

Evaluation of Main Effect of Maximum Fatigue Load on Crack Growth in Magnesium Alloy Using ANOVA

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Abstract: *The analysis of variance (ANOVA) method was adopted to evaluate the main effect of the maximum fatigue load (MFL) on the crack growth at the early crack propagation and at the stage near failure in magnesium alloy. In the early stage of crack propagation, the smaller the MFL, the smaller the crack growth and the larger the MFL, the greater the scatter of grown crack. But at the stage near failure, the smaller the MFL, the larger the crack growth and the scatter of grown crack. The variances of grown crack were equal for three levels of the MFL at the early crack propagation and at the stage near failure. Through the residuals diagnostics for the grown crack, the validity of ANOVA was confirmed by verifying the normal distribution and mutual independence of the residuals. The ANOVA verified the MFL factor is highly significant for crack growth. The significance of the levels of MFL factor was also verified through the ANOM test. It was found that the MFL factor and its level affect significantly on the crack growth at the early crack propagation and at the stage near failure in magnesium alloy.*

Key Word: *ANOVA, Fatigue, Magnesium alloy, Maximum fatigue load (MFL)*

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I. Introduction

Magnesium alloy is excellent in lightweight, specific strength, machinability, and electromagnetic shielding property, so it is widely used in structural materials for an energy saving and an emission reduction. The fatigue property of structural material is very important because the structure is subjected to repeated loads and its study is required. Briffod et al.¹ have presented an analysis of the fatigue behavior of rolled AZ31 magnesium alloy through numerical simulations of polycrystalline aggregates using the crystal plasticity finite element method. Nakai et al.² have reported that mean stress effect on the fatigue strength is similar to its effect for metals without twinning and fatigue cracks are initiated from large grains with large Schmid factor of the basal slip system. Han et al.³ have found that the mechanism of the acoustic emission is the crack extension and the twin at the tip of crack. They are also reported the twin is prime factor of fatigue behavior in magnesium alloy. Ishihara et al.⁴ have studied the effect of load ratio on fatigue life and crack propagation behavior of magnesium alloy and reported that the relation crack propagation rate vs. M parameter is found to be useful in predicting fatigue lives at different R ratios. Zheng et al.⁵ have presented that the specimen orientation affects reasonably the fatigue crack growth rate and the crack path in extruded AZ31B magnesium alloy. Sivapragash et al.⁶ have reported the fatigue life prediction of ZE41A magnesium alloy using Weibull distribution. Choi⁷ has studied the aspects of the cumulative distribution function of the fatigue crack propagation behavior in magnesium alloy. However the statistical fatigue behavior in magnesium alloy has been rarely reported^{6,7}.

The purpose of this study is to find out whether the MFL is the significant influence factor and whether the MFL level has the main effect on the stability of crack growth at the early crack propagation and at the stage near failure in magnesium alloy.

II. Methodology

The one-way ANOVA is used to evaluate the main effect of the MFL factor on the fatigue crack growth. The ANOVA is performed with commercial statistical software MINITAB⁸. The ANOVA assumes that the variances of different populations are equal. To verify ANOVA's assumptions, we first test the homoscedastic of grown crack. The validity of ANOVA can be confirmed by verifying the normal distribution and mutual independence of the residuals of grown crack. The ANOVA is adopted to the grown crack data by three MFL levels to verify the significance of MFL factor. The ANOM test also verifies the significance of the levels of MFL factor. After verifying the significance of MFL factor, the main effect of the MFL on the crack growth is analyzed through the main effect plots.

The statistical grown crack data⁷ used for ANOVA are as listed in Tables 1 and 2. These are the fatigue crack propagation experiment data on the magnesium alloy of AZ31 under the three MFLs (kN) of 2.00, 2.25, and 2.50, the load ratio of 0.2, and the specimen thickness (mm) of 6.60. The grown crack data at the early stage

of crack propagation of 5000 cycle are listed in Table 1. Table 2 are the grown crack data at the stage near failure ($N/N_f=0.95$), where N and N_f are fatigue cycle and failure cycle, respectively.

