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Modal Analysis and Testing on the Vibrational Response of Both Simple and Complex Structural Systems.

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Abstract: *This report demonstrates and verifies the best appropriate method to be used in vibration excitation and response of simple and complex structural systems. The aim was to investigate which method is best for design purposes. Three components were investigated. The simple and complex systems (components) studied in this work are; flexible beam, rectangular plate and steam turbine blade. The methods used were; analytical, numerical and experimental methods from which the mode shapes and natural frequencies for comparison of results were obtained. This study is also aimed at finding frequency response functions, predicting and measuring first four natural frequencies and mode shapes, node locations, tip deflections and anti-node locations of the flexible beam. It also reveals how to measure and estimate damping ratio, modulus of elasticity of the beam, force waveform and reciprocity check of the steam turbine blade and comparison of the impact hammer tips and linearity check of the structures were also done using experimentation, MATLAB codes and Finite Element Simulations. The results obtained showed that, the experimental method gave 97% accurate results which was very close to the analytical method(exact solutions)and the Numerical predictions gave between (85-90)% accurate results. The experimental method was therefore adopted as the acceptable method to be used for design of any simple and complex systems.*

Keywords: *Modal analysis, complex system, cantilever steam turbine blade, experimental Numerical vibration.*

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I. Introduction

Cantilever beams are widely used in the construction of cantilever bridges, fixed wing aircraft design and balconies of buildings [1]. Whereas, the steam turbine blade are used for the extraction of thermal energy from very high pressure steam in order to do Mechanical work in rotating machines [2]. Therefore, considering their areas of applications, vibration (the design problem) is the dominant force that poses serious danger and threats on both users and machines. It should therefore, be designed against and monitored carefully to avoid failure. Also, resonant frequency should be avoided in designing the problem. The turbine blade failure during service, may incur many safety related risks and this leads to revenue losses. Therefore, it is very paramount to maintain the reliability of the blades. Modal analysis basically refers to understanding the dynamic characteristics of a particular structure when subjected to vibration. The mode superposition method is basically a linear dynamic response method, which evaluates mode shapes, superimposes them in order to give us displacement patterns. [3] The mode shapes basically gives us an idea of the configurations into the structure will deform, of which the main concern are the lateral displacements. As the mathematical order of the mode shapes increases, they contribute less to the results and their predictions are less reliable. [4] The reasons for performing a modal analysis especially for a turbine blade are deciding a particular rotational speed, amount of loading and critical operation speeds. A modal analysis also explains the frequency at which the structure will absorbs all the energy and the shape that corresponds to the structure. As far as time and cost are concerned, modal analysis in FEA is performed, so that we obtain predicted results, even without physical testing the structure. Therefore a comprehensive analysis needs to be performed on the turbine blades and flexible beam, in order to find the natural frequencies and mode shapes at which the structure will magnify the load effect.

II. Methodology

The study was divided into two tasks, being the computer lab and the experimental lab.

Cantilever Beam Experimental Set up: The cantilever beam setup is shown below in figure 1. The scale is attached to the top end of the beam, in order to measure the deflections caused by the increasing loads. An accelerometer is attached with the help of cables, to the strut, from where the frequency is supplied to the strut in order to create vibrations and the corresponding natural frequencies, node and anti-node locations are measured.

Experiment 1: The Flexible beam.

- The dimensions (Length (L), width (w) and depth (d)) of a steel cantilever beam were measured while the beam was fixed horizontally in order to obtain the tip deflection (48mm).
- The modulus of elasticity was calculated from the tip deflection obtained.
- The beam was fixed vertically and excited and the first four natural frequencies, node locations, anti-node locations for the four modes were obtained and recorded.
- The damping ratio was finally estimated for the first mode using the half amplitude method.

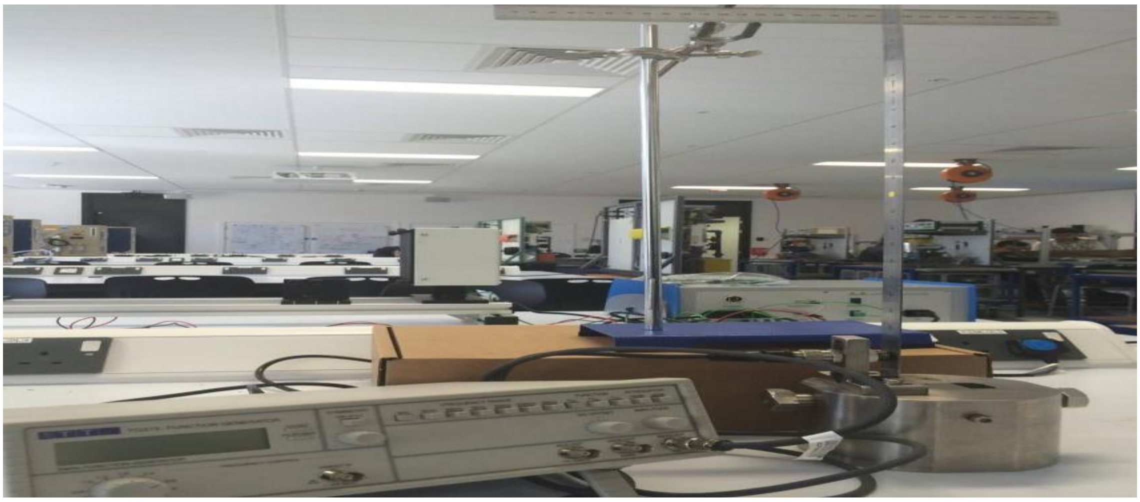


Figure 1: Experimental Setup for the Flexible Beam Experiemnt

Steam Turbine Blade Experiment set up: This set up consisted of an oscilloscope connected to the computer loaded with specific software to measure the force waveforms as shown below in figure 2.
Hard plastic and soft rubber tips hammers were used, with a similar force to measure the force waveform.
The probe was attached to the blade with the help of wax as an adhesive at point 5 and the force and acceleration time histories were recorded for the same. After recording values for all 10 locations, the position of the accelerometer was changed to point opposite the point 5 and the readings were taken accordingly.

Experiment 2: The Steam Turbine blade

a. The accelerometer sensor cables (with wax) were attached to one end of the blade and the opposite end was excited using a hammer with hard and soft tips.
b. The sensitivity of the accelerometer and the impact hammer were noted and recorded.
c. The frequency and voltage readings were adjusted in the Pico Scope 6
d. At a very low impact force from the hammer (for 10 different locations) for both the soft tip and hard tip was applied at one end of the blade. The vibration response was transmitted and received at the opposite end of the blade.
e. The force and acceleration time histories were plotted and saved in the computer.

