IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 16, Issue 2 Ser. II (Mar. - Apr. 2019), PP 65-71
www.iosrjournals.org

Development of Empirical Equations for Queue Management in Vegetable Oil Plants in Imo State

Owuama, K. C.
Mechanical Engineering Department, Anambra State UniversityUli (now, Chukwuemeka Odumegwu Ojukwu University), Nigeria

Abstract: *Management of queue in vegetable oil plants, in Imo State, had been going on without the established parametric relationship to enable guided and more effective management. This research paper presents the development of empirical equations for the queue management in vegetable oil plants in Imo State. The plants form an $(M_1/M_2/s)$: (∞, FCFS) queue model with consequent assumptions. Mathematical analyses of the queue system established that arrival rate and processing rate of palm kernel in the plants follow a Poisson distribution with the plants' mean palm kernel arrival rate and mean palm kernel processing rate as λ_m = 22.5T/day and μ_m = 5.74T/day respectively. Characterization of the plant's queue parameters were carried out. Graphically, the average empirical relationship of mass in system, Ls versus mass in queue (Lq) and waiting time in queue ($W_{q)}$ versus mass in queue (Lq) in the plants are deduced to be Ls = Lq +3.925, Wq = 0.444 Lq respectively. The empirical relation for prediction of waiting time in queue, and queue length proved valid on a test plant where $\chi^2_{\alpha=0.05} \geq \chi^2$ criterion for validity holds.*

Key word: *Equation, management, queue, vegetable oil plant*

Date of Submission: 11-03-2019 Date of acceptance: 27-03-2019

I. Introduction

Industry is an organized economic activity connected with production, manufacturing, or construction of a particular product or range of products. It is said to be an activity that involves many people, especially one that has become commercialized or standardized

Industry has undergone series of transformation.

Early industries involved manufacturing goods for trade. In medieval Europe, industry became dominated by the guilds in cities and towns, who offered mutual support for the member's interests, and maintained standard industry workmanship and ethical conduct[1]..

Industrial revolution led to the development of factories for large scale production with consequent changes in society. Some manufacturing industries are developed around processing the abundant natural resources harvested or extracted in the area: examples include processing of vegetable oil, canning of fish, processing of fruits and berries, etc.

Vegetable oil industry is one of the process industries. It involves the extraction and processing of oil and fats from vegetable sources. Series of activities and machine operations are involved from preparation of fruits to the packaging of the oil for sale.

When this industry carries out its various activities, that is, processing of the raw materials into finished products, either the machines (servers/customers) or the materials (customers/servers) must wait for some time due to delay in arrival or service time. This leads to queue, that is, a situation where there is idle time for the server or customer to wait for service. In production system,both production and transfer lines have a product-flow layout and are used in mass manufacturing. We denote production lines as flow lines with asynchronous part transfer, while transfer lines have synchronous part transfe,.[9]. While these lines operate, queue cannot be eliminated due to the rate of processing and rate of arrivalof raw materials.

There is need for proper understanding of the queue system and development of models suitable for the management of the plants for minimal loss due to queue.

Many researchers had carried out researches applying queueing theory to firms. In the work on queueing problems with two parallel servers, a group of agents waiting for their job to be processed in a facilitywasconsiderd. Assuming the same amount of processing time and waiting cost, two parallel servers, will be able to serve two agents at a time. Interest was much in finding the order to serve the agents and the (positive or negative) monetary compensations they should receive. Two rules were introduced for the problem, the minimal transfer rules and the maximal transfer rule[2]Inanother work, consideration of a relationship between equity and efficiency in queueing problems was made[3]. They showed that under strategy-proofness, anonymity in welfare implies queue-efficiency. They gave a characterization of the equally distributed pair wise pivotal rule, as the only rule that satisfies strategy-proofness, anonymity in welfare and budget- balance.[4]in their study focused on new measures of performance in single-server Markovianqueueing system. These measures depend on the moments of order statistics. The expected value and the variance of the maximum (minimum) number of customers in the system as well as the expected value and the variance of the minimum (maximum) waiting time are presented. Application to an M/M/1 model is given to illustrate the idea and the applicability of the proposed measures. [5] in their work discussed dynamic system performance evaluation in the river port utilizing queuing models with batch arrivals. The general models of the system are developed. This system is modeled by queue with finite waiting areas and identical and independent cargo-handling capacities. The models are considered with whole and part batch acceptance (or whole and part batch rejections) and the inter-arrival and service times are exponentially distributed. Numerical results and computational experiments are reported to evaluate the efficiency of the models for the real system.

