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Regular and Irregular Plate Deflection Analysis using Matrix Method

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Abstract: *This paper presents a simplified method for the calculation of thin-plate deflections for the regular and irregular plates. The method, which deals with the uniform and concentrated loaded plates, uses the stiffness method to obtain plate deflection equations for specified boundary conditions. A case study was presented to demonstrate the use of the proposed method and to illustrate its capabilities. The results obtained were in close agreement with those obtained analytically and with those obtained using the finite element methods. Finally, a user-friendly program for the plate deflection calculations based on the proposed method was developed using the mathematical package MATHCAD.*

Keywords: *Deflection, Finite Element, Mathcad, Plate Analysis, Stiffness*

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I. Introduction

A flat plate is a structural element whose thickness is relatively small compared to its in-plane dimensions. Several analytical methods, such as the equilibrium and energy methods, have been developed for the calculation of plate deflections [1-2]. However, these methods are not always possible, and one must resort to numerical methods such as the finite difference and finite element methods.

The finite difference method requires the solution of a set of simultaneous equations while the finite element methodrequires a mesh generation and a solution for a large stiffness matrix. Even though they are accurate and widely used, these numerical methods are costly since they require longer solution time and can only be implemented by qualified technical people. The high cost of these numerical solutions is not always justified especially in preliminary design cases where low accuracy results are still acceptable [3-4]. For these cases, a less costly simplified method is usually more than adequate [5].

Several experimental tests and analytical techniques have been conducted to determine the short and long-term deflections in the slabs [6]. Tahsin Reza Hossain et.al. [7] presented a simple method to calculate the long-term deflection of multistory building due to the construction loading. Richard and Andrew [8] performed an experimental testing on three one-way reinforced slabs subjected to concentrated load and compare their results with the standard thin plate theory.

This paper presents a simplified method for plate deflection calculations. The proposed method uses the concept of the stiffness method to calculate the deflection of a plate with specified boundary conditions and subjected to a uniform and with concentrated loads [9-11]. A case study is presented to demonstrate the use of the simplified method and to show its capabilities. Finally, a program for plate deflection calculation was developed using mathematical package MATHCAD.

II. Proposed Method

Figure 1 shows a rectangular plate with in-plane dimensions a x b. The plate, which has a specified boundary conditions, is subjected to a uniformly- distributed load x(x,y), the plate deflection equation is derived using the stiffness method. Figure 2 shows the point n1 where the plate deflection is sought. Figure 3 shows the plate structural model. Based on the plate's boundary conditions, the following cases of analysis are considered [5]

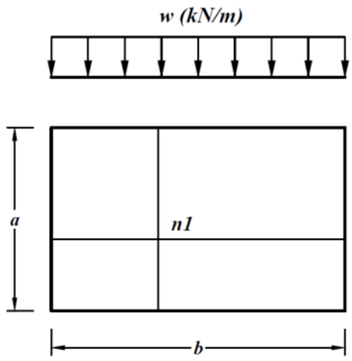


Figure 1: Rectangular plate with dimensions a x b

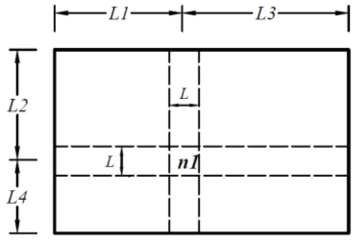


Figure 2: Nodal point Location in-plane

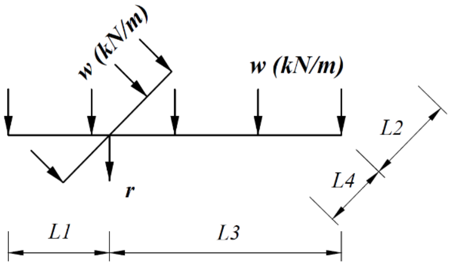


Figure 3: Plate structural model

2.1 Fixed Boundaries

Figure 4 shows the structural model of the plate while figure 5 shows the internal element forces. The displacement transformation matrix [a] is computed using the following equations [5].

$$[\boldsymbol{q}] = [\boldsymbol{a}][\boldsymbol{r}] \quad (1)$$

$$[\boldsymbol{q_1}\ \boldsymbol{q_2 q_3 q_4}\ \boldsymbol{q_5}\ \boldsymbol{q_6}\ \boldsymbol{q_7 q_8}]^{\boldsymbol{T}} = \left[\frac{-1}{L_1}\frac{-1}{L_1}\frac{-1}{L_3}\frac{-1}{L_3}\frac{-1}{L_4}\frac{-1}{L_4}\frac{-1}{L_2}\frac{-1}{L_2}\right]^{\boldsymbol{T}} [\boldsymbol{r}] \quad (2)$$

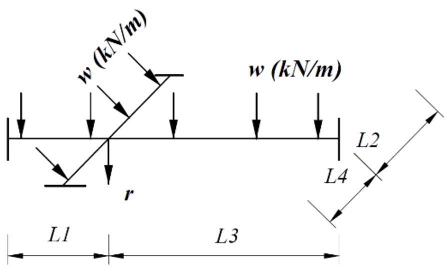


Figure 4: Plate structural model for fixed boundaries

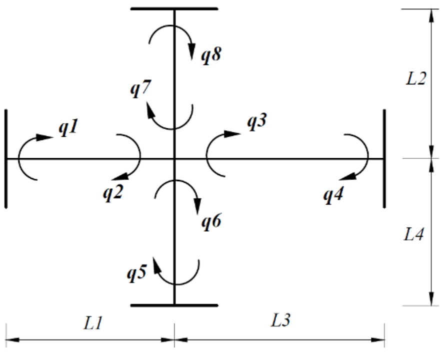


Figure 5: Plate internal element forces for fixed boundaries

The element stiffness matrix **[k]** is equal to:

$$[k] = \frac{D}{L}\begin{bmatrix} 4 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix} \tag{3}$$

The flexural rigidity **D** is given by the following equation:

$$D = \frac{Et^3}{12(1-v^2)} \tag{4}$$

Where t is the plate thickness, E is the modulus of elasticity, v is the Poisson's ratio, and L is the member length. In our case L is taken equal to unity ($L = 1$).
The structure stiffness matrix [K] is assembled using the following equations:

$$[K] = [a]^T[k][a] \tag{5}$$

The load matrix [R} is given by the following equation:

$$[R] = \frac{w}{2}(L_1 + L_2 + L_3 + L_4) \tag{6}$$

Finally, the required displacement matrix $[r]$ is obtained as follows [5]:

$$[r] = [K]^{-1}[R] \tag{7}$$

$$[r] = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[24[D(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]} \tag{8}$$

For the concentrated load

$$[R] = P \tag{9}$$

And the displacement matrix $[r]$ is obtained as follows:

$$[r] = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\;\, P}{[12[D(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]} \tag{10}$$

2.2 Simply-Supported Boundaries

Figure 6 shows the structural model while Figure 7 shows the internal element forces. The required displacement matrix [r1] is given by the following equations [5]:

$$[r_1] = [K]^{-1}[R] \tag{11}$$

$$[r_1] = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[6[D(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]} \tag{12}$$