Figure 2: Steam Turbine Blade Experiment Set up

III. Results

Flexible Cantilever Beam

Using the equation:

The natural frequency, $$F_n = \frac{\lambda_n^2}{2\pi L^2}\left(\sqrt{\frac{EI}{\rho A}}\right)$$ (3.1)

For mode 1

The Modulus of elasticity, $$E = \frac{4\pi^2 L^4 \rho A F_1^2}{I\lambda_1^4} = 2.06\ GPa$$ (3.2)

Where, F_1= 2.23Hz, L = 0.6m, w = 0.025m, d = 0.00104m, λ_1 = 1.875

The moment of inertia, $$I = \frac{wd^3}{12} = 2.343*10^{-12}\ m^4$$ (3.3)

Area, $$A = w * d = 3 *10^{-5} m^2$$ (3.4)

Table1 below shows the comparison between the experimental results and the computational in terms of natural frequencies, the nodes and the anti-nodes location. It was can be seen that, the percentage difference in both values was very small to less than 6%. This means that, the computational was more than 90% accurate. Hence, values are in close agreement and valid. However, the little difference in error may be attributed to cumulative rounding off effect or wrong parameters.

Table 1.measured first four modes and predicted first four modes

EXPERIMENTAL				COMPUTATIONAL				PERCENTAGE DIFF Fn(Hz)
MODE	Fn(HZ)	NODE(m)	ANTI-NODE(m)	MODE	Fn(HZ)	NODE(m)	ANTI-NODE(m)	(%)
1	2.23	0		1	2.35	0		-5.38
2	14.75	0.471	0.234	2	14.72	0.461	0.285	0.2
3	41.9	0.31,0.51	0.151,0.42	3	41.2	0.297,0.527	0.176,0.418	1.67
4	82.49	0.22,0.393,0.545	0.106,0.303,0.472	4	80.74	0.212,0.388,0.546	0.127,0.303,0.473	2.12

Figure 4 below displays the first four beam mode shapes combined into one using MATLAB codes. It can be seen from the figure that, as the beam is exited, the vibrational response increases towards the tip of the beam. Which means that, as the nodes increases, the nodes and anti-nodes also increases and the natural frequency increases towards the tip of the beam due to less damping at the tip.

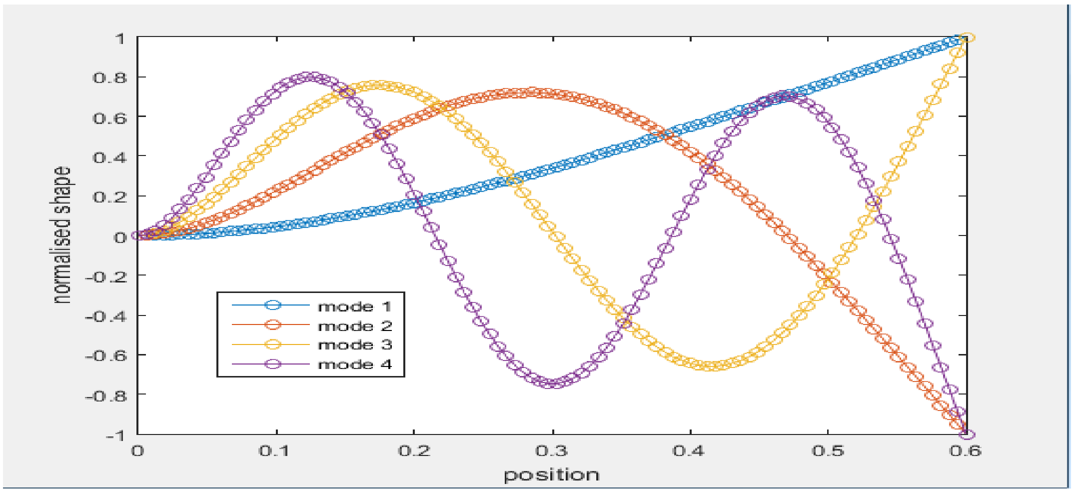


Figure 4. The first four beam mode shapes

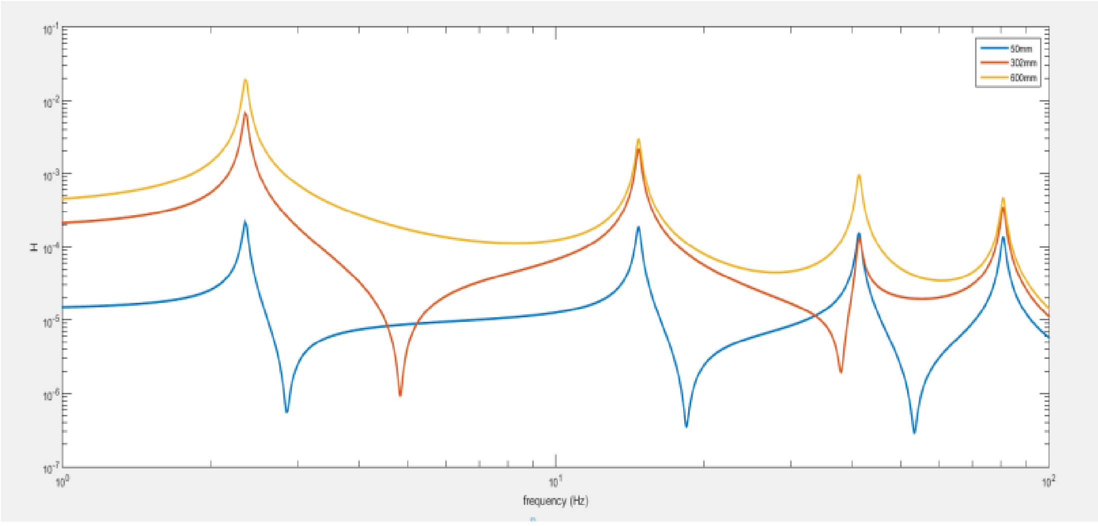


Figure 5. The three simulated beam FRFs

Figure 5 above, shows the frequency response function (FRF) where the amplitude increases towards the tip of the beam. This is because, the damping effect is less at the tip with an increase in resonance of the vibration. That is why the upper yellow line has higher amplitude because it is at 600mm which is the tip of the beam. The red signal curve matches close to the peak of the upper yellow line since its distance is 302mm from the base. This means that, Amplitude increases with less damping and how close it is to the tip. Using the half amplitude method, the estimated damping ratio is estimated below:

The damping ratio,

$$\zeta = \frac{0.11}{n} = 0.003793 \quad (3.5)$$

Where, the number of cycles of half the tip displacement, $\mathbf{n = 29}$

Rectangular Plate

Table 2 below compares the values obtained from MATLAB and ANSYS for the rectangular plate. Both values are in close agreement.