In Nigeria, vegetable oil industry's revolution emerged as a result of 1970-1974 economic plan where industries whose raw materials can be sourced locally are encourage by restricting the importation of finished products of such raw materials [6]The raw materials required for the operation of these plants (palm kernel) are sourced randomly and the arriving quantities are undefined. In time of arrival, some of these raw materials are processed and leaving the unprocessed waiting. This cumulates to queue of the raw materials which sometimes take very long time before being processed, and subsequently degenerate to other problems. Some situations occur where there is scarcity of raw materials, the machines lie idle. It is observed that the queue continues to build up, notwithstanding the effort of the operators to manage the system.

It is noted that proper management of queueuing problems associated with vegetable oil plants, using empirical equations, had not beenestablished in the State. Although, various operation and production systems are instituted in the industry with their consequent queueing arrangements, the industry is still groping with queue system whose is not clear to the investors. In effect management parameters are not available

Following this backdrop, most of these plants experience lots of queueing problems resulting in heavy economic losses. The aim of this work is to develop the empirical equation for the queue management in the plants. To enable proper decision that will enhance cost effective operation of the system.

II. Materials And Method

Data Collection:
Five plants, producing crude vegetable oil from Palm Kernel, are considered for study following the existence of queue in them, operation age not less than ten years and expected to have attained steady state operation. They are tagged plants M, N, O, P and Q.

- The following data were sourced:

Mass of palm kernel arrived per day.
This was generated by measuring the mass of the palm kernel that arrived the company's store per day. This is repeated for 60 days.

Mass of palm kernel processed per day.
Knowing the capacity of the reservoir/hopper, the quantity of raw material processed per day of operation is ascertained by subtracting the remaining quantity (mass) from the initial mass. This was repeated for 60 days.and

Number of machine(s) in operation per day; for 60 days, .

Method of data analysis
The data collected, i.e., mass of palm kernel arriving per day (feed rate), mass of palm kernel processed per day and number of machines engaged per day were statistically analyzed. The mean distribution of the data were evaluated using the equation;

Mean, $\lambda = (\sum fx)/N$. 2.1

where f =frequency; x = variable; and N= sum of frequency..

Variance, $\sigma^2 = \sum(x_i-\lambda)^2/N$. 2.2

Standard deviation, $\sigma = [\sum (x_i-\lambda)^2/N]^{1/2}$. [7, 8] 2.3

The mean distribution of the data was evaluated for unitized masses of arrival and processed palm kernel.This was done using the chi square equation given thus;

$\chi^2 = (o_i - e_i)^2/e_i$ 2.4

Where o_i = observed frequency; e_i = expected frequency; α = significance level chosen = 0.05. Also, v = degree of freedom

Critical region: $\chi^2_\alpha \geq \chi^2$, we cannot reject that the data is from a Poisson distribution.

Ascertaining that the data is from the presumed Poisson distribution for the process system, and examining the other queue system characteristics as well as the arrangement/capacities of production facilities (Hopper feeding the machines and the machines processing the palm kernel), the queue model was defined.
With the model definition, the related queueing mathematical model was adopted for use in the prediction of the queue performance characteristics of the process system. This was established for the industry considering the average operation characteristics of the plants.
From the performance characteristics, of the queue system, obtained, graph showing the mathematical relationship between the number of palm kernel units in the queue, Lq, and number of units of palm kernel in the system, Ls, and waiting time in queue, Wq, were obtained.
Considering graph of Ls against Lq;
Gradient, $a_1 = \Delta Ls/\Delta Lq$.
Limit as ΔLq tends to zero $\Delta Ls/\Delta Lq = dLs/dLq = a_1$
$dLs = a_1 dLq$ (2i)

Integrating equation (3i) i.e., $\int (dLs) = \int a_1 dLq + b_1$.

Where b_1 = constant of integration = intercept on Ls-axis.

$\mathbf{Ls = a_1Lq + b_1}$ 2.5
Considering graph of Wq against Lq;
Gradient, $a_2 = \Delta Wq/\Delta Lq$.
Limit as ΔLq tends to zero $\Delta Wq/\Delta Lq = dWq/dLq = a_2$
$dWq = a_2 dLq$ (2ii)

Integrating equation (3ii) i.e., $\int (dWq) = \int a_2 dLq + b_2$

$\mathbf{Wq = a_2Lq + b_2}$ 2.6

Where b_2 = constant of integration = intercept on Wq-axis.

