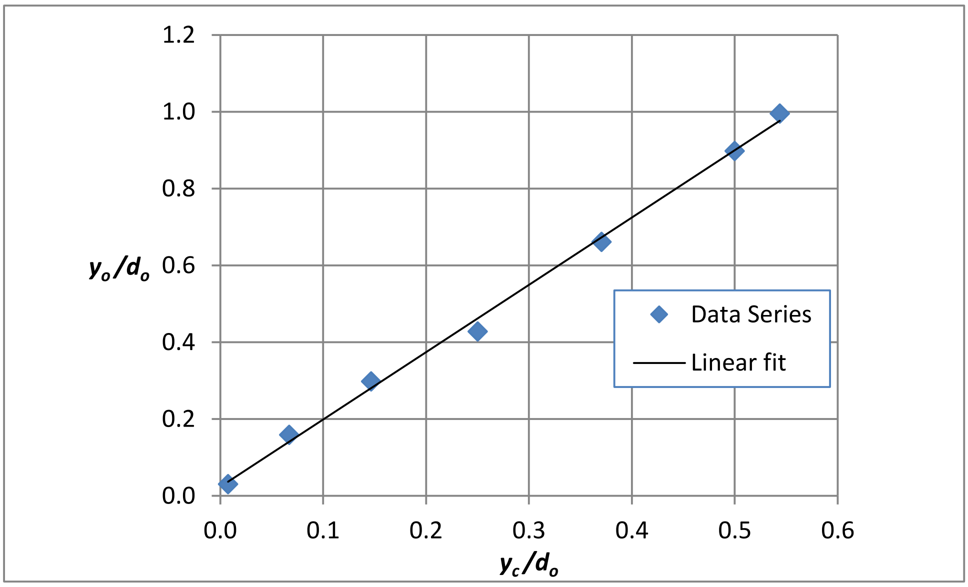

Fig. 2. Circular conduit chart for constant flow with free discharge

III. Skimming Pipe

For the skimming pipe, the edge of the slot in the upright position must be above the maximum water level. The skimmer must be capable of tipping the slot edge the necessary amount below the minimum water level, under which conditions the pipe sector below the weir must have sufficient hydraulic capacity. The skimmer pipe design is thus related to the basin outlet design insofar as the latter affects the water surface level variations.

In some cases the skimming pipe discharges directly and freely to the downstream structure (e.g., a scum manhole). In other cases the skimming pipe *per se* is followed by a freely-discharging pipe which imposes a tailwater depth. Although the relatively short length of the skimming pipe itself is such that friction can generally be neglected (more about which is said below), there is thus the added complexity of having a subcritical downstream depth, i.e., Froude Number $\mathbf{F}_L < 1$. The functional form of the equation in that case in given by Graber [1, Equation (38c)]:

$$\frac{y_o}{d_o} = \Phi\left\{\frac{y_L}{d_o}, \mathbf{F}_L\right\} \text{ for } S_o = 0, f = 0 \qquad (4)$$

$\mathbf{F}_L$ in the above equation can be replaced by $Q_L / \sqrt{g} d_o^{5/2}$, giving a convenient conceptual relationship in terms of pipe capacity Q_L.

Graber [1] mentions a convenient generalized uniform-inflow solution he obtained corresponding to the above equation for the case of horizontal, frictionless channels of circular cross section (such as applicable to skimming pipes). That solution was obtained by applying the momentum equation in the large (i.e., to the entire channel in one step) as presented below.

The author's [1] numerical formulation for open-channel collector conduits can be reduced for zero slope, zero lateral outflow, and zero friction to the following:

$$\Delta y = \frac{Q_1\left(V_1 + V_2\right)}{g\left(Q_1 + Q_2\right)}\left(\Delta V + \frac{V_2 \Delta Q_i}{Q_1}\right) \qquad (5)$$

in which subscripts "1" and "2" denote respectively upstream section 1 and downstream section 2 (upstream being in the direction of the ultimate discharge and downstream being towards the zero inflow end of the conduit); the Δ-delta terms represent the value at section 2 minus the value at section 1, such as $\Delta V = \left(V_2 - V_1\right)$; Δy is increase of water surface elevation between sections 2 and 1; and $\Delta Q_i = q_i \Delta x$ in which $q_i =$ inflow per unit channel length. The flow terms Q_1 and Q_2 are prescribed based on the uniform inflow for which $Q = Q_i\left(x / L\right)$, and V_1 and V_2 are calculated by continuity based on the prescribed flows at the corresponding sections and the flow areas which are known functions of corresponding flow depths.

We consider a single step with the usual conditions of $Q_1 = 0$ and $V_1 = 0$, for which Eq. (5), using the notation of Eq. (4), reduces to:

$$y_o = y_L + \frac{V_L^2 q_i \Delta x}{g Q_L} \qquad (6)$$

in which the terms on the right-hand side are known. Using the terms of Equation (4), the above equation becomes:

$$\frac{y_o}{d_o} = \frac{y_L}{d_o} + \mathbf{F}_L^2 \frac{D_L}{d_o} \qquad (7)$$

in which D_L / d_o is a function of y_L / d_o.

