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Buckling and Postbuckling Analysis of SSSS Rectangular Plates using Modified Iyengar's Equation

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Abstract: *This research work is aimed at modifying the postbuckling equation derived by Iyengar based on classical approach for a plate simply supported alround (SSSS). The equation in its present form lacks numerical application to postbuckling analysis of SSSS rectangular thin plate because of the difficulties of determining the variables 'E^{I}' which is the slope of the stress-strain curve at the inelastic range and 'A' the amplitude of deflection. In order to circumvent this, mathematical manipulation has been carried out on the equation to make the equation easily usable. The new expressions were applied using polynomial shape function to determine the postbuckling load of SSSS plate and the numerical factor β. From the results gotten, comparison was made with those in literature and it was found that the values agreed very closely with those compared with and the numerical factor 'β' was found to be 0.025. Therefore, the conclusion that the new expressions in this work are adequate and an easy expression for postbuckling analysis of SSSS rectangular plates.*

Key Woods: *Postbuckling, Simply supported, rectangular plate, numerical factor, polynomial shape function*

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I. Introduction

The quest for light weight structures in aerospace and ship building industries is on the increase and has catalysed researchers into finding ways of harnessing the strength of materials beyond their initial yield points. They did their analysis mostly based on Airy's stress functions and assumed trigonometric shape function[1,2,3,4,5,6].Of recent, as a result of the difficulties of using assumed trigonometric shape functions, some other researchers formulated new expression based on von Karmon large deflection equation still using stress function. However, they derived all the variables including the shape functions[7.] Their approach even though of great relief as it offer a better understanding of postbuckling behaviour of materials but still involve lengthy and discouraging expressions. The postbuckling equation for a rectangular plate simply supported alround (SSSS) was derived by Iyengar in 1988 as equation (1). The first term in the right hand side (RHS) represent the critical load while the second term represent the additional load due to infinite deflection. The equation in this form lack easy numerical applications as a result of the difficulties of determining the variable E^{I} and A, that is the slope of the stress-strain curve at the inelastic range and the applitude of deflection respectively. The aim of this work is to modify this equation into new simpler expressions which are numerically applicable easily in the postbuckling analysis of SSSS plate and to determine the numerical multiplier factor that account for the elastic behaviour beyond the elastic limit.

II. Methodology of Analysis

Iyengarin 1998 gave the postbuckling equation for SSSS rectangular thin plate as equation (1)

$$N_x = \frac{4D\pi^2}{a^2} + \frac{E^1A^2\pi t}{8a^2} \qquad (1)$$

Where,
E^1 = the modulus of the material at the inelastic range (ie the slope of stress –strain curve beyond elastic limit).
t = the plate thickness
A = the deflection amplitude.
D = the plate rigidity given as equation (2)

$$D = \frac{Et^3}{12(1-v^2)} \qquad (2)$$

Equation (1) cannot be easily determined because of the variables E^1 and A in the second term of the equation. The equation has little or no use at its present form. Therefore, the equation need to be simplified for easy application to obtaining the critical and postbuckling loads of SSSS thin rectangular isotropic plate.

Rearranging this equation becomes

$$N_x = 4\pi^2 \frac{D}{a^2} + \frac{E^1\pi}{8}\left(\frac{A}{t}\right)^2 \frac{t^3}{a^2} \quad (3)$$

$$Let\ E^1 = \beta E \quad (4)$$

Where E is the Young's Modulus or modulus of elasticity

$\beta =$ a numerical factor accounting for the inelastic modulus which is to be determined in this work.