For the concentrated load case;

$$[R] = P$$

And the displacement matrix $[r]$ is obtained as follows:

$$[r_1] = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\;\, P}{[3[D(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]} \tag{13}$$

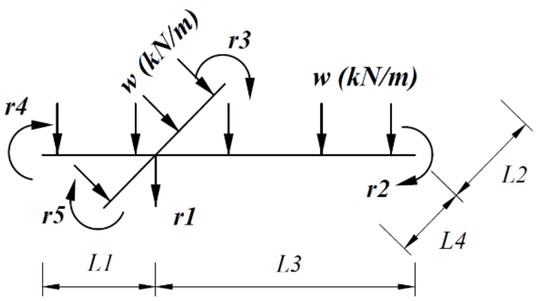


Figure 6: Structural model for simply supported boundaries

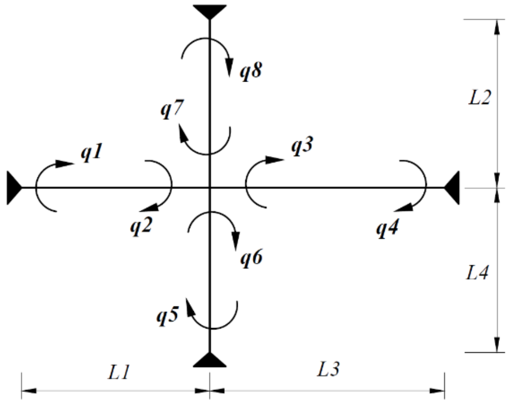


Figure 7: Plate internal element forces for simply-supported boundaries

2.3 Other Boundaries

Tables 1 and 2 summarizes the displacement matrices for several boundary conditions for uniform and concentrated loaded plates [5].

Table 1: Displacement matrices for uniformly loaded plates

Sr. No.	Case S $\Longrightarrow$ Simple F $\Longrightarrow$ Fixed	*Vertical Displacement at any point (r)*
1	F F F F	$r = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[24.[D.(L_3^2\, L_4^2 L_2^2\; + L_1^2 L_4^2 L_2^2\; +\; L_1^2\, L_3^2\, L_2^2\; +\; L_1^2 L_3^2 L_4^2)]]}$
2	S S S S	$r = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[6.[D.(L_3^2\, L_4^2 L_2^2\; + L_1^2 L_4^2 L_2^2\; +\; L_1^2\, L_3^2\, L_2^2\; +\; L_1^2 L_3^2 L_4^2)]]}$
3	F S S F	$r = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[6.[D.(L_3^2\, L_4^2 L_2^2\; + L_1^2 L_4^2 L_2^2\; +\; 4L_1^2\, L_3^2\, L_2^2\; +\; 4L_1^2 L_3^2 L_4^2)]]}$
4	S S S F	$r = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[6.[D.(L_3^2\, L_4^2 L_2^2\; + L_1^2 L_4^2 L_2^2\; +\; 4L_1^2\, L_3^2\, L_2^2\; +\; L_1^2 L_3^2 L_4^2)]]}$
5	F F F S	$r = \frac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, w\,(L_1 + L_2 + L_3 + L_4)}{[6.[D.(4L_3^2\, L_4^2 L_2^2\; + 4L_1^2 L_4^2 L_2^2\; +\; L_1^2\, L_3^2\, L_2^2\; +\; 4L_1^2 L_3^2 L_4^2)]]}$

Table 2: Displacement matrices for concentrated loaded plates

Sr. No.	**Case** **S $\Longrightarrow$ Simple** **F $\Longrightarrow$ Fixed**	***Vertical Displacement at any point (r)***
1	**F** **F** **F** **F**	$r = \dfrac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, P}{[12.[D.(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]}$
2	**S** **S** **S** **S**	$r = \dfrac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, P}{[3.[D.(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]}$
3	**F** **S** **S** **F**	$r = \dfrac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, P}{[3.[D.(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, 4L_1^2\, L_3^2\, L_2^2\, +\, 4L_1^2 L_3^2 L_4^2)]]}$
4	**S** **S** **S** **F**	$r = \dfrac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, P}{[3.[D.(L_3^2\, L_4^2 L_2^2\, + L_1^2 L_4^2 L_2^2\, +\, 4L_1^2\, L_3^2\, L_2^2\, +\, L_1^2 L_3^2 L_4^2)]]}$
5	**F** **F** **F** **S**	$r = \dfrac{L_1^2 L_2^2\, L_3^2\;\, L_4^2\, P}{[3.[D.(4L_3^2\, L_4^2 L_2^2\, + 4L_1^2 L_4^2 L_2^2\, +\, L_1^2\, L_3^2\, L_2^2\, +\, 4L_1^2 L_3^2 L_4^2)]]}$

2.4 Plates with Holes

Figure 8 shows the structural model of a plate with a central hole. Only the plates with fixed boundaries and having holes in the center or very close to the center were considered herein. Table 3 summarizes the displacement matrices for the plates considered [5].

Table 3: Displacement matrices for regular plates with central holes loaded uniformly

Sr. No.	**Case** **S $\Longrightarrow$ Simple** **F $\Longrightarrow$ Fixed**	***Vertical Displacement at any point (r)***
1	**F** **F** **F**	$r = \dfrac{L_1^2 \,.\, L_3^2 \,.\, L_2^2 \,.\, w\; (2.L_1 + L_2 + L_3)}{[24.[D.(L_3^2.L_2^2\, + L_1^2.L_2^2\, +\, L_1^2.L_3^2)]]}$
2	**S** **S** **S**	$r = \dfrac{L_1^2 \,.\, L_3^2 \,.\, L_2^2 \,.\, w\; (2.L_1 + L_2 + L_3)}{[6.[D.(L_3^2.L_2^2\, + L_1^2.L_2^2\, +\, L_1^2.L_3^2)]]}$

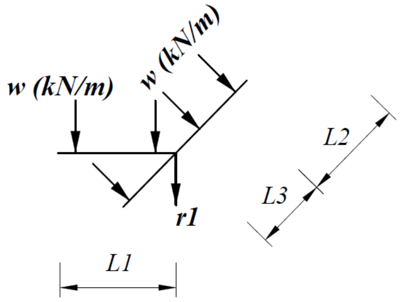


Figure 8: Structural model for a plate with a central hole

III. Case Study

To determine the use of the simplified methodand to verify the accuracy of its results, a case study was conducted. Total of twelve plates, which are shown in Figures 9 to 20, were selected for this study. Six of the selected plates contained holes. The plates were analyzed for uniformly distributed load as well for the concentrated load under different boundary conditions. The uniformly loaded plates (Plate-1 and Plate 3) were analyzed for allofthe five different boundary conditions that are listed in table 1 while the remaining solid plates were investigated only under the fixed boundary conditions which is the case 1 of table 1. Plates with the holes (Plate 2, 4, 6, 7, 10 and 12) were analyzed for fixed boundary conditions that are shown in case-1 of table 3. For the concentrated loaded case, solid plates with the fixed boundary condition (case 1 of table 2) were examined.

The nodal deflection magnitude and the nodes coordinates that are shown in the tables are all in meters. The parameters used for all the plate examples are: Uniform load ($w = 12\ kN/m^2$), Concentrated Load = 50 kN, Poison's ratio ($\upsilon = 0.3$), modulus of elasticity ($E = 302000\ kN/m^2$), thickness ($t = 0.15m$) and material density= $25\ kN/m^3$.

