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Buckling analysis of single-layered FG nanoplates on elastic substrate with uneven porosities and various boundary conditions

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Abstract: *In this investigation the buckling behavior of single-layered functionally graded nanoplates with uneven porosities exposed to hygrothermal loads is modeled for the first time. Using the Eringen's nonlocal elasticity theory with one scale parameters, the small size effect on the buckling behaviour of the functionally graded nanoplates is considered. Based on the new first order shear deformation theory the equations of equilibrium are obtained from the principle of minimum potential energy. Also, elastic Pasternak foundation is adopted to capture the foundation influence on the critical buckling load. The equations of equilibrium are solved for various boundary conditions using Galerkin's method. The impacts of nonlocal parameter, porosity distribution and boundary conditions on the critical buckling load are demonstrated.,*

Keywords: *Buckling, FG nanoplates, Galerkin's method, Nonlocal elasticity theory, Porosities*

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I. Introduction

Functionally graded (FG) plates and nanoplates have an inhomogeneous structure and very good properties under thermal loadings due to the change of mechanical and thermal properties in the thickness direction. FG nanoplates are made from a mixture of ceramic and metal with a change of mechanical properties between two surfaces. FG nanoplates as well as other nanostructures have size dependent mechanical properties and behaviours. In analyzing their mechanical behaviour a conventional form of continuum mechanic which takes into account the influence of size effects is required. The nonlocal theory of elasticity of Eringen's [1,2] is a continuous mechanical model that introduces influence of atomic length scale into the constitutive relations via material (nonlocal) parameter.

Daneshmehr et al. [3] studied the buckling behaviour of FG nanoplates with different boundary conditions using the nonlocal elasticity and higher order plate theories. Sobhy [4] investigated the bending response, free vibration and mechanical buckling of single-layered FG nanoplate embedded in elastic medium using the four-unknown shear deformation theory incorporated in nonlocal elasticity theory. Ansari et al. [5] applied nonlocal three-dimensional theory to investigate to free vibration of FG nanoplates resting on elastic foundation. Khorshidi and Fallah [6] presented the buckling analysis of FG nanoplate based on nonlocal shear deformation theory. Liu et al. [7] performed an analysis of nonlocal vibration and biaxial buckling of double-viscoelastic FG nanoplate on the basis of nonlocal elasticity theory and the Kelvin model. Shahverdi and Barati [8] investigated the vibration beahaviour of porous functionally graded nanoplates using a general nonlocal strain-gradient theory. Barati and Zenkour [9] investigated the wave propagation of nanoporous graded double-nanobeams based on the general bi-Helmholtz nonlocal strain gradient elasticity. She et al. [10] carried out the vibration behaviour of porous nanotubes based on the nonlocal strain gradient theory and refined beam theory which includes effects of shear stress. Shafiei and Kazemi [11] used the modified couple stress theory to analyze the buckling behaviour of functionally graded porous nano-/micro-scaled beams. Radić [12] examined buckling behaviour of double-layered porous FG nanoplates in Pasternak elastic foundation using nonlocal strain gradient theory.

To the authors' best knowledge, the buckling behaviour of a single-layered FG nanoplates with uneven porosities and various boundary conditions has not been studied in the open literature.

II. Theoretical formulation

2.1 Porosities and thickness dependent material properties of FG nanoplate

Consider a single-layered FG nanoplate resting on elastic Pasternak foundation with uneven porosities distribution of uniform thickness *h*, length L_x and width L_y associated with the *z*, *x* and *y*-axes of the coordinate system. The present single-layered FG nanoplate is under in-plane mechanical and hygrothermal compressive load and embedded in Pasternak foundation.