Table 2. Comparison between ANSYS and MATLAB

RECTANGULAR PLATE		
NO. OF MODES	ANSYS Fn(Hz)	MATLAB Fn(Hz)
1	13.855	13.8208
2	26.642	26.5748
3	47.964	47.8233
4	77.818	77.5992

Steam Turbine Blade

Figure 6 below displays the force time history for different hammer tips (Hard and soft tips). The black curve is the hard tip which has high frequency, low energy and small impact time duration. Whereas, the blue curve represent the soft tip which has low frequency, higher energy and longer impact time duration both at same impact. The range of the frequencies is sway by the square root of the contact stiffness and impactor mass [5].

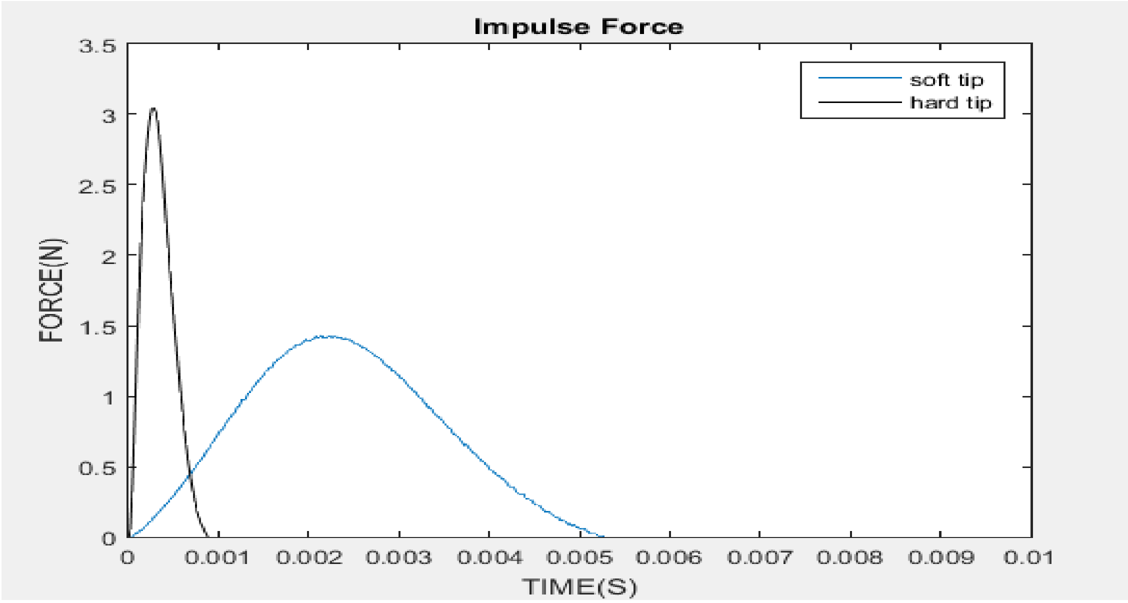


Figure 7. Force Spectra

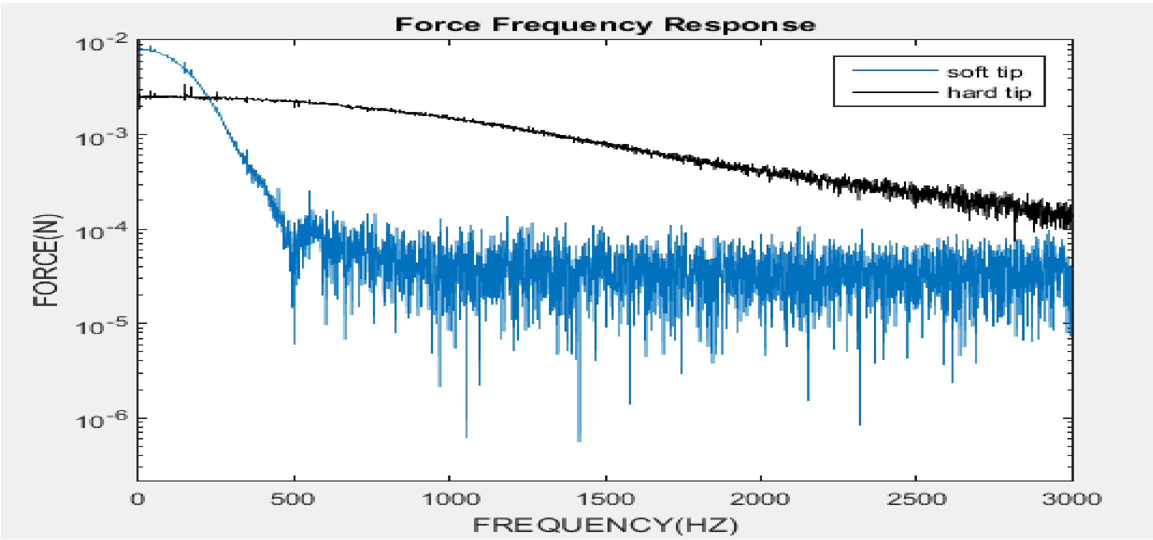


Figure 6. Force time history

Figure 7 above shows the force spectra for both the hard tip and the soft tip and their frequencies responses. The blue curve is the soft tip which has a cut off frequency at 200Hz and after which it is heavily distorted due to noise effects and material properties. Whereas, the black curve represents the hard tip which higher force and higher peak acceleration with smooth frequency response and a cut off frequency of 1000Hz. This is because, the hard tip has good material property such as stiffness and its contact time is shorter.

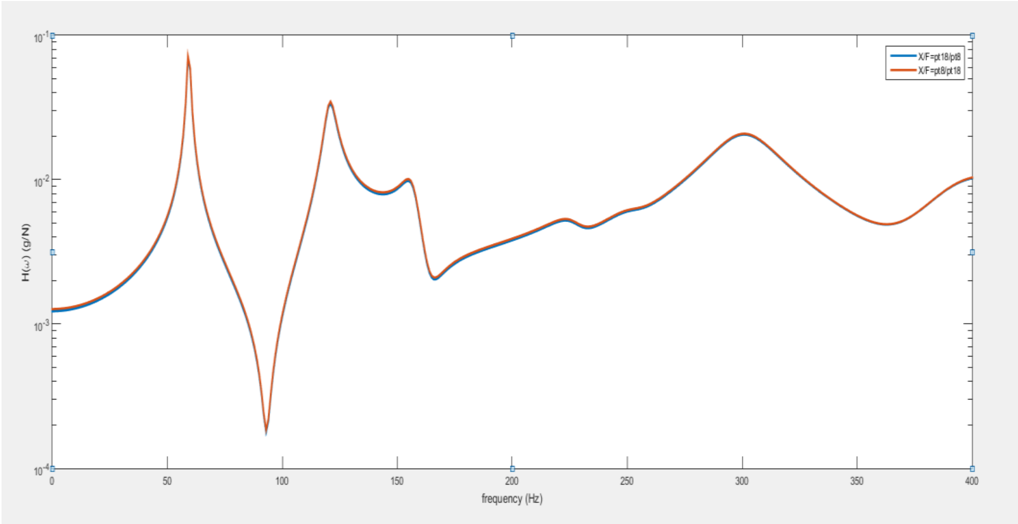


Figure 8. Reciprocity residues between points 8 and 18

Figure 8 above displays the reciprocity check for the turbine blade impact at 18 and 8 for both acceleration and impact. The curves (red and blue) shows agreement in values. The two curves are just the same and no significant difference in their values. This shows linearity.