Substituting equation (4) into equation (3) gives equation (5)

$$N_x = 4\pi^2 \frac{D}{a^2} + \frac{\beta E\pi}{8}\left(\frac{A}{t}\right)^2 \frac{t^3}{a^2} \quad (5)$$

Fromequation (2),

$$t^3 = \frac{12(1-v^2)*D}{E} \quad (6)$$

Substitute equation (6) in equation (5) gives equation (7)

$$N_x = 4\pi^2 \frac{D}{a^2} + \frac{\beta E\pi}{8}\left(\frac{A}{t}\right)^2 \frac{12(1-v^2)*D}{Ea^2}$$

$$N_x = 4\pi^2 \frac{D}{a^2} + \frac{\beta\pi}{8}\left(\frac{A}{t}\right)^2 \frac{12(1-v^2)*D}{a^2}$$

$$N_x = 4\pi^2 \frac{D}{a^2} + 12(1-v^2)\frac{\beta\pi}{8}\left(\frac{A}{t}\right)^2 \frac{D}{a^2} \quad (7a)$$

The first term in the RHS of equation (7a) is the critical (initial buckling load) for aspect S =b/a = 1 while the second term is the additional load the plate can carry before failure.

From equation (7a),

$$(1-v^2)\frac{3\beta\pi}{2}\left(\frac{A}{t}\right)^2 \frac{D}{a^2} = N_x - 4\pi^2 \frac{D}{a^2}$$

$$\left(\frac{A}{t}\right)^2 = \frac{2N_x}{3(1-v^2)\beta\pi}\frac{a^2}{D} - \frac{8\pi}{3(1-v^2)\beta}$$

$$\left(\frac{A}{t}\right)^2 = \frac{2}{3(1-v^2)\beta}\left[\frac{N_x a^2}{\pi D} - 4\pi\right] \quad (8)$$

Also, equation (7a) can be written as

$$N_x = \left[4\pi^2 + 3(1-v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right]\frac{D}{a^2} (7b)$$

$$N_x = \eta \frac{D}{a^2} (9) *$$

$$\eta = \left[4\pi^2 + 3(1-v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right] (10)$$

$$But\ Stress\ \sigma_x = \frac{N_x}{t} (11)$$

Substitute equation (7b) in equation (11) gives

$$\sigma_x = \left[4\pi^2 + 3(1-v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right]\frac{D}{ta^2} (12a)$$

$$\sigma_x = \xi \frac{D}{a^2} (12b) *$$

$$where\ \xi = \left[4\pi^2 + 3(1-v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right]\frac{1}{t} (13)$$

Also, let express equation (7a) as

$$N_x = 4\pi^2 \frac{D}{a^2} + 3(1-v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2 \frac{3}{2}\frac{D^I}{a^2} (14)$$

$$D^I = \frac{t^3}{18(1-v^2)} (15)$$

$$N_x = 4\pi^2 \frac{D}{a^2} + (1 - v^2)\frac{9\beta\pi}{4}\left(\frac{A}{t}\right)^2 \frac{D^I}{a^2} (16)$$

And the stress willbe

$$\sigma_x = 4\pi^2 \frac{D}{ta^2} + (1 - v^2)\frac{9\beta\pi}{4}\left(\frac{A}{t}\right)^2 \frac{D^I}{ta^2} \qquad (17)$$

Equations (16) and (17) can be written as equation (18) and (19)

$$N_x = \eta_1 \frac{D}{a^2} + \eta_2 \frac{D^I}{a^2} (18) *$$

$$\sigma_x = \xi_1 \frac{D}{a^2} + \xi_2 \frac{D^I}{a^2} (19) *$$

$$\eta_1 = 4\pi^2;\ \eta_2 = (1 - v^2)\frac{9\beta\pi}{4}\left(\frac{A}{t}\right)^2;\ \xi_1 = \frac{\eta_1}{t};\ \xi_2 = \frac{\eta_2}{t} (20)$$

Equations (18) and (19) can be written as equation (21) and (22) respectively

$$N_x = N_{cr} + N_{add} \ (21)$$

$$\sigma_x = \sigma_{cr} + \sigma_{add} \ (22)$$

Where subscript 'cr' means critical and 'add' means additional.