Table 4 summarizes the nodal deflection results for uniformly loaded square plates (Plate 1 and 2) while table 5 summarizes the results for the rectangular plates (Plate 3 and4). These plates were computed using the proposed method, the computer program STAAD Pro v8i, SAP 2000 v16 and Timoshenko's method respectively. The rest of the plates were computed only with the simplified plate deflection method (SPD) and the computer program STAAD Pro v8i as their deflection results are depicted in table-6. Table 7 shows the deflection results for the concentrated loaded plates using the SPD method and computer program STAAD Pro v8i.

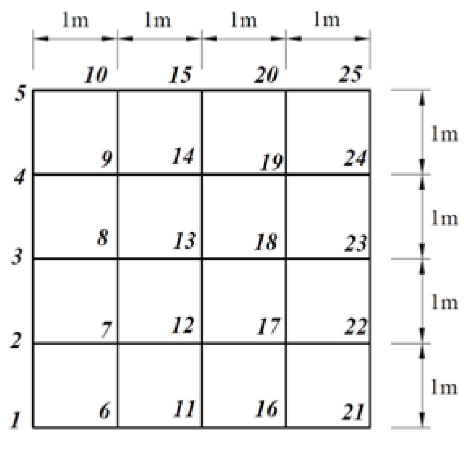


Figure 9: Plate- 1

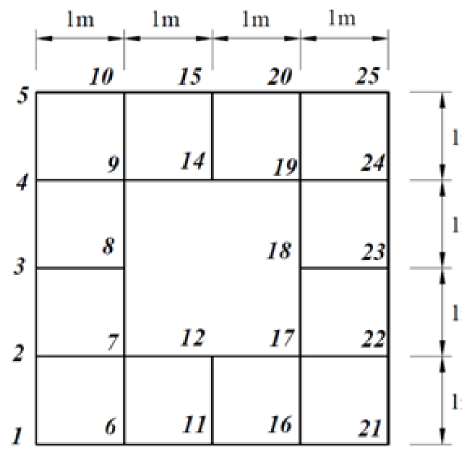


Figure 10: Plate 2

Table 4. Plate Nodal Deflection Results for uniformly loaded square plate

Plate Number	Boundary Conditions	Node No.	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	Vertical Displacement (m) SPD	SAP	STAAD Pro.	TIMOSHENKO
1	1	7	1.00	3.00	3.00	1.00	0.019	0.017	0.014	……..
		8	1.00	2.00	3.00	2.00	0.027	0.029	0.024	……..
		13	2.00	2.00	2.00	2.00	0.043	0.048	0.041	0.042
	2	17	3.00	3.00	1.00	1.00	0.077	0.069	0.070	……..
		18	3.00	2.00	1.00	2.00	0.106	0.096	0.097	……..
		13	2.00	2.00	2.00	2.00	0.171	0.133	0.132	0.134
	3	12	2.00	3.00	2.00	1.00	0.035	0.04	0.035	……..
		13	2.00	2.00	2.00	2.00	0.069	0.069	0.061	0.063
		17	3.00	3.00	1.00	1.00	0.031	0.029	0.027	……..
		18	3.00	2.00	1.00	2.00	0.055	0.05	0.046	……..
	4	7	1.00	3.00	3.00	1.00	0.033	0.036	0.033	…….
		9	1.00	1.00	3.00	3.00	0.067	0.056	0.055	……..
		13	2.00	2.00	2.00	2.00	0.098	0.096	0.090	0.092
		14	2.00	1.00	2.00	3.00	0.088	0.077	0.075	……...
	5	17	3.00	3.00	1.00	1.00	0.029	0.027	0.024	……..
		18	3.00	2.00	1.00	2.00	0.030	0.034	0.029	……..
		19	3.00	1.00	1.00	3.00	0.020	0.018	0.016	……..
		13	2.00	2.00	2.00	2.00	0.053	0.056	0.050	0.052
2	1	8	1.00	2.00	2.00	-----	0.021	0.021	0.020	…….
		14	1.00	2.00	2.00	-----	0.021	0.021	0.020	…….
		18	1.00	2.00	2.00	-----	0.021	0.021	0.020	…….
		12	1.00	2.00	2.00	-----	0.021	0.021	0.020	…….

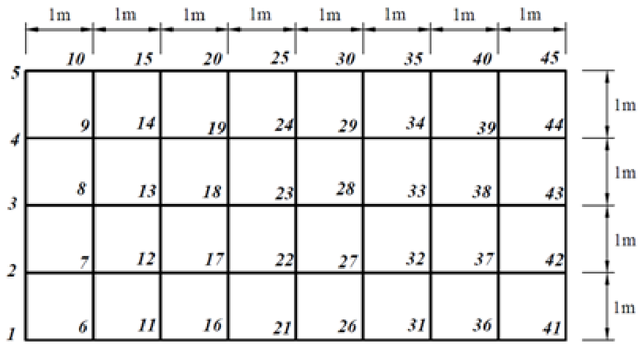


Figure 11: Plate-3

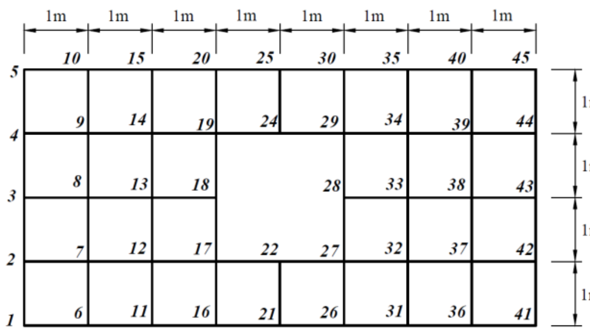


Figure 12: Plate-4

Plate Number	Boundary Conditions	Node No.	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	Vertical Displacement (m)			
							SPD	SAP	STAAD Pro.	TIMOSHENKO
3	1	8	1	2	7	2	0.04	0.03	0.03	------
		19	3	1	5	3	0.05	0.05	0.04	------
		12	2	3	6	1	0.05	0.04	0.04	------
		23	4	2	4	2	0.1	0.09	0.08	0.08
		27	5	3	3	1	0.05	0.05	0.04	------
	2	18	3	2	5	2	0.4	0.31	0.31	-----
		23	4	2	4	2	0.41	0.33	0.32	0.33
		24	4	1	4	3	0.21	0.23	0.23	-----
	3	28	5	2	3	2	0.12	0.09	0.08	-----
		23	4	2	4	2	0.12	0.09	0.08	0.09
		32	6	3	2	1	0.05	0.04	0.04	-----
		33	6	2	2	2	0.11	0.08	0.07	-----
	4	37	7	3	1	1	0.05	0.04	0.04	-----
		23	4	2	4	2	0.19	0.16	0.15	0.16
		32	6	3	2	1	0.06	0.06	0.06	-----
		18	3	2	5	2	0.18	0.15	0.15	-----
	5	12	2	3	6	1	0.1	0.09	0.08	-----
		23	4	2	4	2	0.15	0.15	0.14	0.15
		18	2	2	5	2	0.14	0.14	0.13	-----
		34	1	1	2	3	0.05	0.05	0.05	-----
4	1	18	3	2	2	-----	0.088	0.077	0.081	-----
		22	1	4	4	-----	0.048	0.035	0.029	-----
		24	1	4	4	-----	0.048	0.035	0.029	-----
		28	3	2	2	-----	0.088	0.077	0.081	-----