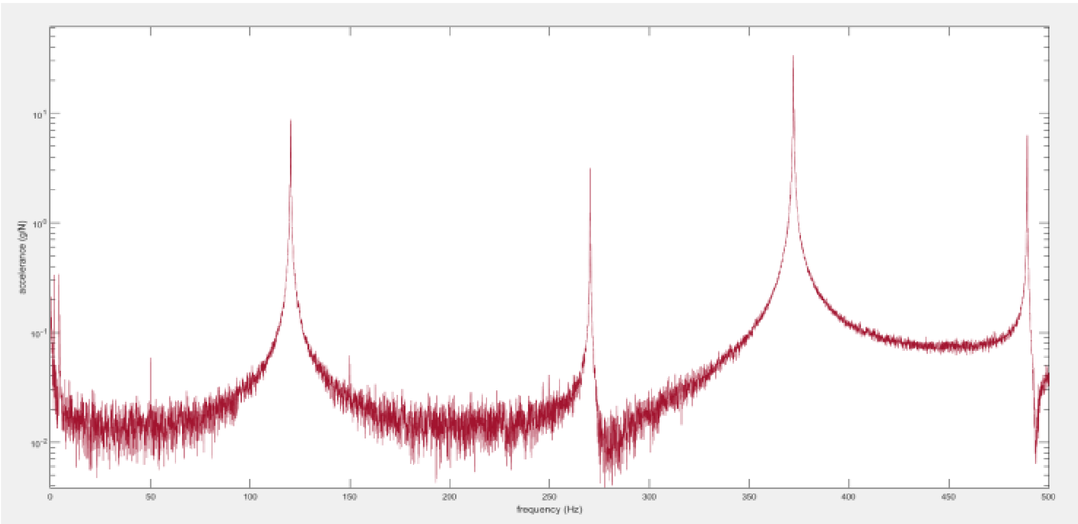


Figure 9. Frequency response function for pt2_pt5

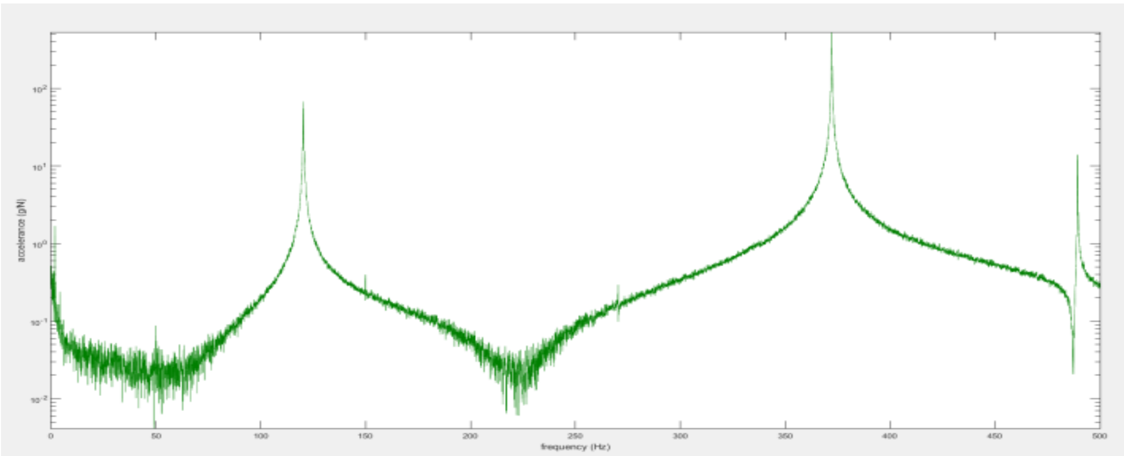


Figure 10. Force response frequency for pt5_pt5

The two Figures 9 and 10 above shows the Frequency Response Function (FRF) at points 2 and points 5 respectively. From figure 8, the shape of signal depicts distortion effect. This could be high noise level from the impact hammer or energy losses due to the travel time for the response to be receive by the accelerometer probe. Also, the peak acceleration is lower than that obtained in figure 9. It could also be caused by stiffness of material and damping effect. But figure 9 is free of noise and the peak is very high since receiving point is same as exciting point and material is lighter than in figure 8.

Table 3 below displays the damping ratio estimated from the FRF of point 1 using Half-Power Method. It can be shown that, the damping ratio is 3.91E-04 which is above the expected damping ratio of 1E-04 [5] with a difference of 0.000291 in value. This difference could be due to non-uniformity in the material properties or perhaps losses due to energy dissipation. However, since the damping ratio is still less than 1, the system is still under damped and result valid.

Table 3. Damping ratio for Frequency Response Function (FRF) of pt1_pt5

MODES	H/SQRT(2)	F1(Hz)	F2(Hz)	Fn (Hz)	DAMPING RATIO
1	2.692	120.196	120.29	120.241	3.91E-04
2	1.125	270.255	270.305	270.266	9.25E-05
3	-32.18	372.036	372.116	372.076	1.08E-04
4	5.414	489.253	489.303	489.282	5.11E-05