NB: a_1, a_2 are gradients; b_1, b_2 are intercepts on vertical (Ls-, Wq-) axis.
These are equivalent to $y = ax + b$ (equation of a straight line).
With these, the unique relationship of Lq with Wq and Ls, for the plants, and the industry were established.

III. Results And Discussion

Table 1: Mean distributions of the mass of palm kernel arriving and mass of palm kernel processed per day.

Plant	Mean mass of palm kernel arriving per day ($\dot{m}_\lambda$)	Mean mass of palm kernel processed per day ($\dot{m}_\mu$)
M	45.6	42.45
N	52.95	47.78
O	32.12	20.95
P	27.28	22.78
Q	25.28	21.78

The mean distributions of the data when unitized/ discretized, considering the initials as zero variable, is presented in table 2. Also, using the same equation 2.1, and table.2, the unitized industry's mean distributions of data is presented in table 2a.

Table 2: Mean distributions of the mass of palm kernel arriving and mass processed when unitized

Plant	Mean mass units of palm kernel arriving per day (λ_{plnt})	Mean mass units of palm kernel processed per day (μ_{splant})	Mean number of machines processing per day ($\dot{s}$)	Mean rate of processing per machine unit per day (μ_{plant})	$\frac{\lambda_{industry}}{\square_{industry}}$
M	1.47	1.10	5	0.22	6.682
N	2.80	2.28	5	0.46	6.087
O	2.72	2.20	3	0.73	3.699
P	1.97	1.93	3	0.64	3.078
Q	2.30	1.63	2	0.82	2.805

Table 2a: The industry's mean distribution of the operation characteristics.

Industry	Mean mass units of palm kernel arriving per day (λ_m)	Mean number of machines processing per day ($\dot{s}_m$)	Mean rate of processing per machine unit per day (μ_m)
Vegetable oil industry in Imo state	2.252	3.6	0.574

Fitting into distribution

Using the values of the unitized mean distribution of the data presented in table 2 above, the fitting into the Poisson distribution was carried out using the chi square test. See equation 2.4 and its decision criterion. The 95 percent confidence interval was chosen (i.e., at a significance of 0.05). The table 3a and 3b show the results obtained in a plant. Similar resuls were obtained in other plants.

Table 3a: Fitting a Poisson distribution to the mass of palm kernel arriving per day (mean distribution, λ_B = 2.8).

Mass of kernel arriving per day (Tonnes)	Number of variable, $x_\lambda = m_\lambda$, (new equivalence, unitized)	Pr{x occurring}	Observed frequency (o_i)	Expected frequency (e_i)	$(o_i-e_i)^2/e_i$
(m_λ)					
20.0 - 29.9	0	0.061	4	4	0.0000
30.0 – 39.9	1	0.170	12	10	0.4000
40.0 – 49.9	2	0.238	12	14	0.2857
50.0 – 59.9	3	0.223	13	13	0.0000
60.0 – 69.9	4	0.156	9	9	0.0000
70.0 – 79.9	5	0.087	5	5	0.0000
80.0 – 89.9	6	0.041	3	2	0.5000
90.0 – 99.9	7	0.016	2	1	1.0000

χ^2 =.2.1857. v = k-1-m = 8-1-1 = 6. Therefore, $\chi^2_{0.05} = 12.592$
Crt. $\chi^2_{0.05} \geq \chi^2$, we cannot reject that the sample is from Poisson distribution.

Table 3b: Fitting a Poisson distribution to the mass of palm kernel processed per day (mean distribution, μ_{sB} = 2.3)

Mass of kernel processed per day (Tonnes)	Number of variable, $x_\mu = m_\mu$, (new equivalence, unitized)	Pr{x occurring}	Observed frequency (o_i)	Expected frequency (e_i)	$(o_i-e_i)^2/e_i$
(m_μ)					
20.0 - 29.9	0	0.100	4	6	0.6667
30.0 – 39.9	1	0.231	12	14	0.2857
40.0 – 49.9	2	0.265	18	16	0.2500
50.0 – 59.9	3	0.203	18	12	3.0000
60.0 – 69.9	4	0.117	5	7	0.5714
70.0 – 79.9	5	0.054	3	3	0.0000