Table 1. Grown crack data of AZ31 magnesium alloy ($N=5000$ cycle)

Observation	Treatment (MFL)		
	2.00 kN	2.25 kN	2.50 kN
1	19.138	19.705	19.724
2	18.994	19.566	19.943
3	19.189	19.786	20.080
4	19.229	19.663	19.854
5	19.221	19.447	19.495
6	19.268	19.350	20.023
7	19.349	19.534	20.002

Table 2. Grown crack data of AZ31 magnesium alloy ($N/N_f=0.95$)

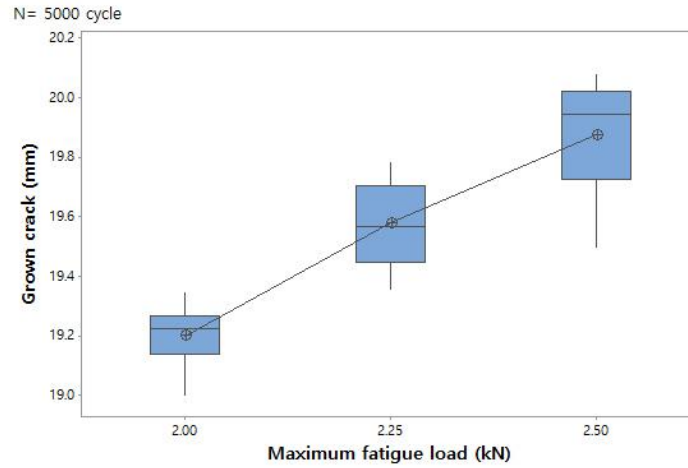
Observation	Treatment (MFL)		
	2.00 kN	2.25 kN	2.50 kN
1	29.155	28.838	28.575
2	29.933	28.787	28.591
3	29.237	28.870	28.622
4	29.307	28.720	28.736
5	29.126	28.983	28.095
6	29.185	28.855	29.037
7	29.789	28.661	28.958

III. Results and Discussions

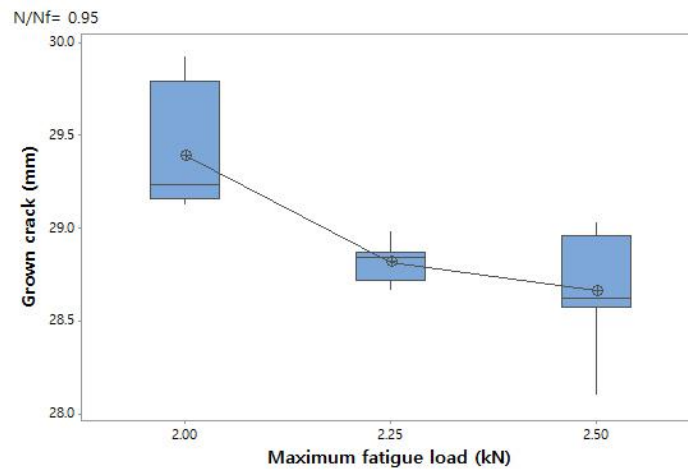
Variability of Grown crack

Figure 1 shows the box plots for the statistical data of grown crack listed in Tables 1 and 2. Figure 1(a) represents the grown crack data at the early stage of crack propagation of 5000 cycle and Figure 1(b) at the stage near failure ($N/N_f=0.95$).