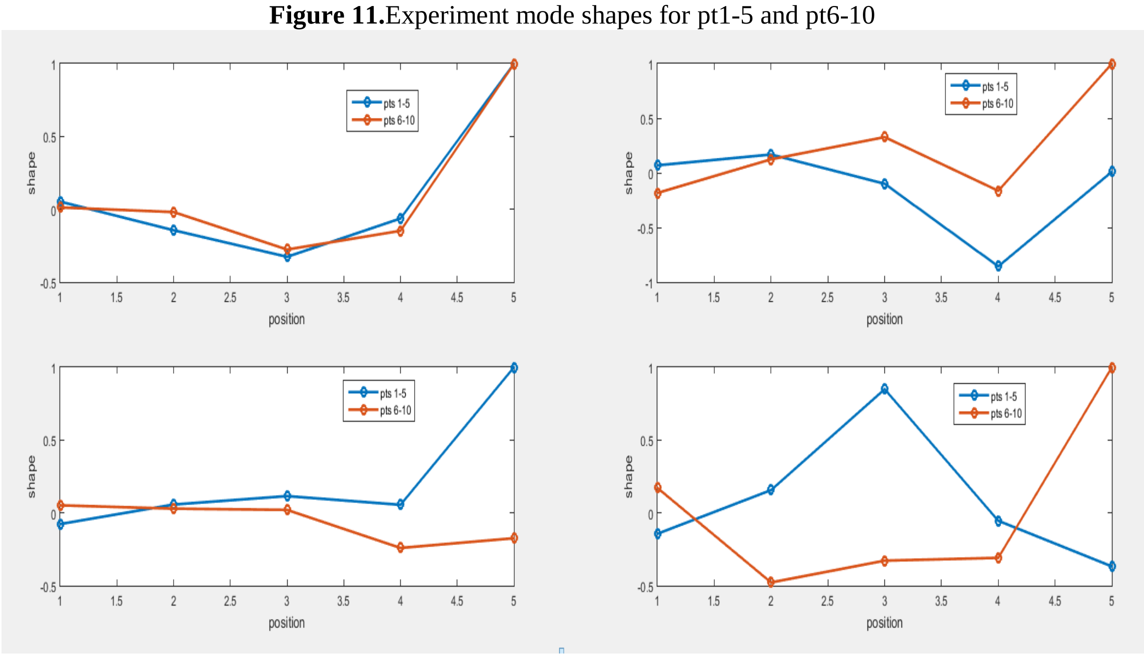
Table 4 below compares the values obtained for the first four natural frequencies from the experiment and the Finite Element solution with less than 15% error difference. This difference in error could be due to wrong parameters in the mathematical model of the Finite Element Method (FEM) [5]. However, the prediction from the FEM was about 88% accurate and values of the natural frequencies were close to the experimentation frequencies. Hence, values are acceptable and result valid.

Table 4. Comparing FEM and Experiment

FEM		EXPERIMENT		Fn(Hz)
MODE	Fn(Hz)	MODE	Fn (Hz)	(%) diff.
1	105.69	1	120.241	12.1
2	253.65	2	270.267	6.1
3	354.03	3	372.075	4.8
4	473.65	4	489.281	3.2

Figure 11 and Figure 12 below displays the mode shapes obtained from the experiment conducted on the steam turbine blade and FEM for points 1 to 5 and points 6-10 for the first four modes respectively. The correlation between this two figures shows that, both shapes looks very similar. Therefore, results is said to be valid.

Figure 11. Experiment mode shapes for pt1-5 and pt6-10



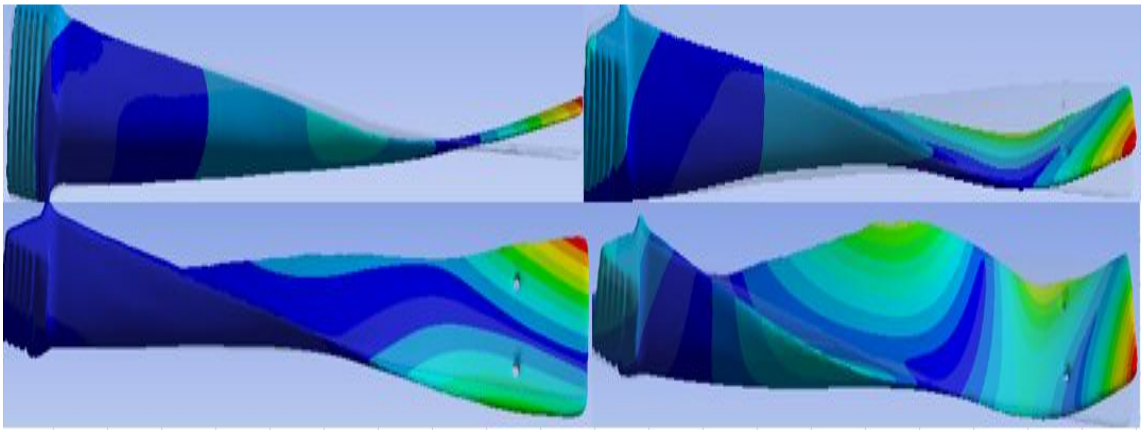


Figure 12. Mode shapes from Finite Element Method (FEM)

Figure 13 below shows the Frequency Response Function (FRF) for experiment and synthesized MATLAB CODES for point 2. As shown, four peaks are represented from the response signal received by the accelerometer probe. The black curve shows the signal contour from the original FRF whereas, the other curves are the synthesized. They only match at the peaks and are almost equally spaced. This variation in shapes could be from errors due to losses in energy dissipation [5]. And also wrong input parameters in the MATLAB codes.

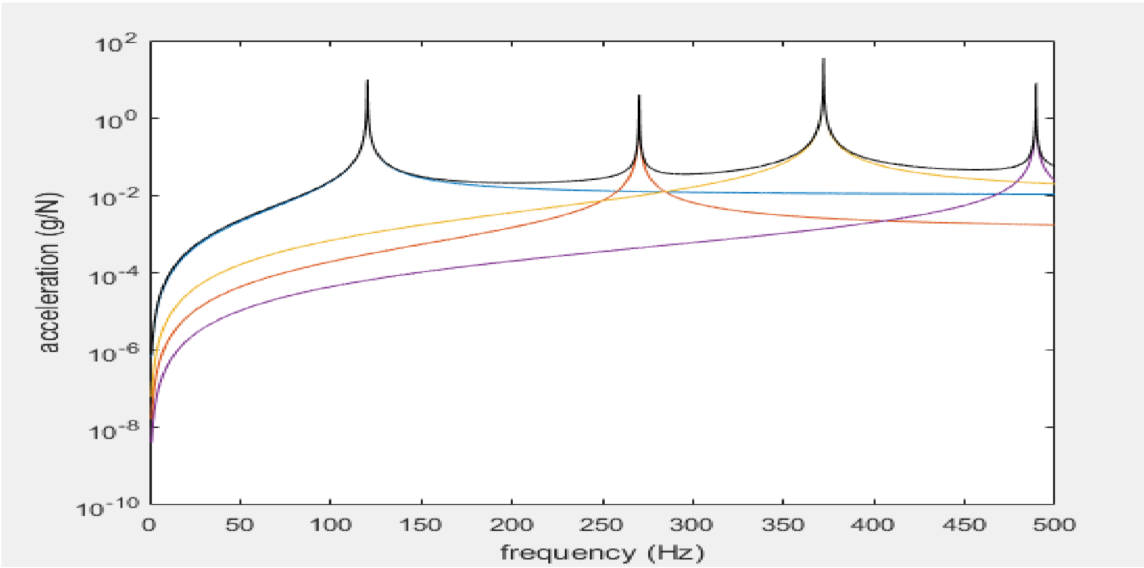


Figure 13. Friquency Response Function (FRF) at pt2-pt5 for both Experiment and Synthesized codes

IV. Discussion

The Flexible Beam

From the experimental demonstrations performed for the flexible beam, the modulus of elasticity was estimated as 206GPa. And was found to have 2.9% error difference when compared to the analytical modulus of elasticity of 200GPa. This means that, the estimated modulus of elasticity gave an accuracy of 97%. The difference in error may be due to wrong measurement of parameters or problem with boundary conditions.