.
χ^2 = 4.7738. v = k-1-m = 6-1-1 = 4. Therefore, $\chi^2_{0.05} = 9.488$
Crt. $\chi^2_{0.05} \geq \chi^2$, we cannot reject that the sample is from Poisson distribution

The mathematical model of queue analysis based on Poisson distribution adopted for the plants implied that the arrangement of machines (servers) and the mass arriving in the system, multiple but identical server arrangement exists (i.e. $M_1/M_2/s$). Single queue formation, constant service rate and infinite population with strict queue discipline (F-C, F-S) also prevail. Where M_1 = Marcovianarrival rate distribution, M_2 = Marcovian service rate distribution, and s = number of servers

In this model, let
P_n = the probability that there are n units in the system,
Ls = expected number of calling units in the system,
L_q = expected number of calling units in the queue,
W_s = expected waiting time a customer spends in the system,
W_q= expected waiting time a customer spends in the queue, s = number of servers,
λ = expected arrival rates of calling units, μ = expected service/processing rates per busy server,
$1/\lambda$ = expected inter arrival time, $1/\mu$ = expected service/processing time,
γ = utilization factor for the service facility = λ/μ.
With these notations and definitions of performance measures, the model equations stated below were used to predict the performance characteristics (queue characteristics).

$P_0 = [\sum_{n=0}^{s-1} \frac{1}{n!}(\frac{\lambda}{\mu})^n + \frac{1}{s!}(\frac{\lambda}{\mu})^s . \frac{s\mu}{s\mu-\lambda}]^{-1}$ 3.1

$L_q = \left[\frac{1}{(s-1)!}.(\frac{\lambda}{\mu})^s.\frac{\lambda\mu}{(s\mu-\lambda)^2}\right] P_0$ 3.2

$L s = L_q + \frac{\lambda}{\mu}$ 3.3

$W_q = \left[\frac{1}{(s-1)!}.(\frac{\lambda}{\mu})^s.\frac{\mu}{(s\mu-\lambda)^2}\right] P_0 = \frac{L_q}{\lambda}$ 3.4

$W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu}$ 3.5

$\gamma_s = \frac{\lambda}{s\mu}$ 3.6.

Average performance characteristics of the industry

Considering the mean distribution of the mean arrival rate of palm kernel and processing rate of the machines in the sampled plants, as presented in table 2a, the mean performance characteristics of the industry (vegetable oil processing industry) in Imo state were evaluated and presented in table 4

Table 4; Calculated mean performance characteristics: Palm kernel processing industry in Imo state

(s)	(γ_m)	(P_0)	(L_q)	(L_s)	(W_q)	(W_s)
0 – 3	$\frac{\lambda}{s\mu} > 1$					
4	0.981	0.0019	50.0784	54.0017	22.237	23.979
5	0.785	0.0146	1.9131	5.8451	0.850	2.592
6	0.654	0.0182	0.5030	4.4260	0.223	1.965
7	0.560	0.0193	0.1589	4.0819	0.071	1.813
8	0.490	0.0196	0.0515	3.9745	0.023	1.765

Relationship between number of units of palm kernel in the queue (Lq) and: waiting time in queue (Wq), and number of units of palm kernel in the system(Ls)

Using the table of the calculated performance characteristics of the industry's average. The graphical representations are presented in figure 1

Lq	Ls	Wq
50.0784	54.0017	22.237
1.9131	5.8451	0.85
0.503	4.426	0.223
0.1589	4.0819	0.071
0.0515	3.9745	0.023

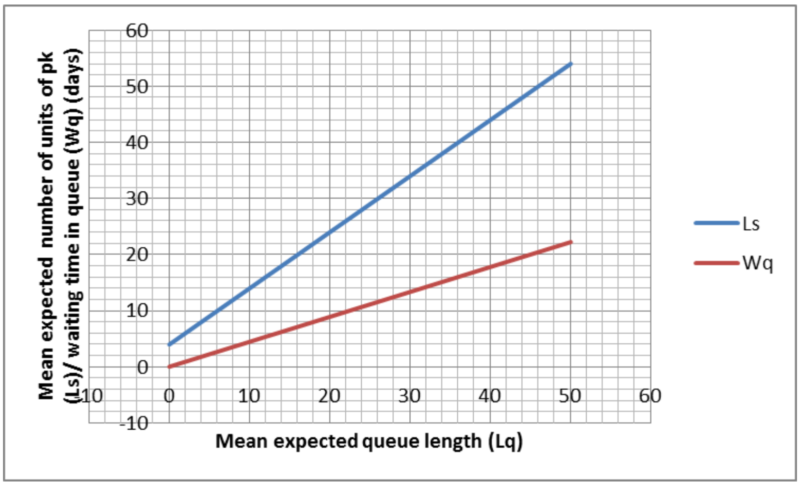

Figure 1: Relationship between expected queue length (Lq), and: waiting time in queue (Wq) and number of units of palm kernel in the system (Ls) , for the industry.

