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Diffused Distructionof a Spherical Vesselunder Internal Pressure

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Abstract: *The failure front equations and formulas of initial failure time of an isotropic spherical layer under the action of monotonically increasing and cyclically changing internal pressure were derived. The numerical calculations were conducted and the failure front motion curves were constructed for monotonically increasing pressure. A formula of critical number of loading cycles to the first failure was derived.*

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I. Introduction

In the paper [4], a problem on investigation of scattered failure of an isotropic spherical layer under the action of time invariable internal pressure is solved. However, the cases of emergency of pressure spherical vessels at initial period of liquid or gas product injection therein are known. In this connection, there arises a necessity to study the process of deformation and damage of spherical pressure vessels in this initial period characterized by the factor of monotone growth or cyclically changeable internal pressure. In this paper we estimate the influence of these factors.

II. Problem Statement

Consider a spherical layer under the action of internal pressure $p(t)$ monotonically increasing in time and uniformly distributed over the surface. We'll assume that the external surface of the spherical layer is forces-free.

Accept that the pipe's material is isotropic and elastically damageable. In place of deformation equations we take [1]:

$$\begin{cases} e_{ij} = \dfrac{1}{2\mu}\left(1+M^*\right) S_y \\ \varepsilon = \dfrac{1}{3K}\sigma \end{cases} \tag{1}$$

where ε, σ are spherical parts $e_{i\,j,}$ $S_{i\,j}$ are stress and strain tensors deviations:

$$\varepsilon = \varepsilon_{i\,i}; \quad \sigma = \sigma_{i\,i}; \quad e_{i\,j} = \varepsilon_{i\,j} - \frac{\varepsilon}{3}\delta_{i\,j}; \quad s_{i\,j} = \sigma_{i\,j} - \frac{\sigma}{3}\delta_{i\,j}.$$

In relations (1) the volume damage is neglected. In (1) M^* is an integral damage operator of hereditary type

$$M^* s_{i\,j} = \sum_{k=1}^{n} \Phi\left(\sigma_i\left(t_k^+\right)\right) \int_{t_k^-}^{t_k^+} M\left(t_k^+ - \tau\right) s_{i\,j}(\tau) d\tau + \int_{t_{n+1}}^{t} M(t-\tau)\, s_{i\,j}(\tau) d\tau, \tag{2}$$

where $\left(t_k^-;\, t_k^-\right)$ are the damage growth time integrals, $\Phi\left(\sigma_i\left(t_k^+\right)\right)$ is the deficiency healing function [1]. For the stress state monotonically changeable in time, the damage operator (2) goes into ordinary hereditary viscoelasticity operator:

$$M^* s_y = \int_0^t M(t-\tau)\, s_y(\tau) d\tau. \tag{3}$$

At complicated stress states, for damageable isotropic medium, we accept the strength criterion as in [1]:

$$\sigma_i + M^* \sigma_i = \sigma_0 , \quad (4)$$

where σ_i is stress intensity, σ_0 is ultimate strength of defectless material.

As in the problem under investigation the stress increases monotonically in time, then the damage operator has the form of (3). Then the deformation relations are identical to appropriate relations of viscoelasticity theory. The stresses are determined on the basis of the correspondence method and have the same form for the stresses in the case of elastic problem, more exactly:

$$\begin{cases} \sigma_r = \dfrac{pc^3}{1-c^3}\left[1-\left(\dfrac{R}{r}\right)^3\right] \\ \sigma_\theta = \sigma_{0=} \dfrac{pc^3}{1-c^3}\left[1+\left(\dfrac{R}{r}\right)^3\right] , \end{cases} \quad (5)$$

where r is the current radius, and

$$c = \frac{R_0}{R} \quad (6)$$

is a relative thickness of the spherical layer, R_0 and R are the radii of the internal and external surfaces of the spherical layer, respectively. The stress intensity determined by the known formula

$$\sigma_i = \frac{1}{\sqrt{2}}\left[(\sigma_r - \sigma_\varphi)^2 + (\sigma_r - \sigma_\theta)^2 + (\sigma_\varphi - \sigma_\theta)^2\right]^{1/2},$$

allowing for representations (5) for stresses, will be

$$\sigma_i = \frac{15pc^3}{1-c^3}\left(\frac{R}{r}\right)^3 \quad (7)$$

Here the internal pressure p is a time function $- p = p(t)$, $\{1\}$and is monotonically increasing.