The estimated damping ratio was calculated using the half amplitude method and value with value as 0.00379. This method is not a good method to use since it considers just the peaks and one fundamental mode only. This has significant effect on the results as there will be loss in energy dissipation due to damping. The best method to be considered here if the half power method. This method can handle multiple modes and it has the ability to contain both the force and response signal to produce the Frequency Response Function (FRF).

When the individual dimensions (length, width and thickness) are increased, the natural frequencies of the beam decrease. This is because; the natural frequency is inversely proportional to the length, cross sectional area and density.

Figure 4 above shows that, the higher the resonance towards the tip of the beam, the more the modes, anti-nodes, nodes and the higher the amplitude and natural frequencies.

From Table 1, it can be seen that the error difference estimated between the experimental frequency and the computational frequency is less than 6% which means that values are in close agreement. Thus, the computational prediction is 94% accurate. However, the experimental values are a little higher than the computational.

The Rectangular Plate

From the table (Table 2) of results obtained under the rectangular plate section. It can be seen that the values obtained for both MATLAB and FEM are almost the same and the modes shape similar when viewed visually. This closeness in the values shows that the results are valid. This is because, they are using two different methods to solve the same problem and arriving at same results.

From the results obtained in both the FEM and the MATLAB, it was observed that mode 1 has the highest effective mass followed by mode 3. Then mode 2 and mode 4 had zero effective mass. During excitation, the centre of gravity of the mode with the highest effective mass will experience notable translation whereas; those with zero effective masses will remain static. The mass of the modes with zero effective masses are negligible [6].

In Finite Element Model (FEM), the number of modes to be calculated depends on the number of modes you want to achieve. And you can calculate as many modes as you want.

If the plate is excited at constant acceleration, the dynamic stress will be evaluated in terms of the displacement. Which means that the greater the displacement, the higher the amount of the dynamic stress [5]. Therefore, using the relationship;

$$\text{Acceleration, } \ddot{\boldsymbol{x}} = \boldsymbol{\omega_n^2} \tag{4.1}$$

$$\boldsymbol{x} = \frac{\ddot{x}}{\boldsymbol{\omega_n^2}} \tag{4.2}$$

Where ω_n is the natural frequency, x is the displacement and $\ddot{x}$ is acceleration

From equation (4.2) it can be seen that, an increase in the natural frequency, ω_n will cause a decrease in the displacement, x and a corresponding increase in the dynamic stress. From the results obtained, Mode 4 has the highest frequency. Therefore, Mode 4 will give the lowest stress.

The Steam Turbine Blade

The range of the input excitation frequency is regulated mainly by the hardness of the tip selected. The harder the hammer tip, the wider the range of the frequency. When a soft tip was selected, it was observed that, all modes were not excited satisfactorily to get a better result [7]. The hard tip produces higher peak force and higher acceleration compare to the soft tip that uses longer time duration to produce very low frequency.

From the time domain force signal plots, it can be observed that, the hard tip produces high frequency, low energy and short contact time. Whereas, the soft tip produces low frequency, high energy and longer contact time. In the force signal plots, the hard tip produces high frequency (1000Hz) before the cut off frequency. Whereas; the soft tip produces a low frequency (200 Hz) before cut off frequency.

For a light structure, a soft tip will be preferable to use in order to avoid double impact. Soft tip also have longer impact time.

Yes, the hammer tip used was acceptable within certain ranges. The soft tip was acceptable between 0 and 400Hz and the hard tip was acceptable from 400Hz and above.

The cut off frequency is the inverse of the impulse duration [5]. From the time domain, the soft tip was observed to be the inverse of 5e-3 which gave 200Hz and the hard tip was the inverse of 1e-3 which gave 1000Hz. When compare to the Force spectra, the cut off frequency end at 200Hz before the noise distorted areas and the hard tip was 1000Hz which is to the end of the signal.

For the reciprocity check, the yes it agreed with what was expected. This is because, a linear behaviour was assumed and from Figure 8, the relationship for the impact force and the acceleration, were linearly related as there no significant difference between them.

From Figures 9 and 10 it was observed that point 5 was the response point with the acceleration probe and point 2 was the excitation point. Point 2 dissipates lots of energy for its response to be receive due to travel distance and damping effects. But the travel distance of point 5 is shorter so no too much energy is dissipated since the travel distance is shorter. And the damping effect at point 5 is less than that of point 2 since the material appears lighter at point 5.

The expected damping is 1e-4 [5]but the measured damping gotten is 3.9e-4 which is above what was expected. Although, the measured damping was beyond the expected but since it was not equal or greater than 1 it means that the system was not over-damped but has considerable amount of damping which in the real sense is correct [1].

From figure 13 it can clearly be seen that the synthesized FRF matched the original only at the peak which means there were losses due to energy dissipation which arises from damping effects. However, the peaks were almost equally spaced which means no coupling effects occurred [5].

Comparing Figures 12 and 11, it can clearly be seen that both mode shapes display similar behaviours which means agreement in result and confirming both results to be valid.

Considering Figure 4 which comprises of all mode shapes in one figure, it can be clearly seen that as the modes increases, the number of nodes and anti-nodes increases as well. This is because, there is an increases in resonance, increase in natural frequencies, increase in acceleration and amplitudes but decrease in the damping.

V. Conclusion

After performing three different methods in solving the same problems. The investigation concludes that, the results from the experimental method were 97 percent (%) accurate with few errors emanating from boundary condition problems, equipment and data processing errors (measurement errors). However, it is quicker to accomplish and represents real life problems.

The numerical predictions were (85 -90) percent (%) accurate. They all pointed towards the direction of the experimental results but were not on same level but close. This confirms and verifies the extent to which the numerical results were valid and acceptable. Although, results were affected by wrong input parameters, boundary condition problems and approximations.

The analytical results are the ideal condition or method where everything is completely based on assumptions, theories and approximations. However, the results obtained in this method also pointed in same direction as experiments'. Showing agreement and validation but cannot be totally relied upon because it overestimates results.

Nevertheless, the aims and objectives of the study were achieved based on the validity in the comparison of the results obtained from the three methods. And also due to the fact that, they were extremely in close agreement. This confirmed the results to be acceptable and recommendable for future investigations.

Conclusively, the best recommended method for design purposes is the experimental method. This is because, it represents real life scenarios and do not require leaning of any software.