In the early stage of crack propagation, the scatter of the case of MFL(2.00) is smallest and the larger the MFL, the greater the scatter of grown crack. The smaller the MFL, the smaller the crack growth, and the larger the MFL at the same stage, the more the crack growth. Because the case of MFL(2.00) has a smaller fatigue load amplitude than MFL(2.50), so that the crack grows slower and smaller, resulting in smaller crack scatter. At the stage near failure as shown in Figure 1(b), the smaller the MFL, the larger the crack growth. The case of MFL(2.00) has the largest scatter. In case of MFL(2.00), the mean stress is less than MFL(2.50), so even if it is close to the failure stage, the crack can grow larger without breaking. Instead, the longer the crack grows, the larger the scatter of grown crack is due to crack growth instability near the failure stage.



(a) At early crack propagation stage (N=5000cycle)



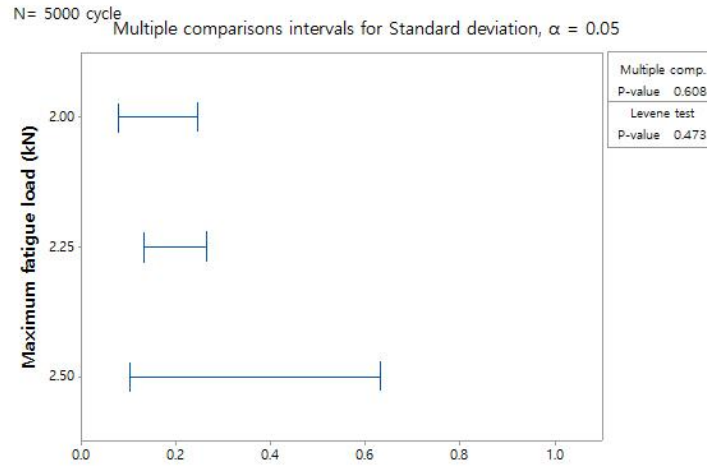
(b) At stage near failure (N/Nf=0.95)

Figure 1. Boxplot of Grown crack

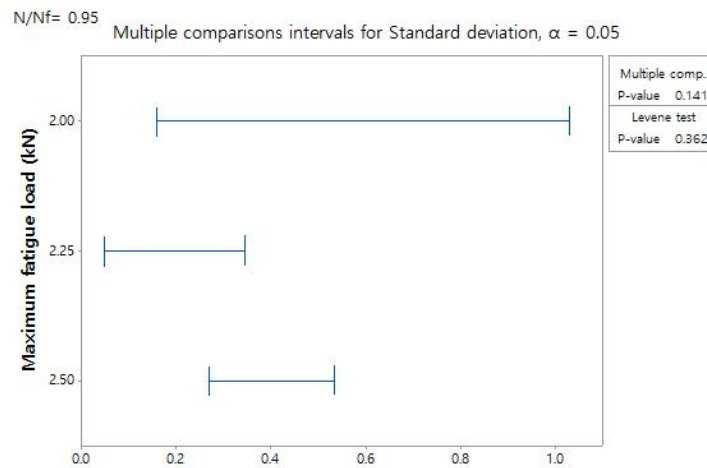
Diagnostic for Equality of Variance

ANOVA assumes that the variances of different populations are equal. Since the heteroscedasticity can affect the estimate, the diagnostic for equality of variance is required. MINITAB provides the p-value through the multiple comparisons test (the MC test) and the Levene test. If the p-value is smaller than the selected significant level (also known as alpha), the null hypothesis can be rejected. In other words one or more standard deviations are significantly different from other standard deviations. The multiple comparisons intervals are used to identify which standard deviations are significantly different. If 95 percent confidence intervals for two treatments are slipped from each other, those standard deviations are significantly different, i.e. the populations are heteroscedastic⁹.

Figures 2(a) and 2(b) represent the multiple comparisons intervals for standard deviation and p-value calculated through the MC test and the Levene test for the grown crack data at the early stage of crack propagation and near the failure stage, respectively. Since the p-values of 0.608 and 0.473 obtained from MC test and Levene test at early crack propagation stage, respectively, are greater than the selected significant level of 0.05, the null hypothesis can be accepted and there are no significant differences between any pairs of standard deviations. The same results are also found in Figure 2(b). As all of the confidence intervals on the graphical display in Figures 2(a) and 2(b) are not slipped but overlapped each other, it is also confirmed that there are no significant differences between the standard deviations. Therefore, the variances of grown crack are equal for each level of MFL factor. This result also indicates the experiment was performed properly¹⁰.