The amplitude of deflection per thickness of plate ($\frac{A}{t}$) is related to deflection per thickness of plate ($\frac{w}{t}$) by

$$\frac{w}{t} = \frac{Ah}{t} (23)$$

$$Hence, \frac{A}{t} = \frac{w}{ht} (24)$$

Where h is a shape parameter of the deflected plate.

Numerical Application using Polynomial Shape Function

For a plate simply supported alround,theexpression for h is given as [10].

$$h = (R - 2R^2 + R^4)(Q - 2Q^2 + Q^4) \ (25)$$

where R and Q are distance along X and Y axis of the plate respectively

At maximum deflection, which is at the center of the plate, for R = Q = 0.5.

The values of the load and stress coefficients in equations (8), (12), (18) and (19) are presented in Tables 1 and 2 as shown below.

III. Results

The new simplified equations are presented in in Table no1 for both postbuckling load and postbuckling stress.

Tableno1: Iyenyar Equations and the New Equations from the present work

IYENGAR(1988) EQUATION	PRESENT EQUATIONS
$N_x = \frac{4D\pi^2}{a^2} + \frac{E^1A^2\pi t}{8a^2}$	$N_x = \left[4\pi^2 + 3(1 - v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right]\frac{D}{a^2}$
	$N_x = 4\pi^2\frac{D}{a^2} + (1 - v^2)\frac{9\beta\pi}{4}\left(\frac{A}{t}\right)^2\frac{D^I}{a^2}$
	$\left(\frac{A}{t}\right)^2 = \frac{2}{3(1 - v^2)\beta}\left[\frac{N_x a^2}{\pi D} - 4\pi\right]$
$\sigma_x = \frac{4D\pi^2}{ta^2} + \frac{E^1A^2\pi}{8a^2}$	$\sigma_x = \left[4\pi^2 + 3(1 - v^2)\frac{\beta\pi}{2}\left(\frac{A}{t}\right)^2\right]\frac{D}{ta^2}$
	$\sigma_x = 4\pi^2\frac{D}{ta^2} + (1 - v^2)\frac{9\beta\pi}{4}\left(\frac{A}{t}\right)^2\frac{D^I}{ta^2}$
	$where\ \beta = 0.025;\ D^I = \frac{t^3}{18(1 - v^2)}$

The results obtained from numerical application using polynomial shape function are present in Table no2 and Table no3 using equations 9 and 12b (approach 1).

Table no 2: Values of Coefficients of Critical Load, Post-buckling Load, Critical Stress and Post-buckling Stress using approach 1

		S =b/a = 1	v = 0.3	$\beta = 0.025$	t = 20mm
w/t	A/t	$N_x = \eta \frac{D}{a^2}$	$N_x = \eta^1 \pi^2 \frac{D}{a^2}$	$\sigma_x = \xi \frac{D}{a^2}$	$\sigma_x = \xi^1 \pi^2 \frac{D}{a^2}$
		η	η^1	ξ	ξ^1
0	0	39.47842	4.00000	1.97392	0.20000
0.25	2.56	40.18101	4.07119	2.00905	0.20356
0.5	5.12	42.28878	4.28475	2.11444	0.21424
0.75	7.68	45.80173	4.64069	2.29009	0.23203
1	10.24	50.71987	5.13900	2.53599	0.25695
1.25	12.8	57.04319	5.77968	2.85216	0.28898
1.5	15.36	64.77169	6.56274	3.23858	0.32814
1.75	17.92	73.90537	7.48818	3.69527	0.37441
2	20.48	84.44423	8.55599	4.22221	0.42780
2.25	23.04	96.38827	9.76617	4.81941	0.48831
2.5	25.6	109.73750	11.11873	5.48687	0.55594
2.75	28.16	124.49191	12.61367	6.22460	0.63068
3	30.72	140.65149	14.25098	7.03257	0.71255
3.25	33.28	158.21626	16.03066	7.91081	0.80153
3.5	35.84	177.18622	17.95272	8.85931	0.89764
3.75	38.4	197.56135	20.01715	9.87807	1.00086
4	40.96	219.34166	22.22396	10.96708	1.11120

Note: N_{cr} occurs when $\frac{w}{t} = 0$

Tableno 3: Values of Buckling and Postbuckling Load and Stress from approach 1.