According to L.M.Kachanov [3] express the stress intensity by means of the unique function $\beta(t)-$ a dimensionless radius of failure front, having accepted the denotation:

$$\frac{r}{R} = \beta(t); \quad c = \frac{R_0}{R} = \beta(r); \quad 0 < r < 1. \quad (8)$$

Then for stress intensity we get:

$$\sigma_i(t;\tau) = 1{,}5\,p(\tau)\frac{\beta^3(\tau)}{\beta^3(\tau)\left[1-\beta^3(\tau)\right]}. \quad (9)$$

Since σ_i accepts its greatest value on the inner surface of the spherical layer, where $r = R_0$, the failure first happens namely at that place.

Let t_0 be an initial time of failure of internal surface of the spherical layer determining the incubation period of scattered failure. Then for $\tau < t \le t_0$ when $\beta(\tau) = \beta(t) = \beta_0$, where $\beta_0 = R_0 / R$,for stress intensity we have

$$\sigma_i = \frac{1{,}5}{1-\beta_0^3} = p(t) \quad (10)$$

In order to determine the initial failure time t_0, we substitute (8) in strength criterion expression (4) and accept

$$p(t) = p_0 f(t) \quad (11)$$

where $f(t)$ is a dimensional time function. Then we get:

$$f(t_0) + \int_0^t M(t_0 - \tau) f(\tau) d\tau = g(1-\beta_0^3) \quad (12)$$

Here we accept the denotation:

$$g = \frac{\sigma_0}{1{,}5 p_0}. \quad (13)$$

After failure of the internal surface of the spherical layer, there happens redistribution of stresses with consequent failure of layer adjoined to internal surface and so on. Thus, a new generated internal boundary surface, the failure front moves to the side of external boundary surface.

Character of its motion and velocity determine the basic parameters of the scattered failure of a spherical vessel and its long term strength.

Assuming that behind the failure front, the material exhausts its load bearing capacity, the stress intensity in the remaining unfailed part of a spherical vessel will be determined by the same formula (9). For obtaining the failure front motion equation, we substitute (9) in strength criterion (4) and get:

$$\frac{f(t)}{1-\beta^3(t)} + \frac{1}{\beta^3(t)} \int_0^t M(t-\tau) \frac{f(\tau)\,\beta^3(\tau)}{1-\beta^3(t)} d\tau = g. \quad (14)$$

It holds for $t > t_0$. For $t \geq t_0$ the dimensionless radial coordinate of the failure front $\beta(t) = \beta_0$.

III. Problem solution

In order to estimate the effect of liquid or gas product injection period on a spherical vessel when the pressure changes from zero to the given one, we analize the situation qualitatively and quantitatively for a simple case of damage operator kernel:

$$m(t-\tau) = m = const: \qquad f(t) = t^{\alpha}; \;\; \alpha > 0. \quad (15)$$

In what follows we'll consider the time t non-dimensionalized by means of the constant, m and therefore in further notations we accept $m = 1$.

In this case, from equation (12) for the initial incubation period of failure we get the following algebraic equation:

$$t_0^{\alpha} + \frac{1}{1+\alpha} t_0^{+\alpha} = \left(1 - \beta_0^3\right). \quad (16)$$

For obtaining the differential equation of failure front motion, we multiply integral equation (14) by $\beta^3(t)$, differentiate with respect to time and solve the found one with respect to the derivative function $\beta(t)$ in time. For convenience we accept the denotation:

$$z(t) = \beta^3(t) \quad (17)$$

Then for this function we get the following ordinary differential equation of first order:

$$\frac{dz}{dt} = \frac{z(1-z)}{g(1-z)^2 - f} \cdot \left(f + \frac{df}{dt} \right). \quad (18)$$

For the concert from of the function $f(t)$ given by formula (15) this equation accepts the form:

$$\frac{dz}{dt} = \frac{z(1-z)}{g(1-z)^2 - t^{\alpha}} \cdot (\alpha + t)\, t^{\alpha-1}. \quad (19)$$

Relation (16) will be an initial condition for it, where $\beta^3(t) = z_0$ should be assumed.

Consider a special case when $\alpha = 1$. Then condition (1) accepts the form:

$$t_0^2 + 2t_0 - 2g(1 - z_0) = 0.$$

whence for initial failure time we find:

$$t_0 = \sqrt{1 + 2g(1 - z_0) - 1}. \quad (20)$$

It follows from this formula that the incubation period holds always except when $\alpha = 0$, when the pressure $p = p_0$ is constant. In the last case, the incubation period holds only subject to definite restrictions imposed on pressure p_0, initial thickness of the spheric layer β_0 and instant ultimate strength σ_0.