(a) At early crack propagation stage (N=5000cycle)



(b) At stage near failure (N/Nf=0.95)

Figure 2. Diagnostic for equality of variance of grown crack

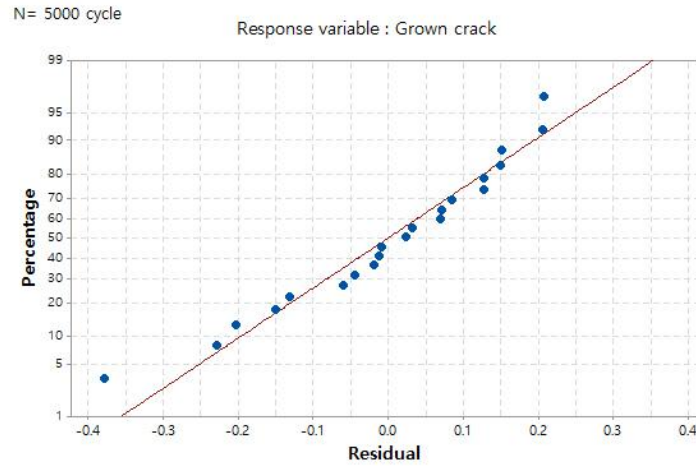
ANOVA residuals diagnostics for Grown crack

The residuals diagnostic graphs in Figure 3 and Figure 4 are created following the one-way ANOVA of the grown crack data obtained at the early stage of crack propagation and near failure stage, respectively.

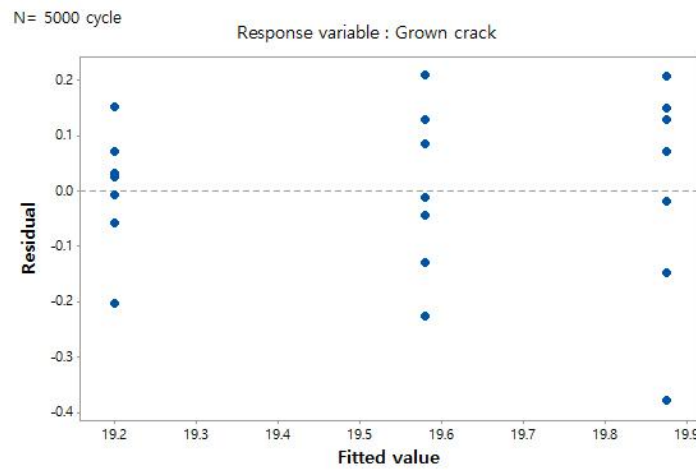
In the early stage of crack propagation, the normal probability plot of the residuals in Figure 3(a) shows that they fall along an approximately red straight line, which supports the claim that the residuals are normally distributed. Figure 3(b) shows a plot of the residuals versus the fitted values(that is, the treatment means). The amount of variation in the residuals appears to be comparable whether the magnitude of the response is large or small so the residuals are probably homoscedastic with respect to the fitted values. Figure 3(c) shows a run chart of the residuals. The run chart does not show any patterns so the residuals are probably mutually independent. The residuals of grown cracks also appear to be homoscedastic with respect to run order⁹.

At the stage near failure, as the residuals also fall along an approximately straight line in the normal probability plot as shown in Figure 4(a), they are normally distributed. Figure 4(b) also indicates that the variation in the residuals shows to be comparable and is symmetrical about the fitted values so the residuals are random and probably homoscedastic with respect to the fitted values. Since any pattern is not shown in Figure 4(c), the residuals are probably mutually independent and appear to be homoscedastic with respect to the run order.