Table 6. Plate Nodal Deflection Results for uniformly loaded irregular plates

Plate Number	Boundary Conditions	Node No.	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	Vertical Displacement (m)	
							SPD	STAAD Pro.
5	1	7	1.00	2.60	3.00	0.87	0.015	0.016
		12	2.00	2.60	2.00	0.87	0.020	0.020
		13	2.00	1.73	2.00	1.73	0.034	0.033
		14	2.00	0.87	2.00	2.60	0.020	0.020
6	1	8	1.00	1.73	1.73	-----	0.018	0.015
		13	2.00	1.73	1.73	-----	0.044	0.044
7	1	11	1.73	1.00	2.00	-----	0.022	0.017
		16	1.73	2.00	1.00	-----	0.048	0.048
8	1	9	1	1.732	4	0.866	0.015	0.010
		11	1	1.732	2	2.6	0.023	0.021
		16	2	3.464	2	1.732	0.054	0.036
		21	3	1.732	1	1.732	0.023	0.021
9	1	14	2	1	2.667	3	0.031	0.030
		18	3	2	1.8	2	0.051	0.046
		22	4	3	0.667	1	0.014	0.010
10	1	13	2	2	2	------	0.057	0.065
		18	1.8	2	2	------	0.05	0.053
11	1	6	1	0.866	2	2.6	0.013	0.016
		10	2	1.732	2	1.732	0.034	0.028
		14	2	2.6	1	0.866	0.013	0.015
12	1	6	0.866	1	2	------	0.01	0.010
		11	0.866	2	1	------	0.01	0.011
		10	1.732	2	2	------	0.048	0.038

Table 7. Plate with Concentrated Load Nodal Deflection Results

Plate Number	Boundary Conditions	Node No.	L_1 (m)	L_2 (m)	L_3 (m)	L_4 (m)	Vertical Displacement (m) SPD	STAAD Pro.
		13	2	2	2	2	0.045	0.047
1	1	8	1	2	2	3	0.028	0.028
		17	3	3	1	1	0.02	0.0196
		23	4	2	4	2	0.066	0.06
3	1	13	2	2	6	2	0.057	0.053
		34	6	1	2	3	0.032	0.29
5	1	13	2	2	2	2	0.045	0.048
8	1	16	2	2	2	2	0.045	0.049
9	1	13	2	2	3	2	0.0518	0.0517
11	1	10	2	2	2	2	0.045	0.044

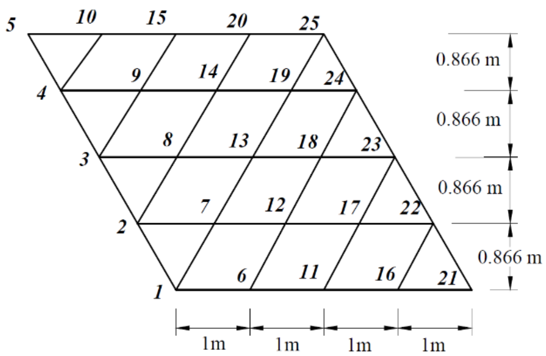


Figure 13: Plate-5

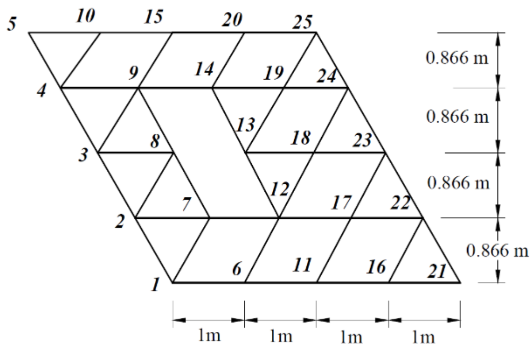


Figure 14: Plate-6

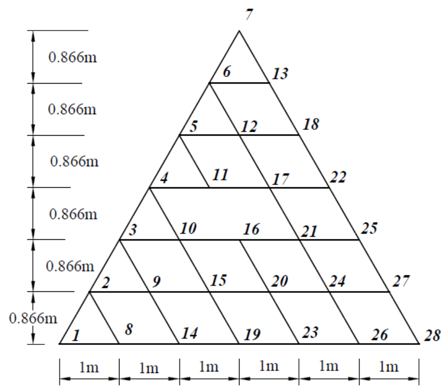


Figure 15: Plate 7

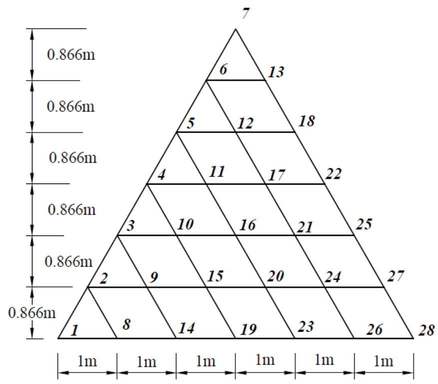


Figure 16: Plate-

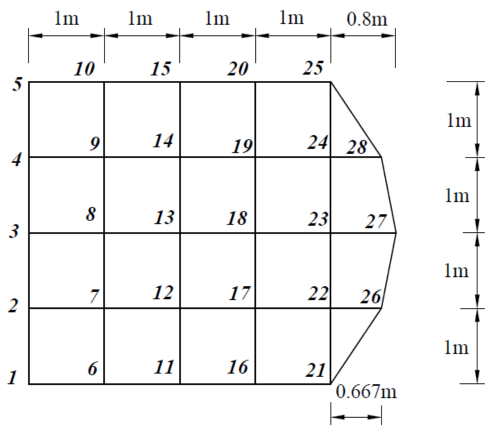


Figure 17: Plate -9

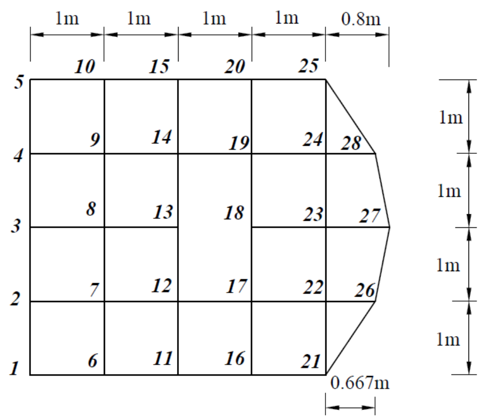


Figure 18: Plate 10

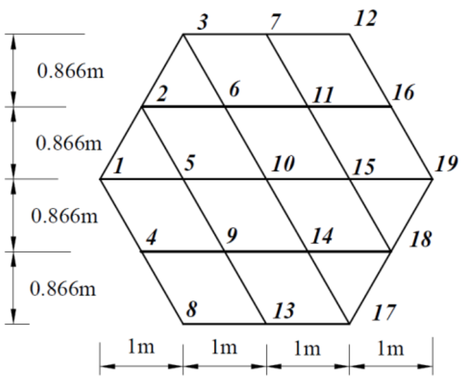


Figure 19: Plate 11

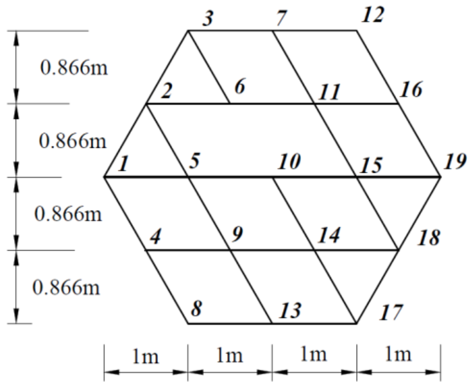


Figure 20: Plate 12

IV. Results and Discussions

The result shows that the proposed method yielded good and accurate results as compared to those obtained using finite-element programs and as compared to those obtained analytically.