Thus for the function of a cube of dimensionless radial coordinate of the failure front we have the Cauchy problem: first order differential equation (19) for $\alpha = 1$ and initial condition (20) to it. Since according to its physical sense the function $z = z(t)$should be a monotonically increasing function on the interval $[z_0\,;1]$then (19) yields necessity of fulfillment of the condition:

$$g(1-z_0)^2 - t_0^2 > 0\,, \tag{21}$$

providing the positivity of the derivative function $z(t)$. For $\alpha = 1$ this condition is written as

$$\sqrt{1+2g(1-z_0)} - g(1-z_0)^2 < 1. \tag{22}$$

In place of initial numerical data we accept $\beta_0 = 0{,}5;\ g = 2,$ for which condition (22) is fulfilled.

According to formula (20) for the initial failure period we get $t_0 = 1,\ 12$. At initial condition $t_0 = 1,\ 12\ ;\quad z_0 = 0{,}125$ the differential equation (19) having for $\alpha = 1$the from:

$$\frac{dz}{dt} = \frac{z(1-z)}{2(1-z)^2 - t}\cdot(1+t)\,, \tag{23}$$

was solved numerically by the Runge-Kutta method .Note that since $(1-z)^2$ in the course of time decreases, then the critical failure time t_{cr} is the time when the denominator of the right side of equation (23) vanishes, i.e. the derivative function $z(t)$ turns into infinity. This means the beginning of the catastrophic stage of failure. It is determined as a solution of the implicit algebraic equation :

$$2(1-z(t_{cr}))^2 = t_{cr}\,.$$

The calculation was carried out from time to time close to t_{cr} .

The results of the calculations are given in thefigure where the failure front curve for the constant pressure $p = p_0$ when $\alpha = 0$ is also depicted. For this case the Cauchy problem for the function $z(t)$ has the form:

$$\begin{cases} \dfrac{dz}{dt} = \dfrac{z(1-z)}{2(1-z)^2 - t}\ , \\ z_0 = 0{,}125 : t_0 = 0{,}75 \end{cases}$$

The given curves indicate the essential effect of taking account of the filling period of the spherical vessel on its load bearing capacity.

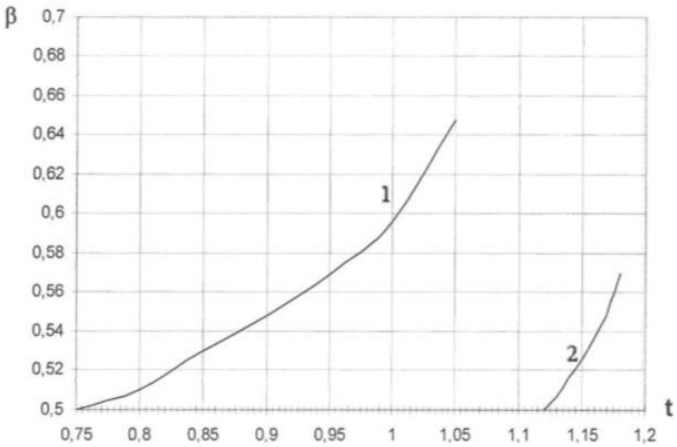

Fig.1. Failure front curves 1) $\alpha = 0$, 2) $\alpha = 1$

Now let's consider the case of cyclic changes of internal pressure:

$$f(x) = 1 + \alpha \sin \omega t \tag{24}$$

where α is a dimensionless amplitude of internal pressure change, ω is the frequency of this change.
Substituting (24) in (11) and (10) and then to failure criterion (3), allowing for the form of damage operator (2), we obtain the following equation to determine the incubation period t_0 :

$$1+\alpha\sin\omega t_0+\sum_{k=1}^{n}\Phi_k\left(t_k^+\right)\int_{t_k^-}^{t_k^+}M\left(t_k^+-\tau\right)\left(1+\alpha\sin\omega\tau\right)d\tau+$$
$$+\int_{t_{n+1}^-}^{t_0}M\left(t_0-\tau\right)\left(1+\alpha\sin\omega\tau\right)d\tau=g\left(1-\beta_0^3\right), \qquad (25)$$

where of g as before is determined by formula (11)
The failure front motion equation will be of the form:

$$\frac{1+\alpha\sin\omega\tau}{1-\beta^3(t)}+\frac{1}{\beta^3(t)}\left\{\sum_{k=1}^{n}\Phi_k\left(t_k^+\right)\int_{t_k^-}^{t_k^+}M\left(t_k^+-\tau\right)\left(1+\alpha\sin\omega\tau\right)\frac{\beta^3(t)}{1-\beta^3(t)}d\tau+\right.$$
$$\left.+\int_{t_{n+1}^-}^{t_0}M\left(t_0-\tau\right)\left(1+\alpha\sin\omega\tau\right)\frac{\beta^3(t)}{1-\beta^3(t)}d\tau\right\}=g;\quad t>t_0 \qquad (26)$$