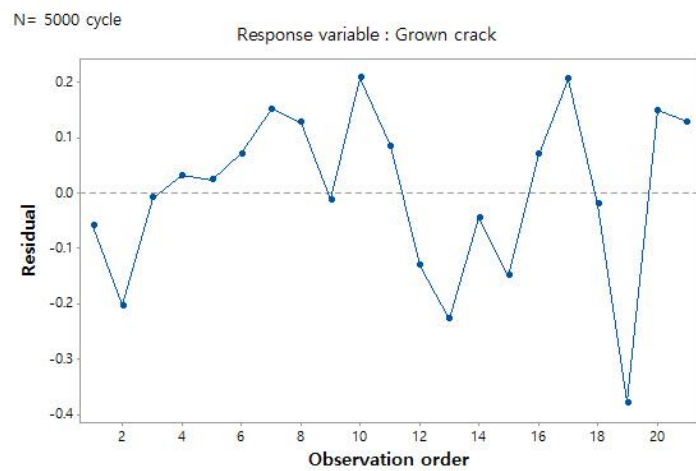
The residuals diagnostic plots in Figure 3 and Figure 4 indicate that the residuals are normally distributed and homoscedastic with respect to the fitted values and the run order, as required by the ANOVA. Thus, it is confirmed that the ANOVA assumptions are satisfied by using the graphs in Figures 3 and 4.