Figure 21 shows the comparison of the nodal displacements for the uniformly loaded plates. It reflects that the values obtained from three different approaches are relatively close to each other.

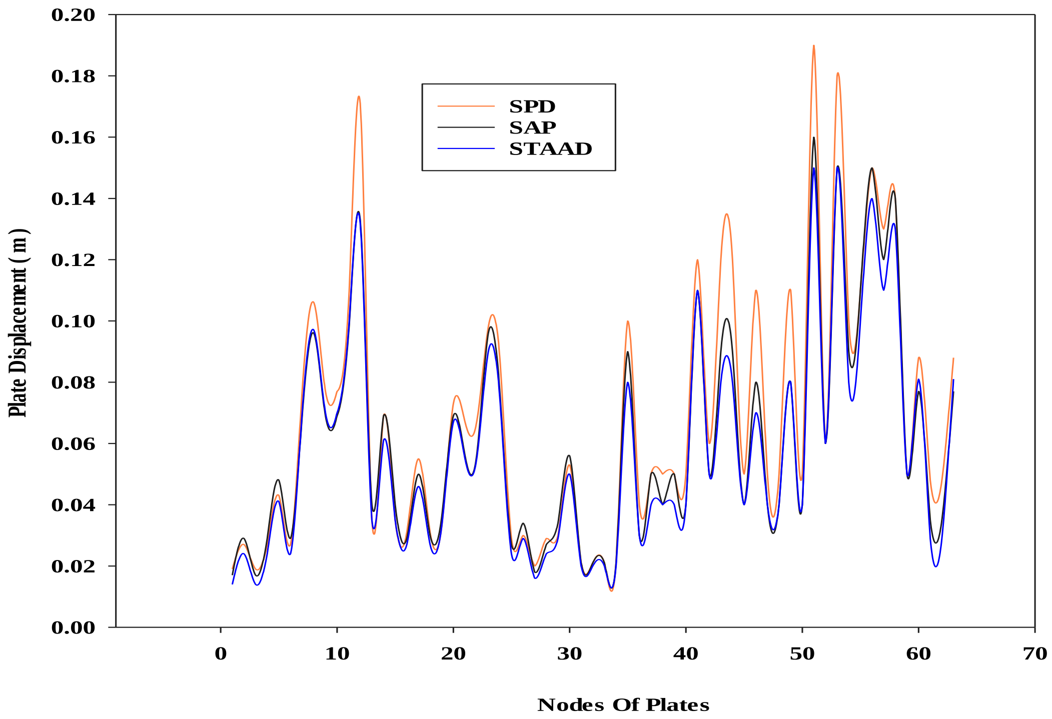


Figure 21: Nodal Displacements Comparison of Plates Loaded uniformly

The bar charts in Figure 22and 23depicts the nodal deflection results of plates 1 and3 with five different boundary conditions (BC-1 to BC-5) loaded uniformly. The deflection results showed that the proposed method generated good and accurate results to the ones obtained from different numerical and theoretical approaches.

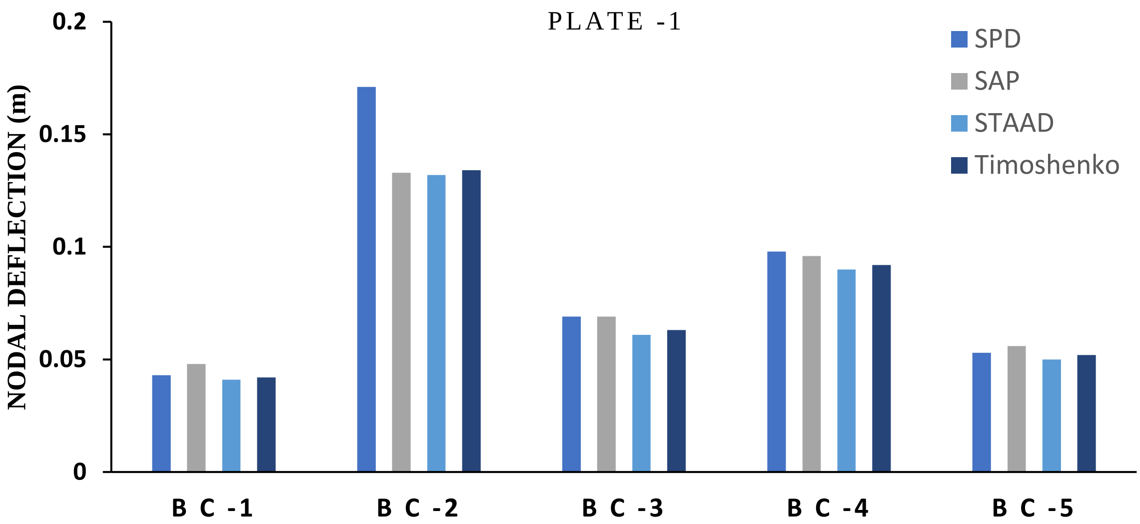


Figure 22: Nodal displacement comparison of Plate -1 loaded uniformly

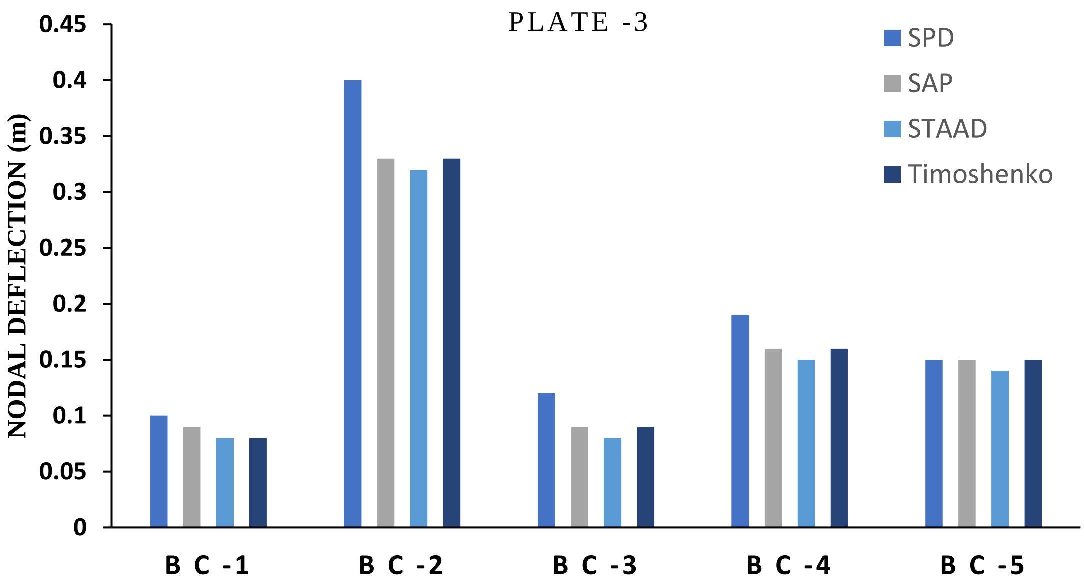


Figure 23: Nodal displacement comparison of Plate -3 loaded uniformly

Figure 24 shows the deflection results for the remaining irregular plates using the theoretical approach (SPD) with the finite element program (STAAD). The deflection values give a good agreement between the values obtained from the theoretical approach with the ones using the computer program.