	S =b/a	**V**	**E(Mpa)**	**a(mm)**	**t(mm)**	π	$\boldsymbol{\beta}$		
	1	**0.3**	**200,000**	**4000**	**20**	**3.141592654**	**0.025**		
w/t	0	0.25	0.5	0.75	0.1	1.25	1.5	1.75	2.0
A/t	0	2.56	5.12	7.68	10.24	12.8	15.36	17.92	20.48
N_x	361.52	367.96	387.26	419.43	464.47	522.37	593.15	676.79	773.30
σ_x	18.08	18.40	19.36	20.97	23.22	26.12	29.66	33.84	38.66

While those results obtained from equations 18 and 19 are presented in Table no4 and Table no5 for postbuckling load and postbuckling stress respectively that approach 2.

Table no 4: Values of Critical Load, Post-buckling Load, using Approach 2

V	**h**	**E(Mpa)**	**a(mm)**	$\boldsymbol{\beta}$	π	**t(mm)**
0.3	**0.09765625**	**200000**	**4000**	**0.025**	**3.141592654**	**20**
w/t	**A/t**	$\boldsymbol{\eta_1}$	$\boldsymbol{\eta_2}$	$\boldsymbol{N_{cr}}$	$\boldsymbol{N_{add}}$	$\boldsymbol{N_x}$
0	0	39.47842	0.00000	361.52397	0.00000	361.52397
0.25	2.56	39.47842	1.05389	361.52397	6.43398	367.95795
0.5	5.12	39.47842	4.21554	361.52397	25.73593	387.25990
0.75	7.68	39.47842	9.48498	361.52397	57.90584	419.42981
1	10.24	39.47842	16.86218	361.52397	102.94371	464.46768
1.25	12.8	39.47842	26.34716	361.52397	160.84954	522.37351
1.5	15.36	39.47842	37.93990	361.52397	231.62334	593.14731
1.75	17.92	39.47842	51.64042	361.52397	315.26511	676.78908
2	20.48	39.47842	67.44872	361.52397	411.77483	773.29880
2.25	23.04	39.47842	85.36478	361.52397	521.15252	882.67649
2.5	25.6	39.47842	105.38862	361.52397	643.39818	1004.92215
2.75	28.16	39.47842	127.52023	361.52397	778.51179	1140.03576
3	30.72	39.47842	151.75961	361.52397	926.49337	1288.01734
3.25	33.28	39.47842	178.10677	361.52397	1087.34292	1448.86689
3.5	35.84	39.47842	206.56170	361.52397	1261.06042	1622.58439
3.75	38.4	39.47842	237.12440	361.52397	1447.64589	1809.16987
4	40.96	39.47842	269.79487	361.52397	1647.09933	2008.62330

Table no 5: Values of Critical Stress and Post-buckling Stress using Approach 2

V	**h**	**E(Mpa)**	**a(mm)**	$\boldsymbol{\beta}$	π	**t(mm)**
0.3	**0.09765625**	**200000**	**4000**	**0.025**	**3.141592654**	**20**
w/t	**A/t**	ξ_1	ξ_2	σ_{cr}	σ_{add}	σ_x
0	0	1.97392	0.00000	18.07620	0.00000	18.07620
0.25	2.56	1.97392	0.05269	18.07620	0.32170	18.39790
0.5	5.12	1.97392	0.21078	18.07620	1.28680	19.36299
0.75	7.68	1.97392	0.47425	18.07620	2.89529	20.97149