(a) Normal probability plot for Residual of Grown crack

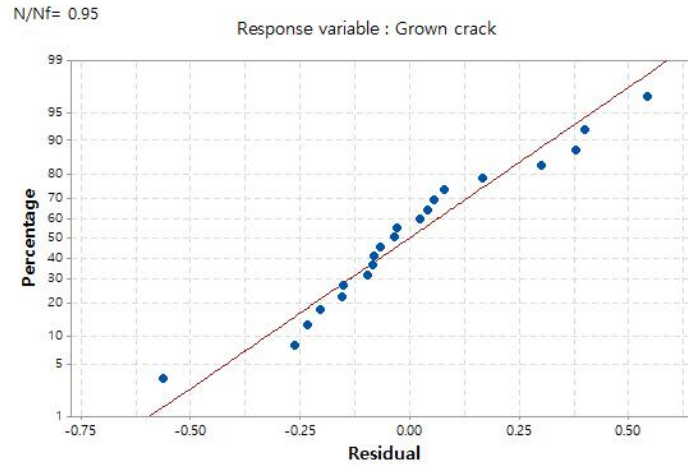


(b) Residuals versus Fitted values

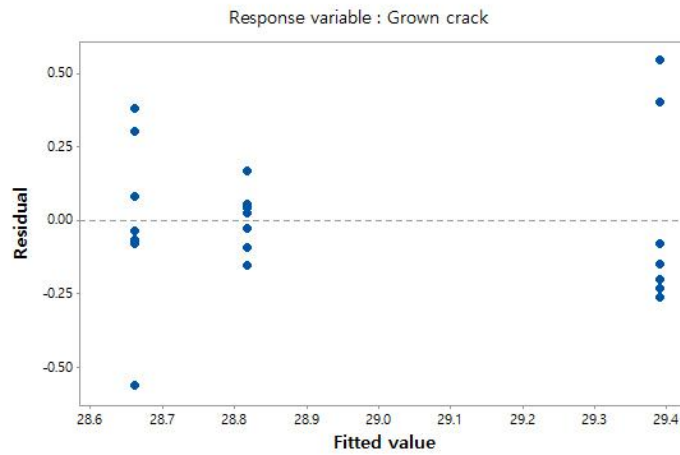


(c) Residuals versus Observation order

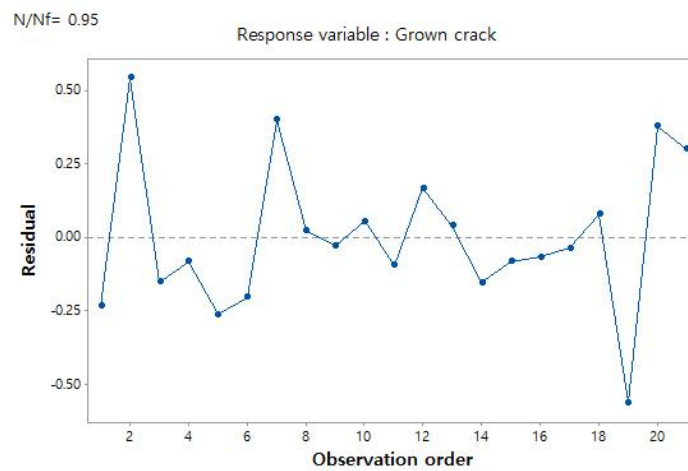
Figure 3. ANOVA residuals diagnostics for Grown crack at early stage of crack propagation (N=5000cycle)



(a) Normal probability plot for Residual of Grown crack



(b) Residuals versus Fitted values



(c) Residuals versus Observation order

Figure 4. ANOVA residuals diagnostics for Grown crack at stage near failure (N/Nf=0.95)

One-way ANOVA

The conditions⁹ required to validate the use of the ANOVA method are:

- The populations being sampled are normally distributed.
- The populations being sampled are homoscedastic.

- The observations are independent.

The diagnostic for the equality of variance of the grown crack in Figure 1 is performed as shown in Figure 2, so it is confirmed that the variances of grown crack are equal for three levels of the MFL at the early stage of crack propagation and at the stage near failure. The residuals diagnostic in Figures 3 and 4 confirms that the residuals are normally distributed, homoscedastic, and mutually independent. Since the assumptions required to validate the use of the ANOVA are satisfied, the two ANOVAs are performed on the grown crack data at the early crack propagation and at the stage near failure, respectively, with MINITAB and the results are presented in Tables 3 and 4.

The first ANOVA in Table 3 shows that the MFL effect is highly significant with P-value of 0.000 less than the significant level of 0.05. The second ANOVA in Table 4 also shows that there is significant MFL effect. Thus, the MFL factor affect significantly on the crack growth in the structure.

Table 3. One-way ANOVA : Grown crack versus MFL (N=5000cycle)

Source	DF :Degree of Freedom	Adj SS :Adj. Sum of Squares	Adj MS :Adj. Mean Square	F-value	P-value
Treatment (MFL)	2	1.6085	0.80423	31.16	0.000
Error	18	0.4646	0.02581		
Total	20	2.0731			

Table 4. One-way ANOVA : Grown crack versus MFL (N/Nf=0.95)

Source	DF :Degree of Freedom	Adj SS :Adj. Sum of Squares	Adj MS :Adj. Mean Square	F-value	P-value
Treatment (MFL)	2	2.074	1.03686	14.50	0.000
Error	18	1.288	0.07153		
Total	20	3.361			

One-way ANOM(Analysis of Means) test

ANOM test¹¹ provides a graphical method of testing

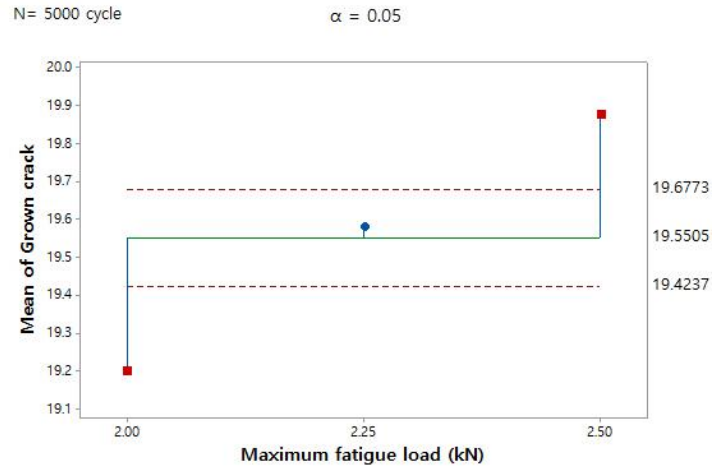
Null hypothesis, Ho : $\mu_1 = \mu_2 = \dots = \mu_k$, where μ denotes the mean of population.