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7. APPENDIX

7.1 Codes for calculating first four beam modes

```
clear,clc,close all

Uinput=inputdlg({'width:',...
            'thickness:',...
            'length:',...
            'density:',...
            'modulus of elasticity:'},...
            'input',1,{'.025','1.04e-03','.6','7900','2e11'});
w=eval(Uinput{1});
t=eval(Uinput{2});
L=eval(Uinput{3});
rho=eval(Uinput{4});
E=eval(Uinput{5});

lambda=[1.8751 4.6941 7.8548 10.9955];
sigma=[0.734096 1.018467 0.999224 1.00003];
I=(w*(t^3))/12;
m=w*L*t*rho;
```

```
for i=1:4
fn=lambda(i)^2/2/pi/L^2*sqrt((E*I*L)/m);
x=linspace(0,L,100);
phi=(cosh((lambda(i).*x)./L))-(cos((lambda(i).*x)./L))-(sigma(i).*((sinh((lambda(i).*x)./L))-
(sin((lambda(i).*x)./L)))); %Blevins
phin=phi/(max(abs(phi)));
```

7.2 codes for calculating FRF versus frequency (Different mode shapes)

```
clear,clc,close all

Uinput=inputdlg({'width:',...
        'thickness:',...
        'length:',...
        'density:',...
        'natural frequency 1:',...
        'natural frequency 2:',...
        'natural frequency 3:',...
        'natural frequency 4:',...
        'damping ratio:'},...
        'input',1,{'.025','1.04e-03','.6','7900','2.3481','14.7152','41.2033','80.7406','.01'});
w=eval(Uinput{1});
t=eval(Uinput{2});
L=eval(Uinput{3});
rho=eval(Uinput{4});
f1=eval(Uinput{5});
f2=eval(Uinput{6});
f3=eval(Uinput{7});
f4=eval(Uinput{8});
zn=eval(Uinput{9});
mnum=eval(Uinput{6});
%%
%Enter the frequencies
fn=[f1 f2 f3 f4];
kn=((2*pi.*fn).^2)*(L*w*t*rho);
zn=[zn zn zn zn];
phif=[0.01105 0.06268 0.1589 0.294];    %mode shapes for the forcing location
phir=[0.01105 0.06268 0.1589 0.294];    %mode shapes for the response location
f=linspace(0,100,5000);
for n = 1:length(fn),
  r = f/fn(n);
  hi(:,n) = phif(n)*phir(n)/kn(n)./(1-r.^2+j*2*zn(n)*r);
end
h=abs(sum(hi,2));
loglog(f,h)
hold on

phif2=[0.01105 0.06268 0.1589 0.294];    %mode shapes for the forcing location
phir2=[0.3454 0.7112 -0.008335 -0.7455];    %mode shapes for the response location
f2=linspace(0,100,5000);
for n = 1:length(fn),
  r = f2/fn(n);
  hi2(:,n) = phif2(n)*phir2(n)/kn(n)./(1-r.^2+j*2*zn(n)*r);
end
h2=abs(sum(hi2,2));
loglog(f2,h2)
hold on
phif3=[0.01105 0.06268 0.1589 0.294];    %mode shapes for the forcing location
phir3=[1 -1 1 -1];    %mode shapes for the response location
```

```
f3=linspace(0,100,5000);
for n = 1:length(fn),
   r = f3/fn(n);
   hi3(:,n) = phif3(n)*phir3(n)/kn(n)./(1-r.^2+j*2*zn(n)*r);
end
h3=abs(sum(hi3,2));
loglog(f3,h3)
legend('0.05m', '0.30m','0.6m')
ylabel('FRF (m/s^2N)')
xlabel('frequency (Hz)')
```

7.3 codes for impuse function and force frequency function
Here we use comparison.m file

```
clear,clc,close all
format long
%we change the sensitivity to 2.248
sensitivity=2.248;                    %hammer sensitivity (in N/mV)
%%
%Soft Tip
[fnam pnam] = uigetfile({'*.txt'},'Select soft tip file');
fnam = [pnam fnam];
soft = importdata(fnam);
Fs=(soft(2,1)-soft(1,1))^-1;            %sampling frequency
t=soft(:,1);
y=soft(:,2)*sensitivity;
fsoft=(Fs*(0:length(y)-1)/length(y))';  %frequency range
Ysoft=(abs(fft(y)/Fs/(max(t)/2)));
%%
%Hard Tip
[fnam pnam] = uigetfile({'*.txt'},'Select hard tip file');
fnam = [pnam fnam];
hard = importdata(fnam);
Fs=(hard(2,1)-hard(1,1))^-1;
t=hard(:,1);
y=hard(:,2)*sensitivity;
fhard=(Fs*(0:length(y)-1)/length(y))';
Yhard=(abs(fft(y)/Fs/(max(t)/2)));
%%
%PLOTTING
figure(1),plot(soft(:,1)-.0029,(soft(:,2)*sensitivity))
hold on
plot(hard(:,1)-.0047,(hard(:,2)*sensitivity),'-k')
axis([0 0.01 0 0.35])
title('Impulse Force')
xlabel('Time(s)')
ylabel('Force(N)')
legend('soft tip','hard tip')

figure(2),semilogy(fsoft,Ysoft,fhard,Yhard,'-k')
axis([0 1000 0 10e-3])
title('Force Frequency Response')
xlabel('Frequency(Hz)')
ylabel('Force(N)')
legend('soft tip','hard tip')
```

7.4 code for reciprocity residues between points 8 and 18.
Here we use the reciprocity.m file

```
clear,clc,close all
format long
```

```
sensimpact=0.002248; %sensitivity of impact hammer is mV/lbf need to convert to V/N to match data
sensaccel=0.010; %sensitivity of accelerometer is mV/g need to convert to V/g to match data
%%
%Impact Frequency Response
[fnam pnam] = uigetfile({'*.txt'},'Select impact pt 8 data file');
fnam = [pnam fnam];
impact = importdata(fnam);
Fs=(impact(2,1)-impact(1,1))^-1;
t=impact(:,1);                  %time
y=impact(:,2)/sensimpact;       %amplitude
f=(Fs*(0:length(y)-1)/length(y))';  %frequency range
Y=(abs(fft(y)/Fs/(max(t)/2)));
%%
%Accelerometer Frequency Response
[fnam pnam] = uigetfile({'*.txt'},'Select accelerometer pt 18 data file');
fnam = [pnam fnam];
accel = importdata(fnam);
Fs=(accel(2,1)-accel(1,1))^-1;          %sampling frequency
t=accel(:,1);                   %time
y=accel(:,(2))/sensaccel;       %amplitude
f=(Fs*(0:length(y)-1)/length(y))';  %frequency range
X=(abs(fft(y)/Fs/(max(t)/2)));
%%
%FRF 1
Hpt1=[f(:,1) (X(:,1)./Y)];
%%
%%
%Impact Frequency Response
[fnam pnam] = uigetfile({'*.txt'},'Select impact pt 18 data file');
fnam = [pnam fnam];
impact = importdata(fnam);
Fs=(impact(2,1)-impact(1,1))^-1;
t=impact(:,1);                  %time
y=impact(:,2)/sensimpact;       %amplitude
f=(Fs*(0:length(y)-1)/length(y))';  %frequency range
Y=(abs(fft(y)/Fs/(max(t)/2)));
%%
%Accelerometer Frequency Response
[fnam pnam] = uigetfile({'*.txt'},'Select accelerometer pt 8 data file');
fnam = [pnam fnam];
accel = importdata(fnam);
Fs=(accel(2,1)-accel(1,1))^-1;          %sampling frequency
t=accel(:,1);                   %time
y=accel(:,(2))/sensaccel;       %amplitude
f=(Fs*(0:length(y)-1)/length(y))';  %frequency range
X=(abs(fft(y)/Fs/(max(t)/2)));
%%
%FRF 2
Hpt2=[f(:,1) (X(:,1)./Y)];
%%
%Plot
semilogy(Hpt1(:,1),Hpt1(:,2),Hpt2(:,1),Hpt2(:,2))
axis([0  500  1e-4 0.1])
xlabel('frequency (Hz)')
ylabel('H(\omega) (g/N)')
legend('X/F=pt18/pt8','X/F=pt8/pt18')
```