1	10.24	1.97392	0.84311	18.07620	5.14719	23.22338
1.25	12.8	1.97392	1.31736	18.07620	8.04248	26.11868
1.5	15.36	1.97392	1.89700	18.07620	11.58117	29.65737
1.75	17.92	1.97392	2.58202	18.07620	15.76326	33.83945
2	20.48	1.97392	3.37244	18.07620	20.58874	38.66494
2.25	23.04	1.97392	4.26824	18.07620	26.05763	44.13382
2.5	25.6	1.97392	5.26943	18.07620	32.16991	50.24611
2.75	28.16	1.97392	6.37601	18.07620	38.92559	57.00179
3	30.72	1.97392	7.58798	18.07620	46.32467	64.40087
3.25	33.28	1.97392	8.90534	18.07620	54.36715	72.44334
3.5	35.84	1.97392	10.32808	18.07620	63.05302	81.12922
3.75	38.4	1.97392	11.85622	18.07620	72.38229	90.45849
4	40.96	1.97392	13.48974	18.07620	82.35497	100.43117

IV. Discussions

It is observed that at w/t = 0, the load correspond to the initial buckling load or critical load of the plate. To validate this values, comparison was made between the critical load of this present study with existing values in some literatures as shown in Table no6. The results of the study agreed with all the ones compared with, signifying that the new simplified equation are accurate.

Table no 6: Comparison of Coefficient of Nx of this Study with those in Literatures (v = 0.3)

Present	Iyengar (1988)	Adah (2016)	Ibearugbulem et at (2014)	Iwuoha (2016)	Ventsel&Krauthemer (2001)	Timoshenko & Gere (1963)
η_a	η_b	η_c	η_d	η_e	η_f	η_g
39,478	39.478	39.508	39.508	39.508	39.488	39,478
	% Difference $100(\eta_a - \eta_b)/\eta_i\ (i = b, c, d, e, f, g)$					
0.000	0.000	0.0759	0.0759	0.0759	0.0253	0.000

Furthermore, the values of the coefficients of the postbuckling load obtained from the present study were compared with those obtained by Levy (1945) as shown in Table no7. The results indicate closed agreement from the beginning but begin to widen out as w/t increases. It is also observed that the SSSS plate possess additional strength beyond the initial yield point and it may not fail on the basis of geometry but rather material failure. It is also, seen that the value of the numerical multiplier or factor $\boldsymbol{\beta}$ is 0.025. This was gotten by trial approach.

Table no 7: Comparison between Coefficients of Nx of this study with those of Levy (1945)

S = b/a = 1	Present	Levy	$\beta = 0.025$
v = 0.316	$N_x = \eta_a \frac{D}{a^2}$	$N_x = \eta_b \frac{D}{a^2}$	%Difference
w/t	η_a	η_b	$100(\eta_a - \eta_b)/\eta_b$
0	39.47842	39.53432	0.14
0.250	40.17340	40.18243	0.02
0.498	42.23615	42.77484	1.26
0.743	45.61704	46.87950	2.69
0.984	50.24513	52.60442	4.48
1.220	56.02898	59.51752	5.86
1.450	62.85759	68.05089	7.63
1.673	70.60167	77.98848	9.47
1.889	79.15707	89.00624	11.07
2.101	88.56301	101.32021	12.59
2.303	98.45518	114.60633	14.09
2.498	108.86539	129.51272	15.94
2.687	119.76230	145.60729	17.75
2.871	131.13411	161.70187	18.90
3.044	142.51284	181.36101	21.42
3.212	154.19973	202.74843	23.95
3.376	166.21381	231.69707	28.26

V. Conclusion

The present study has modified the postbucklingequation derived by Iyengar (1988) and presented simplified ones which are easier to use in analysing buckling and postbuckling strength of SSSS plate. It has offer two approaches or expressions to use by analysts and designers. The results obtained from the modified expressions agreed closely with those in literature. Therefore, the conclusion that the new obtained expressions are better equations and offer a simpler way of analysis SSSS rectangular plate. The expressions are recommended for use by analyst in aerospace and ship building industries and others.

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