versus

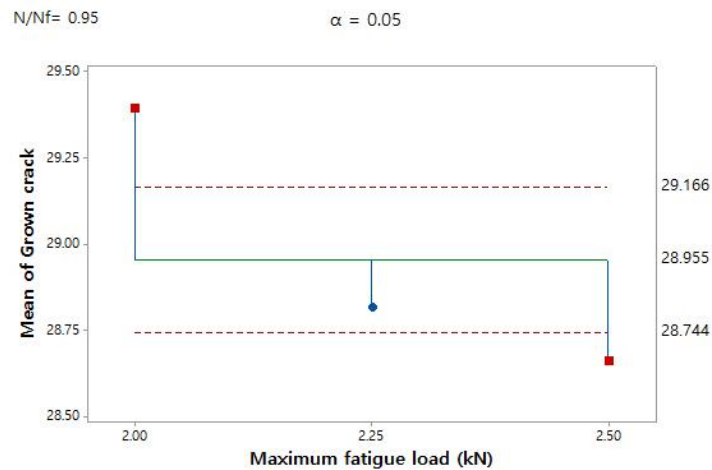
Alternative hypothesis, Ha : “Ho is false”

The graphical outputs from one-way ANOM for the grown crack at the early stage of crack propagation and at the stage near failure are shown in Figure 5(a) and Figure 5(b), respectively. The grown crack data follow a normal distribution. The green solid line is called the center line that indicates the total mean of grown crack. The lines drawn as red dotted lines are called the upper decision limit(UDL) and lower decision limit(LDL), respectively. The treatment means are plotted against the corresponding factor level and connected to the center line by vertical line segments. If a point falls outside the decision limits, reject the null hypothesis, Ho : $\mu_1 = \mu_2 = \dots = \mu_k$ ^{10,11}.

The UDL and LDL as shown in Figure 5(a) are 19.6773 and 19.4237 at the early stage of crack propagation, respectively. Those at the stage near the failure are 29.166 and 28.744, respectively, as shown in Figure 5(b). Since at least one mean falls outside the decision limits as shown in Figures 5(a) and 5(b), we conclude that the three treatment(MFL) means are not all equal and reject Ho. The treatment means are significantly different from the total mean. Thus, the factor levels of MFL(2.00) and MFL(2.50) are significant at the early crack propagation and at the stage near failure. And the MFL factor affects crack growth in magnesium alloy.



(a) At early crack propagation stage (N=5000cycle)



(b) At stage near failure (N/Nf=0.95)

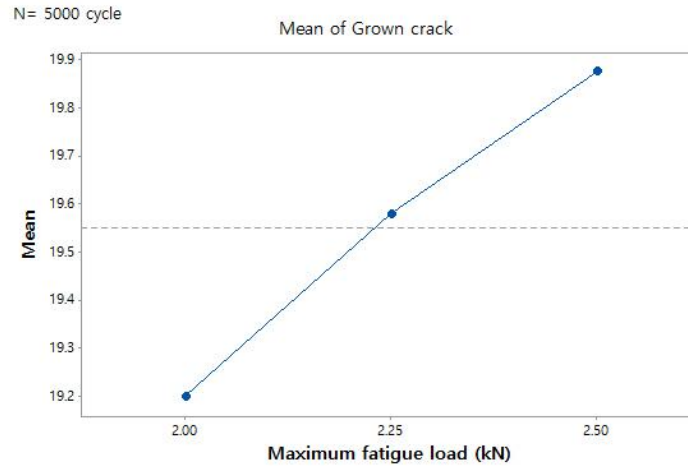
Figure 5. One-way ANOM for Grown crack following Normal distribution