The graph shows straight line relation, i.e., y = ax + b where a = gradient and b = intercept on y-axis. Hence the empirical equivalent equations are $Ls = a_1Lq + b_1$; $Wq = a_2Lq + b_2$. i.e., equations 2.5 and 2.6

From the graphs, the values of "a"and "b" for industry's average are deduced as: $a_1 = 1$ and $b_1 = 3.9252$; $a_2 =$ 0,444 and $b_2 = 0.0002$.

Therefore,substituting the deduced values of "a" and "b" into the empirical relationship of Lq with Wq and Ls for the industry, the following empirical equations result:

Ls = Lq + 3.9252

Wq = 0.444Lq + 0.0002 = 0.444Lq

NB: The intercept on Wq is neglected, since infinitesimal

Application of the result to plant for validation
Plant, H, was selected for the application. The data observed and measured are waiting time in queue and queue length. They are presented in table 5a below.
In comparison, the data of the observed and predicted waiting time in queue and queue length are presented in table 3b and depicted in figures 2.

Table 5b:Comparison of observed and. predicted waiting time in queue, and queue length in the plant

Observed queue length, Lqo	Observed waiting time, Wqo	Predicted waiting time, Wqpred.
2	0.74	0.888
4	1.55	1.776
4	1.62	1.776
5	1.98	2.22
6	2.35	2.664

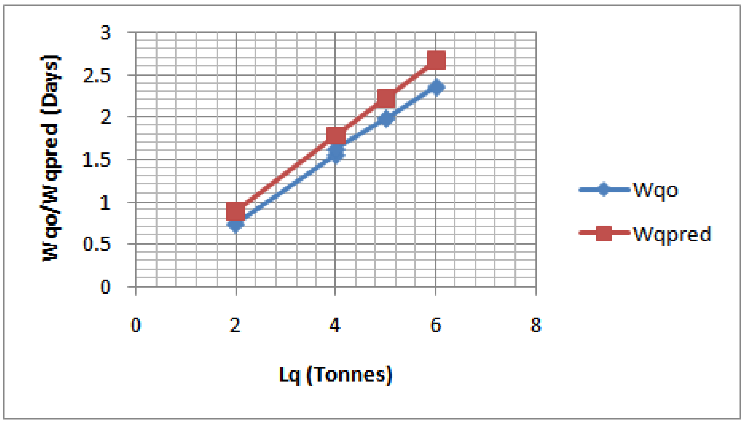

Figure 2: Comparison of the observed and predicted relationship of Lq and Wq for plant H

The gradient of the observed and predicted relationship of the queue length and waiting time in queue are 0.404 and 0.444 respectively.

Test for correctness of result
Using the data presented in table 5b, the correlation between the predicted and observed waiting time in queue was established in figure 3 below. The coefficient of correlation, R^2 was found to be 0.997 (i.e., > 0.8), hence the result is correct.