A plot of Eq. (7) reveals an excellent fit by straight lines according to:

$$\frac{y_o}{d_o} = \mathrm{m}(\mathbf{F}_L)\frac{y_L}{d_o} \qquad (8)$$

Further, a parabola provides a good fit of m as a function of $\mathbf{F}_L$: $\mathrm{m} = 1.339\mathbf{F}_L^2 + 1$. Eq. (8) can then be written as:

$$\frac{y_o}{d_o} = \left(1.339\mathbf{F}_L^2 + 1\right)\frac{y_L}{d_o} \qquad (9)$$

IV. First Example

A skimming pipe is to be designed (Tallman Island WWTP, New York City) to skim surface scum in a deep channel 3 ft-6-in. wide with the water surface elevation varying from 14.43 ft to 13.72 ft. The slot along the pipe will be 3 ft long. The pipe will discharge freely into a scum manhole, and will have other characteristics of skimming pipes as described above. Provide for 1 inch of freeboard F and 3/4-inch head h of flow over the pipe slot, and determine the required pipe diameter and centerline elevation. The outer diameter of the pipe equals 1.072 times the inner diameter. Referring to the geometric relationships of Fig. 3, we have:

d_o > 1.072[14.43 – 13.72 + 1/12 + y_o + (3/4)/12] = 0.917 + 1.072 y_o in units of ft.

The required flow capacity is $Q = CLh^{3/2} \cong$ 3.3(3)(0.75/12)$^{3/2}$ = 0.155 cfs. Assume d_o = 14 inches ÷ 12 = 1.167 ft. Then:

$$\frac{Q}{d_o^{2.5}\sqrt{g}} = \frac{Z}{d_o^{2.5}} = \frac{0.155}{(1.167)^{2.5}\sqrt{32.2}} = 0.0186$$

and, from a geometric elements table (e.g., [3, Appendix A], noting that the section factor $Z = Q/\sqrt{g}$), we obtain y_c / d_o = 0.132. For $y_L / d_o = y_c / d_o$ = 0.132 and $\mathbf{F}_L$ = 1, Eq. (9) gives y_o / d_o = 2.339(0.132) = 0.309 and $y_o = 0.309(1.167)$ = 0.360 ft = 4.32 in. From the above inequality, d_o > 0.917 + 1.072(4.32)/12 = 1.303 ft = 15.6 in.

The 14-inch pipe is too small. Assume d_o = 16 inches ÷ 12 = 1.333 ft; then:

$$\frac{Q}{d_o^{2.5}\sqrt{g}} = \frac{Z}{d_o^{2.5}} = \frac{0.155}{(16/12)^{2.5}\sqrt{32.2}} = 0.0133$$

and, from geometric elements, y_c / d_o = 0.112. For $y_L / d_o = y_c / d_o$ = 0.112 and $\mathbf{F}_L$ = 1, Eq. (8) gives y_o / d_o = 2.339(0.112) = 0.262 and $y_o = 0.262(1.333)$ = 0.349 ft = 4.19 in. From the above inequality, d_o > 0.917 + 1.072(4.19)/12 = 1.291 ft = 15.5 in.

Then, from Fig. 3:

$$14.43 + \frac{1}{12} - \frac{\sqrt{3}}{4}\left(\frac{16}{12}\right) < ℄ \text{ El.} < 13.72 - \frac{0.75}{12} - 0.227 + \frac{16/12}{2}$$
$$13.94 < ℄ \text{ El.} < 14.10$$

A centerline elevation of 13.95 ft was chosen. The skimming pipe as designed and constructed is shown on Fig. 3.

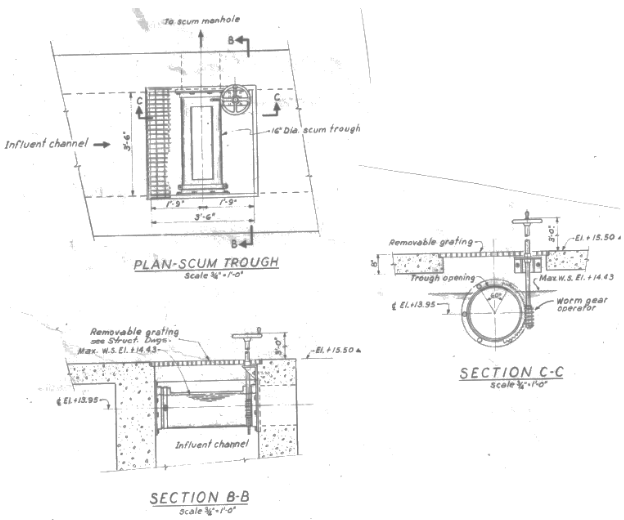

Fig. 3. Skimming pipe with 3-foot slot length

V. Second Example

Two adjacent rectangular final settling tanks operating in parallel, each of 60-ft width, are to have a line of cast iron skimming pipes connected in series and discharging freely into a scum manhole (Tallman Island WWTP, New York City). Each tank has two intermediate flight support beams. The skimming pipes will span the flight support beams and tank walls in such a way that six 20-ft skimming pipes will make up the series. Each skimming pipe will have a total slot length approaching 20 ft, and will be required to operate with a water surface elevation varying from 16.10 to 15.77 ft. The pipes are to have the typical characteristics of skimming pipes described above, and provide for a 3/4-inch head *h* of flow over the pipe slot. The scum manhole has a decant device and can drain continuously into the plant drain system, so any small positive value of freeboard is acceptable. Determine the required pipe diameter and centerline elevation.