Analyze the incubation period equation. Rewrite it in the form

$$1+\alpha\sin\omega t_0\int_0^{t_0}M(\tau)d\tau+\alpha\left\{\sum_{k=1}^{n}\Phi_k\left(t_k^+\right)\int_{t_k^-}^{t_k^+}M\left(t_k^+-\tau\right)\left(1+\alpha\sin\omega\tau\right)d\tau+\right.$$
$$\left.+\int_{t_{n+1}^-}^{t_0}M\left(t_0-\tau\right)\sin\omega\tau\,d\tau\right\}=g\left(1-\beta_0^3\right); \qquad (27)$$

As before, accept the damage operator kernel to be constant, $M(t-\tau)=m=\cos t$, then equation (27) will be :

$$1+\alpha\sin\omega t_0+mt_0+\alpha m\left\{\sum_{k=1}^{n}\Phi_k\left(t_k^+\right)\int_{t_k^-}^{t_k^+}\sin\omega\tau\,d\tau+\right.$$
$$\left.+\int_{t_{n+1}^-}^{t_0}\sin\omega\tau\,d\tau\right\}=g\left(1-\beta_0^3\right); \qquad (28)$$

Determine the intervals of active loading time as the intervals of time of positivity and growth of additional cyclically changing part of internal pressure:

$$\left(t_k^-;\,t_k^+\right)=\left(\frac{2\pi(k-1)}{\omega};\,\frac{\frac{\pi}{2}+2\pi(k-1)}{\omega}\right).$$

Taking this into account in (28), and taking integration, we find :

$$1+mt_0+\alpha\sqrt{1+\left(\frac{m}{\omega}\right)^2}\sin\left(\omega t_0-\varphi\right)+\frac{\alpha m}{\omega}\left(1+\sum_{k=1}^{n}\Phi_k\right)=g\left(1-\beta_0^3\right), \qquad (29)$$

where $tg\,\varphi=\frac{m}{\omega}$.

The defects healing characterizes the functions $\Phi_k\left(t_k^+\right)$ that is accepted in the form:

$$\Phi_k\left(t_k^+\right)=\begin{cases}1\;; & in\ \ non-availability\ of\ defect\ \ healing\\ 0 & for\ \ total\ \ defect\ \ healing\,.\end{cases}$$

Assume that the defect healing is not available, i.e. $\Phi_k = 1$. Then for t_0 we approximately accept that $t_0 = \frac{2\pi n}{\omega}$ from (29), find the approximate estimate of the loading cycles corresponding to the incubation periodic:

$$n_{cr} = \frac{g\left(1-\beta_0^3\right)-1}{m\left(2\pi+\alpha\right)} \cdot \omega \qquad (30)$$

IV. Result

By formula (30) to small values of the means pressure P_0 there corresponds a finite number of loading cycles to the first failure

$$n_{cr} = \frac{\sigma_0\left(1-\beta_0^3\right)}{\sqrt{2mP_1}} \cdot \omega \quad (31)$$

where $P_i = \alpha P_0$ is the finite amplitude of additional, cyclically changing part of internal pressure.
Immediate failure occurs for

$$P_{0,cr} = \frac{\sigma_0\left(1-\beta_0^3\right)}{\sqrt{2}} \quad (32)$$

Qualitatively, the curve of dependence of n_{cr} on P_0 has the form indicated in the figure.

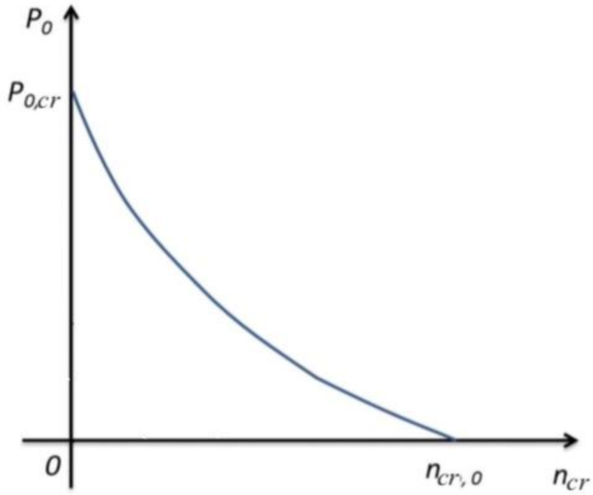

Fig 2. The curve of hidden stage of failure in incubation period.

This curve divides the quarter of the plane $\left(P_0\,;\,n_{cr}\right)$ into the area of realized scattered form of failure and immediate failure area.

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