Main effect of Maximum fatigue load

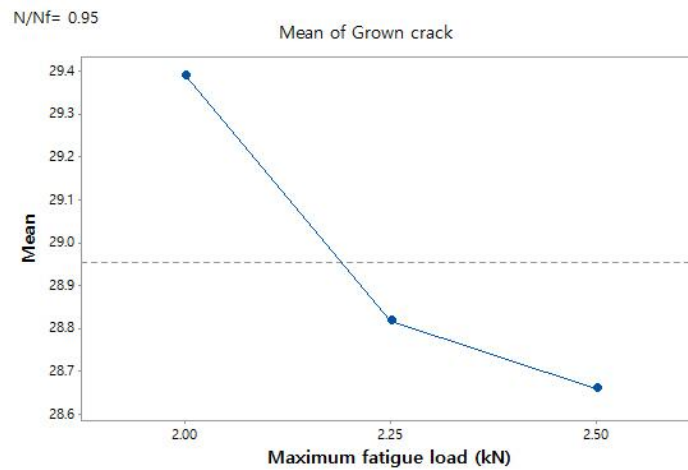
The main effects of the MFL on the crack growth in magnesium alloy are analyzed with MINITAB and presented in Figure 6. The dotted line means the total mean of grown crack and the point indicates the mean of grown crack at given MFL level.

At the early stage of crack propagation as indicated in Figure 6(a), the crack growth is largest at MFL(2.50) of the three MFL levels. The largest grown crack in the same fatigue cycle among the three levels of the MFL is considered to be due to the largest fatigue load amplitude and mean stress of MFL(2.50). The crack growth also seems to be the fastest since those conditions of MFL(2.50) are largest of the three levels of the MFL. If the crack grows rapidly, the structure is dangerous, so the crack growth must be slow for the integrity of the structure. Thus the main effect of slow crack growth occurs at MFL(2.00) level.

At the stage near failure, the crack growth in case of MFL(2.00) is largest of the three MFL levels as shown in Figure 6(b). Due to the smallest fatigue load amplitude and mean stress, the cracks grow more stably and longer and larger. The MFL(2.50) result, on the other hand, shows that the cracks do not grow more and can be broken quickly. In case of MFL(2.50), the failure occurs more quickly, which can be more dangerous for the safety of the structure. The main effect on the stability of crack growth at the stage near failure occurs at MFL(2.00) level. Therefore, the main effects plot suggests that there might be significant effects associated with MFL.



(a) At early crack propagation stage (N=5000cycle)



(b) At stage near failure (N/Nf=0.95)

Figure 6. Main effect of Maximum fatigue load on Crack growth

IV. Conclusion

The ANOVA method was adopted to evaluate the main effect of the MFL on the crack growth at the early crack propagation and at the stage near failure in magnesium alloy. In the early stage of crack propagation, the scatter of the case of MFL(2.00) was smallest and the larger the MFL, the greater the scatter of grown crack. The smaller the MFL, the smaller the crack growth. At the stage near failure, the smaller the MFL, the larger the crack growth. The smaller the MFL, the smaller the mean stress, so that the crack can grow larger without breaking even near the failure stage. The variances of grown crack were equal for three levels of the MFL at the early stage of crack propagation and at the stage near failure. The validity of ANOVA was confirmed by verifying the normal distribution and mutual independence of the residuals of grown crack. The ANOVA verified that the MFL factor is highly significant for crack growth in magnesium alloy. The significance of the levels of MFL factor was also verified through the ANOM test. It was found that the MFL factor and its level affect significantly on the crack growth at the early crack propagation and at the stage near failure in magnesium alloy.

Acknowledgements

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