The maximum depth in the skimmer occurs when the upstream-most skim pipe is rotated and its flow is conducted to the scum manhole via the five downstream pipe segments totaling 5 x 20 = 100 ft in length. The flow in the 100 ft length is constant, and Figure 2 can be used to determine the backwater depth. The flow entering the upstream section is given by $Q = CLH^{3/2} \cong$ 3.3(20)(0.75/12)$^{3/2}$ = 1.03 cfs. Assume d_o = 16 inches = 1.33 ft. Then:

$$\frac{Q}{d_o^{2.5}\sqrt{g}} = \frac{1.03}{(16/12)^{2.5}\sqrt{32.2}} = 0.0885$$

From geometric elements, at the downstream end of the pipe $y_c / d_o =$ 0.294, $R_c / d_o =$ 0.168, and $A_c / d_o^2 =$ 0.193. Determine f at the downstream end as follows:

For cast iron, $\dfrac{\varepsilon}{4R} = \dfrac{0.00085}{4(0.168)(16/12)} = 0.00095$

$$V_c = Q / A = 1.03 / \left[0.193(16/12)^2\right] = 3.00 \text{ ft/sec}$$

$$\mathbf{R} = \frac{4RV}{\upsilon} = \frac{4(0.168)(1.33)(3.00)}{10^{-5}} = 2.68(10)^5$$

From a Moody Diagram [7] or, e.g., Wood's [8] trivariate regression relationship, we have f = 0.022. Then for the constant flow segment $fL / d_o =$ 0.022(100)/1.33 = 1.65. From Figure 2 for $y_c / d_o =$ 0.294 and $fL / d_o =$ 1.65, we obtain $y_o / d_o =$ 0.46, giving y_o = 0.46(16/12) = 0.61. (For comparison, this is within 1% of the value of 0.603 ft calculated using the Newton-Raphson method employed above.) The value of y_o at the upstream end of the 100-ft constant-flow segment becomes the y_L at the downstream end of the spatially-increasing flow segment. Neglecting friction in that 20-ft segment, Eq. (8) can be employed in the following fashion: for $y_L / d_o =$ 0.46, a geometric elements table gives A / d_o^2 = 0.3527 and D / d_o = 0.3538. Then, A = 0.3527(16/12)^2 = 0.627 sq ft, D = 0.3538(16/12) = 0.472 ft, $V = Q / A$ = 1.03/0.627 = 1.64 ft/sec, $\mathbf{F}_L = V\sqrt{gD} = 1.64 / \sqrt{32.2(0.472)} = 0.421$. From Eq. (8) for $y_L / d =$ 0.46 and $\mathbf{F}_L$ = 0.42, we obtain $y_o / d_o = [1.339(0.421)^2 + 1](0.46) = 0.57$ and y_o = 0.57(16/12) = 0.76 ft. Then, from the geometric relationships analogous to those shown on Fig. 3, d_o > 1.072(16.10 – 15.77 + 0 + 0.76 + 0.75/12) = 1.235 ft x 12 = 14.8 inches. Therefore, the 16-inch skimmer is the best selection. The centerline elevation is then determined as follows:

$$16.10 + 0 - \frac{\sqrt{3}}{4}\left(\frac{16}{12}\right) < \text{℄ El.} < 15.77 - \frac{0.75}{12} - 0.69 + \frac{16/2}{12}$$

$$15.52 < \text{℄ El.} < 15.68$$

A centerline elevation of 15.56 was chosen

It is informative in this case to compare the results of the conservative frictionless analysis above to a more precise numerical analysis. This is the best example with which to do this because the flow is fully developed due to the downstream length. For the numerical analysis, the bisection method is used [9, 10]. Intervals of 20 ft/100 = 0.2 ft were used. The resulting calculated upstream value is 0.694 ft, compared to 0.76 given by the aforementioned conservative analysis. The latter is greater than the numerical analysis by (0.76 – 0.694)/0.694 = 0.0868 or approximately 9 percent, which is reasonable for practical purposes. Momentum exchange is the dominant factor for the spatially increasing flows.

VI. Conclusions

Generalized solutions are presented for horizontal, circular channels with a free or submerged discharge with constant or spatially-increasing flow. Those solutions are applied to the design of skimmer channels at wastewater treatment plants. Two examples from the author's practice are presented.

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