The bar chart in Figure 25 displays the maximum nodal deflection results for a concentrated loaded case. The nodes with a maximum deflection results were taken in these plates and by looking at the results, it clearly shows that the deflection results values obtained from the theoretical approach are relatively close to the ones obtained from the computer program. Moreover, Figures 26 and 27 shows the deflection contours in all the plates loaded for the uniformly loaded case.

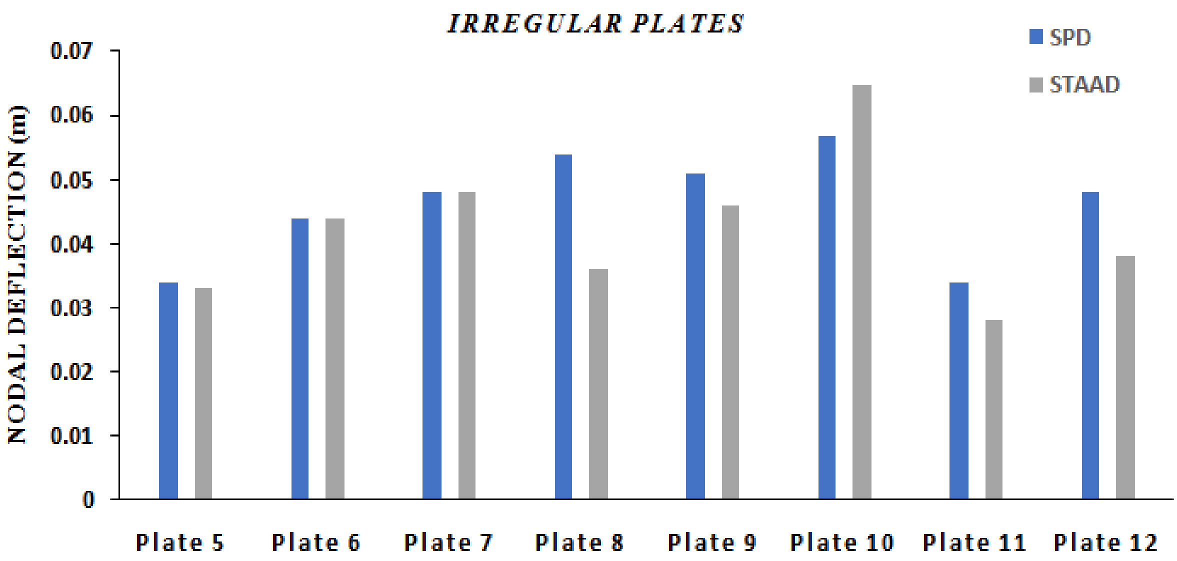


Figure 24: Nodal displacement comparison for Irregular plates loaded uniformly

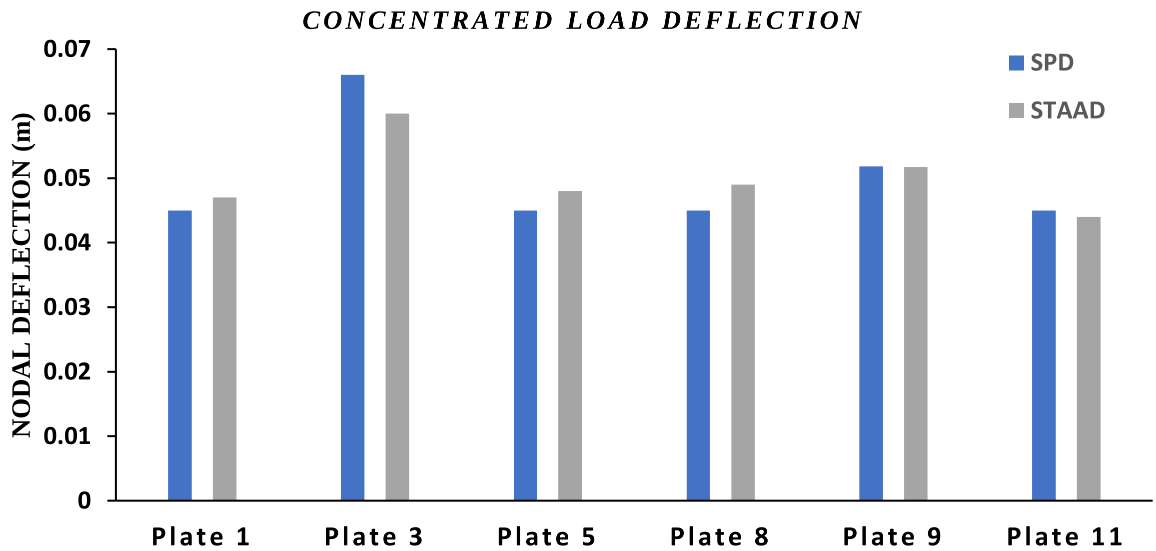
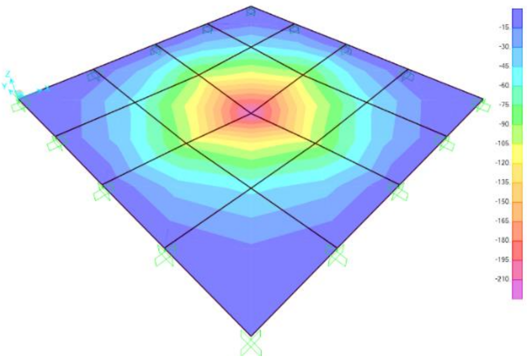


Figure 25: Nodal deflection results for concentrated loading case



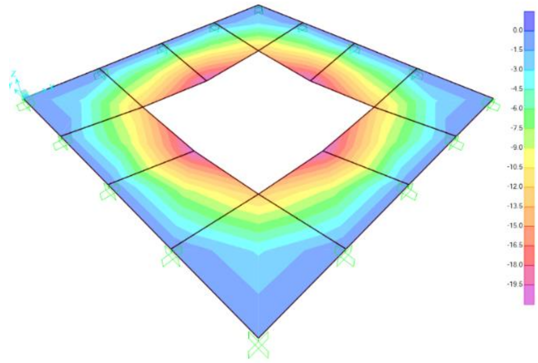
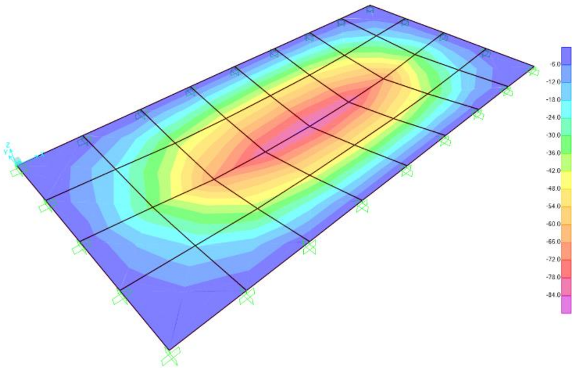