7.5 code for mode shapes from experiment for points 1-5 and 6-10
Here we use normal mode shape .m file

```
clear,clc,close all

[fnam pnam] = uigetfile({'*.txt'},'Select experiment mode shape');
fnam = [pnam fnam];
modes = importdata(fnam);

for i=1:4
   col=i;
   b=abs(modes(:,col));
   div=max(b);
   I=find(b==div);
   extr=modes(I,col);
   nor(:,i)=modes(:,col)./extr;
end

for i=1:4;
   col=i;
   subplot(2,2,i),plot(nor(1:5,i),'-o')
   hold on
   plot(nor(6:10,i),'-o')
   xlabel('position')
   ylabel('shape')
   legend('pts 1-5','pts 6-10')
end
```

7.6 codes for frequency response function from experiment and synthesised frequency response function
Here we use synthesised modes.m file

```
clc,close all

%GET DATA FILE
[fnam pnam] = uigetfile({'*.txt'},'Select impact data');
fnam = [pnam fnam];
fid = importdata(fnam); %time (s), force (V), acceleration (mV)
fid=fid.data;
hammer_sens=2.248;   %mV/N (this was an error and was corrected)
accel_sens=10;  %mV/g (this was an error and was corrected)
t=fid(:,1);
t=abs(min(t))+t;
x=fid(:,2)*1000/hammer_sens;    %hammer signal converted to N (changed from 1000 to 1 for points 5-10. stopped converting mV to V)
y=fid(:,3)*1/accel_sens;   %accelerometer signal converted to g

%select data
plot(x),ylabel('force (N)')
dcm_obj=datacursormode;
set(dcm_obj,'DisplayStyle','datatip','SnapToDataVertex','off','Enable','on')
pause
c_info=getCursorInfo(dcm_obj);
start=c_info.DataIndex
close all, plot(y), ylabel('acceleration (g)')
dcm_obj=datacursormode;
set(dcm_obj,'DisplayStyle','datatip','SnapToDataVertex','off','Enable','on')
pause
c_info=getCursorInfo(dcm_obj);
finish=c_info.DataIndex
```

```
t=t(start:finish,1);
x=x(start:finish,1);
y=y(start:finish,1);
t=t-min(t);
zerosc=2;
%tzero=(linspace(0,(2^nextpow2(length(t))),(2^nextpow2(length(t)))))';
tzero=((linspace(0,(length(t)*zerosc),(length(t)*zerosc)+1)*t(2))+max(t))';
xzero=tzero*0;
yzero=xzero;
t=vertcat(t,tzero);
x=vertcat(x,xzero);
y=(vertcat(y,yzero));
trunc=[t x y];

XF=[9.8431  3.6389  34.8635 7.9352]';        %amplitudes
fnhz=[120.3 270.3 372.1 489.3]';      %natural frequencies
z=[3.3265e-4 9.435e-5 1.1961e-4 4.598e-5]';        %damping ratios
f=(1:0.1:500)'

mnum=1;
d=z(mnum,1);
fn=fnhz(mnum,1);
B1=(f/fn);
H1=((B1.^2)./(sqrt(((1-(B1.^2)).^2)+((2.*d.*B1).^2))));
H1=(H1/max(H1))*XF(mnum,1);
fn1=[B1(:,1) H1(:,1)];

mnum=2;
d=z(mnum,1);
fn=fnhz(mnum,1);
B2=(f/fn);
H2=((B2.^2)./(sqrt(((1-(B2.^2)).^2)+((2.*d.*B2).^2))));
H2=(H2/max(H2))*XF(mnum,1);
fn2=[B2(:,1) H2(:,1)];

mnum=3;
d=z(mnum,1);
fn=fnhz(mnum,1);
B3=(f/fn);
H3=((B3.^2)./(sqrt(((1-(B3.^2)).^2)+((2.*d.*B3).^2))));
H3=(H3/max(H3))*XF(mnum,1);
fn3=[B3(:,1),H3(:,1)];

mnum=4;
d=z(mnum,1);
fn=fnhz(mnum,1);
B4=(f/fn);
H4=((B4.^2)./(sqrt(((1-(B4.^2)).^2)+((2.*d.*B4).^2))));
H4=(H4/max(H4))*XF(mnum,1);
fn4=[B4(:,1),H4(:,1)];

close all

semilogy(f(:,1),fn1(:,2))
hold on
plot(f(:,1),fn2(:,2))
hold on
plot(f(:,1),fn3(:,2))
```

```
hold on
plot(f(:,1),fn4(:,2))
hold on
semilogy(f(:,1),(fn1(:,2)+fn2(:,2)+fn3(:,2)+fn4(:,2)),'-k')
hold on

Fs=(t(2,1)-t(1,1))^-1;      %sampling frequency
f=(Fs*(0:length(x)-1)/length(x))';  %frequency range
halfpt=floor(length(f)/2);
Y=(abs(fft(x)/Fs/(max(t)/2)));
Y2=(abs(fft(y)/Fs/(max(t)/2)));
h=smooth(Y2./Y);

semilogy(f(1:halfpt,1),h(1:halfpt,1))
axis([0 500 0 max(h)])
FRF=[f(1:halfpt,1) h(1:halfpt,1)];



xlabel('frequency (Hz)')
ylabel('acceleration (g/N)')
legend('Mode 1','Mode 2','Mode 3','Mode 4','Synthesised','Original')
```

Engr. Stephen Tambari. “Modal Analysis and Testing on the Vibrational Response of Both Simple and Complex Structural Systems.” IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) , vol. 16, no. 3, 2019, pp. 13-28