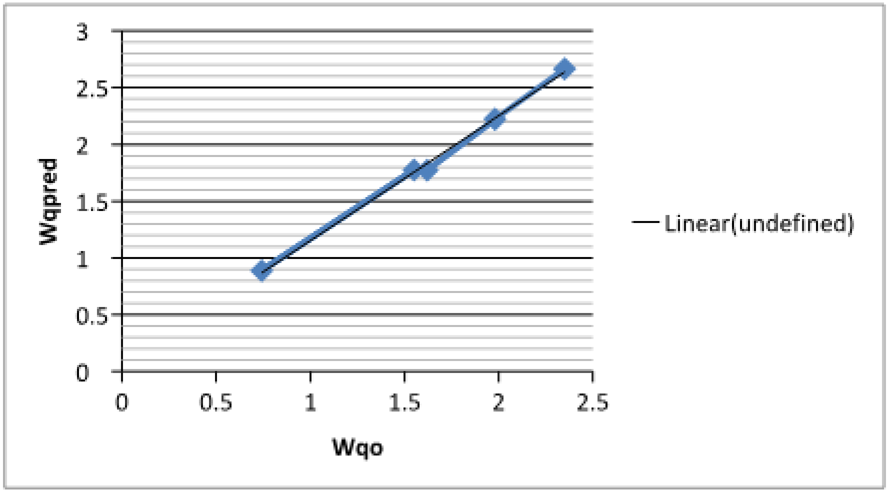

Figure 3: Correlation of predicted and observed waiting times in queue

IV. Discussion

For the industry (vegetable oil plants in Imo state), considering figure 1, the average relationship existing between Lq and Ls, as well as Lq and Wq are straight line, that is direct and proportional. The intercept on the Ls-axis is 3.9252. This imply that the minimum number of processing units that must be engaged for steady state operation for the industry or the utilization factor. The gradient of Wq vs. Lq for the industry is 0.444days which imply the inter-arrival time. The intercept on Wq-axis is approximated zero since value is very small. In effect, the empirical equation relationships for the industry are thus:

Ls = Lq + 3.9252 **4.2.1**
Wq = 0.444Lq. **4.2.2**

Generally, for the industry, Ls tend to Lq as the utilization factor tends to zero or the number of processing units for steady state tends to zero. Also, Wq proportionally varies with Lq in the industry, notwithstanding the value of the inter-arrival time. All the same, waiting time in queue and quantity of palm kernel in the system can be predicted, with knowledge of queue length.

Application of the result to plants for validation

Considering the result obtained and presented in table 5b, it is observed that there is very small variation in the waiting time predicted using the result of the evaluation and that based on observation. Also, the accuracy of prediction increases as the queue length decreases, i.e., as one moves away from exploded queue environment.

Furthermore,the coefficient of correlation R2 = 0.997.This implies that the result of the evaluation is correct and can be used to predict the waiting time in queue as well as queue length of the queue system of vegetable oil plants in Imo state, for management decisions

V. Conclusion

This work is a case study of palm kernel vegetable oil producing plants in Imo State. From the equation; $\frac{\lambda_m}{s\mu_m} < 1$, the steady state operation zone for the industry starts with 4 machines, and minimum of 5 machines are required to operate in a stable zone of zero waiting time in queue.

The relationship of Ls versus Lq, and, Wq versus Lq are direct and proportional relations with the empirical equations as

Ls = Lq + 3.9252

Wq = 0.444

References

[1]. Beck, B. R. World history: Patterns of Interaction. Evanston Illinois: McDouglasLittel.1999

[2]. Young, S. C.&Eun, J. H. "*Queueing problems with 2 parallel Servers*" ISER Discussion Paper 0683, institute of Social and Economic Research, Osaka University .Japan. 2007

[3]. Kazuhiko, H. & Hiroki, S. "Strategy Proof & Anonymous Rules in Queueing Problems: A Relationship Between Equity and Efficiency *Discussion Paper in Economics and Business*. 2008. . 08-17, Japan: Osaka University Graduate School of Economics and Osaka School of International Public Policy (OSIPP).

[4]. Yousry, H. A. &Maram, A. Computing the Performance Measures in Queueing Models via the Method of Order Statistics. *Journal of Applied Mathematic.2011*(ID 790253).

[5]. Branislav, D., Nam-Kyu, P., Nenad, Đ. Z. & Romeo, M. (2012).*Mathematical Models of Multiserver Queuing System for Dynamic Performance Evaluation in Port*. Retrieved from http://dx.doi.org/10.1155/2012/710834

[6]. Owuama, K.C.&Anyanwu, E. E. Waste Disposal and Sanitation Management in Vegetable Oil Producing Industries in Southern Eastern Nigeria. *African Journal of Engineering Research and Development*,2013. *6*(1), 154-159.

[7]. Murray, R. S.& Larry, J. S.. *Theory and Problems of Statistics: Schaum'sOutlines*(3rd ed.).New Delhi. Tata Mcgraw – Hill Publishing Company Limited. 2006

[8]. Richard, J., John, F. & Irwin, M..Probability and Statistics for Engineers. (8thed.).U.S.A.:Prentice Hall Pearson Education Inc., 2011.

[9]. Papadopoulos H. T., Heavey C. Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines. *European Journal of Operational Research*,.1996 92(1), 1–27

Owuama, K. C.. "Development of Empirical Equations for Queue Management in Vegetable Oil Plants in Imo State." IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) , vol. 16, no. 2, 2019, pp. 65-71.