Plate 1 ***Plate 2***



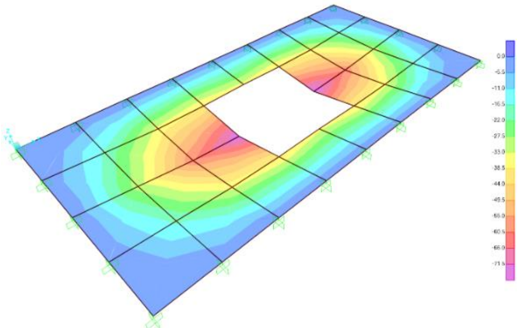
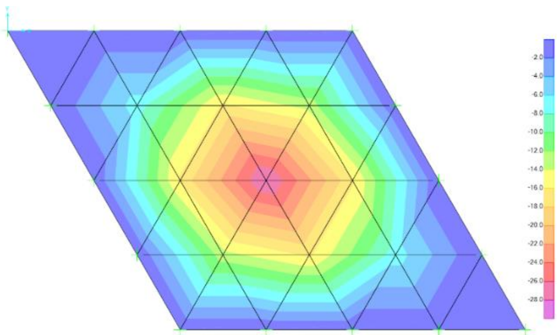


Plate 3 ***Plate 4***



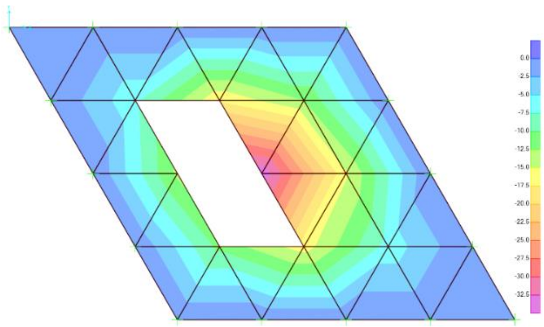


Plate 5 ***Plate 6***

Figure 26: Deflection Contours of uniformly loaded plates

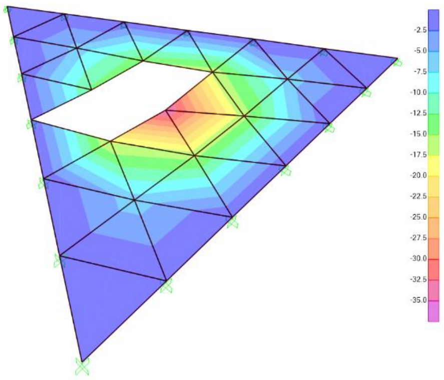


Plate 7

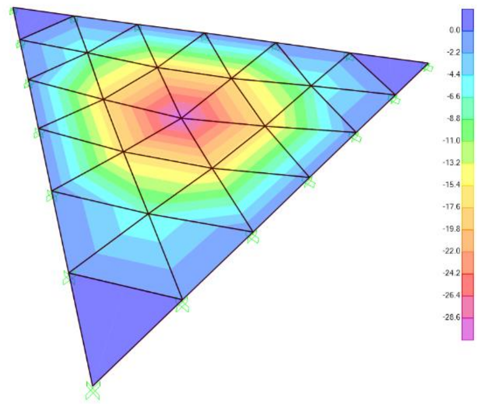


Plate 8

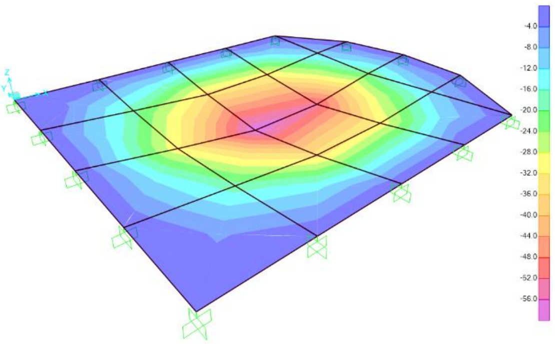


Plate 9

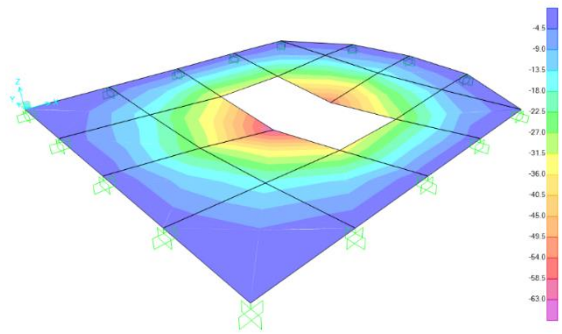


Plate 10

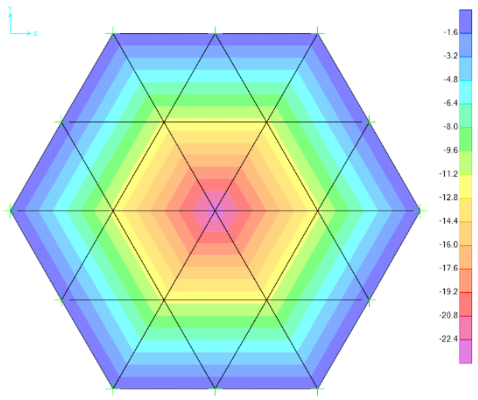


Plate 11

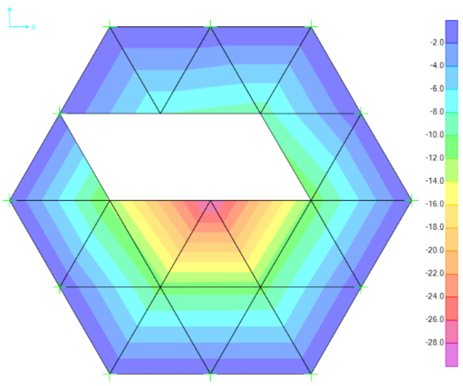
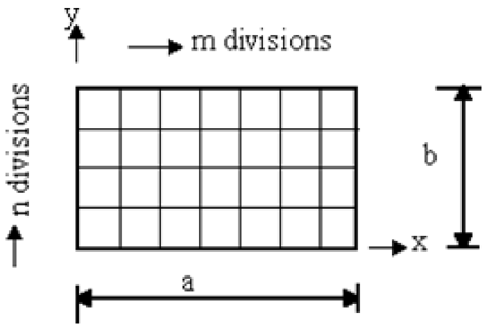


Plate 12

Figure 27: Deflection Contours of uniformly loaded plates (continued)

V. Program Description

A computer program has been written using the mathematical package Mathcad to implement the computational procedure of the proposed method [5]. Figure 28 shows the sheet of the Mathcad program for a fixed - supported plate loaded uniformly while figure 29 shows the sheet of Mathcad program for a concentrated loaded fixed-supported plate. The input data for the program consists of the dimensions $\boldsymbol{a}$ and $\boldsymbol{b}$ of the plate, the intensity of the uniformly distributed loading $\boldsymbol{w}$, the number of divisions $\boldsymbol{n}$ in the y-axis, the number of divisions m in the x-axis, and the plate flexural rigidity D. The number of divisions $\boldsymbol{n}$and $\boldsymbol{m}$ dictate the number of nodal points where the plate deflections are to be computed. The output of the program consists of the nodal deflection matrix $\boldsymbol{\delta}$ which indicates the deflection of the plate at all the nodal points.



INPUT DATA:

a => length in x - axis
b => length in y - axis
w => uniform loading
n => # of divisions in y - axis
m => # of divisions in x - axis
D => flexural rigidity

OUTPUT :

=> nodal deflection matrix

$a := 4 \qquad b := 4 \qquad n := 4 \qquad m := 4 \qquad w := 12 \qquad D := 93.3379$

$L1 = x_i \qquad L2 = b - y_j \qquad L3 = a - x_i \qquad L4 = y_j$

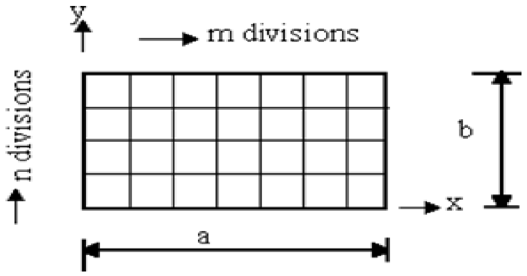
$i := 1..n \qquad j := 1..m \qquad x_i := \frac{a}{n}\cdot i \qquad y_j := \frac{b}{m}\cdot j$

Fixed Supported Boundaries:

$$\delta_{i,j} := \frac{\left[(x_i)^2\cdot(b-y_j)^2\cdot(a-x_i)^2\cdot(y_j)^2\right]\cdot w\cdot\left[(x_i+y_j)+(a-x_i)+(b-y_j)\right]}{24\cdot\left[D\cdot\left[(a-x_i)^2\cdot(y_j)^2\cdot(b-y_j)^2+(x_i)^2\cdot(y_j)^2\cdot(b-y_j)^2+(a-x_i)^2\cdot(x_i)^2\cdot(b-y_j)^2+(a-x_i)^2\cdot(x_i)^2\cdot(y_j)^2\right]\right]}$$

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.019 & 0.027 & 0.019 & 0 \\ 0 & 0.027 & 0.043 & 0.027 & 0 \\ 0 & 0.019 & 0.027 & 0.019 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 28: *MATHCAD Program Sheet for Fixed – Supported Plate (Uniform Loading)*



INPUT DATA:
a => length in x - axis
b => length in y - axis
P => Concentrated Load
n => # of divisions in y - axis
m => # of divisions in x - axis
D => flexural rigidity

OUTPUT :
=> nodal deflection matrix

$a := 4 \quad b := 4 \quad n := 4 \quad m := 4 \quad P := 50 \quad D := 93.3379$

$L1 = x_i \quad L2 = b - y_j \quad L3 = a - x_i \quad L4 = y_j$

$i := 1..n \quad j := 1..m \quad x_i := \frac{a}{n} \cdot i \quad y_j := \frac{b}{m} \cdot j$

Fixed Supported Boundaries:

$$\delta_{i,j} := \frac{\left[(x_i)^2 \cdot (b - y_j)^2 \cdot (a - x_i)^2 \cdot (y_j)^2\right] \cdot P}{12 \cdot \left[D \cdot \left[(a - x_i)^2 \cdot (y_j)^2 \cdot (b - y_j)^2 + (x_i)^2 \cdot (y_j)^2 \cdot (b - y_j)^2 + (a - x_i)^2 \cdot (x_i)^2 \cdot (b - y_j)^2 + (a - x_i)^2 \cdot (x_i)^2 \cdot (y_j)^2\right]\right]}$$

$$\delta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0.028 & 0.02 & 0 \\ 0 & 0.028 & 0.045 & 0.028 & 0 \\ 0 & 0.02 & 0.028 & 0.02 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 29: MATHCAD Program Sheet for Fixed – Supported Plate (Concentrated Load)

VI. Conclusion

A simplified method for the calculation of plate deflections (SPD) has been developed. The method, which deals with uniformly as well as concentrated loaded plates, uses the stiffness method to obtain plate deflection equations for specified boundary conditions. The results obtained using the proposed method were in close agreement with those obtained analytically and with those obtained using the finite element methods.

The proposed method does not require mesh generation and a digital computer to solve a large stiffness matrix as in the case of finite element method, or to solve a set of simultaneous equations as in the case of the finite difference method. It only consist of one equation to determine the nodal deflections for any plate with specific boundary conditions. Locating the nodal coordinates (L1, L2, L3 and L4) and using the right deflection equation based on the plate's boundary conditions is all that one needs to determine the plate nodal deflection.

The proposed method relates only to the plates and cases of boundary conditions that are analyzed in this study. The limitations of the plate size for this proposed method (SPD)are square plates up to 6m x 6m, rectangular plates of the ratio 1:2 up to 5m x10m and irregular plates up to 6m x 6m.The SPD method appears to be yielding good results conservative with mean error of 8%, 10%, and 8%to SAP, STAAD, and Timoshinko respectivelyfor the plates analyzed in this paper.

References

[1]. Timoshenko, S. and Woinowsky, S., (1959), "Theory of Plates and Shells",McGraw-Hill Book Company, New York, USA.
[2]. Wang, C. K., 1983. Indeterminate Structural Analysis, McGraw-Hill BookCompany, New York, USA.
[3]. STAAD Pro V8i, 2014. Finite Element Program, Research Engineers Inc.,Orange, California 92667, USA.
[4]. SAP 2000 v16, "Integrated Software for Structural Analysis and Design", Computers and Structures Inc. CSI., Berkeley, California.
[5]. Mohammed S. Al-Ansari 1997. "Simplified Method for Plate DeflectionCalculation", Engineering Journal of University of Qatar, Vol. 10, 1997, p.51-67.
[6]. American Concrete Institute (2000). "Control of Deflection in ConcreteStructures", ACI 435 R-95.
[7]. Tahsin Reza Hossain, Robert Vollum and Salah Uddinj Ahmed 2011."Deflection Estimation of Reinforced Concrete Flat Plates Using ACI Method". ACI Structural Journal, V 108, No. 4.
[8]. Richard C. Fenwick and Andrew R. Dickson (1989), Slabs Subjected toConcentrated Loading. ACI Structural Journal, V86, No.6.
[9]. R.C Hibbeler, Structural Analysis, Pearson Prentice Hall Inc. 2015, 9thEdition in SI units,United States.
[10]. Mohammed Bin Salem 2015, Structural Analysis & Selected Topics, Partridge, Singapore.
[11]. AslamKassimali, Structural Analysis, Cengage Learning., 2011, Fourth Edition in SI units,Stamford, CT 06902, U.S.A.

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