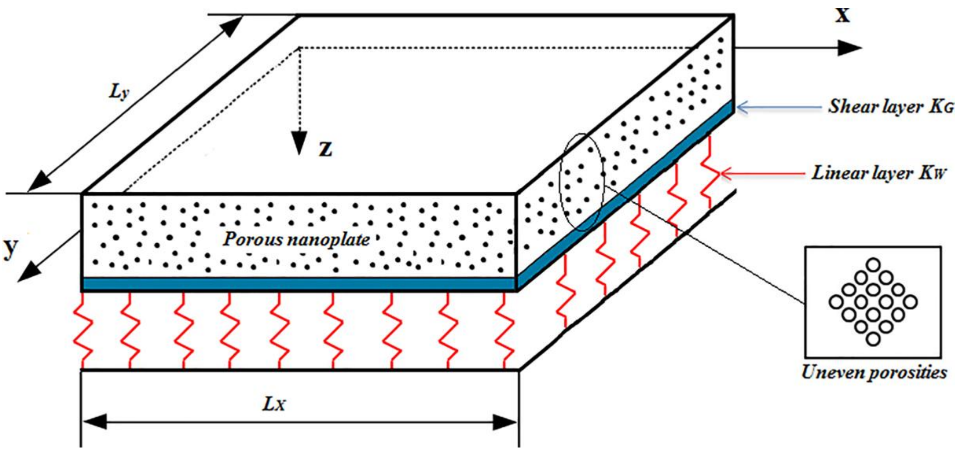

Figure1: Configuration of uneven porous single-layered FG nanoplate

The FG nanoplate is assumed to be composed of mixture of Al and Al_2O_3 and exposed to hygrothermal environment. For uneven distribution of porosities the Young's modulus *E* is changed continuously in the thickness direction by the following form.

$$E(z) = (E_c - E_m)\left(\frac{z}{h}+\frac{1}{2}\right)^p + E_m - \frac{\xi}{2}(E_c + E_m)\left(1-\frac{2|z|}{h}\right)$$

(1)

where *p* is the non-homogeneity or power-law index (*p* is a non-negative parameter) which determine the material distribution across the nanoplate thickness.

2.2 New First Order Shear Deformation Theory (NFSDT)

The present displacement field of new first order shear deformation theory can be expressed as [12]

$$u_x(x,y,z) = u(x,y) - (z-z_0)\frac{\partial \theta}{\partial x}$$

$$u_y(x,y,z) = v(x,y) - (z-z_0)\frac{\partial \theta}{\partial y}$$

$$u_z(x,y,z) = w(x,y)$$

(2)

where u, v and *w* are the displacement of mid-plane of FG nanoplate along *x, y* and *z*-axis respectively, and θ is rotation parameter.

The position of the physical neutral surface z_0 can be determined from the condition that the integral of the first momentum of elasticity modulus $E(z)$ in the direction of *z*-axis is equal to zero.

$$\int_{-h/2}^{h/2} E(z)(z-z_0)dz = 0$$

(3)

Nonzero strains components of the observed nanoplate model can be deduced as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x} \end{Bmatrix} - (z-z_0)\begin{Bmatrix} \dfrac{\partial^2 \theta}{\partial x^2} \\ \dfrac{\partial^2 \theta}{\partial y^2} \\ 2\dfrac{\partial^2 \theta}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w}{\partial y} - \dfrac{\partial \theta}{\partial y} \\ \dfrac{\partial w}{\partial x} - \dfrac{\partial \theta}{\partial x} \end{Bmatrix}$$

(4)

For the present buckling case the principle of minimum of potential energy it given as

$$\delta(U+V) = 0$$

(5)

Where the strain energy is defined by U and work of in-plane loads and elastic foundations is defined by V.

The virtual strain energy δU for the new first-order shear deformation theory can be written as

$$\delta U=\int_V(\sigma_{xx}\delta\varepsilon_{xx}++\sigma_{yy}\delta\varepsilon_{yy}+\sigma_{xy}\delta\gamma_{xy}+\sigma_{xz}\delta\gamma_{xz}++\sigma_{yz}\delta\gamma_{yz})dV$$
(6)

Substituting Eqs. (2) and (4) in Eq. (6) yields

$$\delta U=\int_A\left[N_{xx}\frac{\partial\delta u}{\partial x}-M_{xx}\frac{\partial^2\delta\theta}{\partial x^2}+N_{yy}\frac{\partial\delta u}{\partial y}-M_{yy}\frac{\partial^2\delta\theta}{\partial y^2}+N_{xy}\left(\frac{\partial\delta u}{\partial y}+\frac{\partial\delta u}{\partial x}\right)-2M_{xy}\frac{\partial^2\delta\theta}{\partial x\partial y}\right.$$
$$\left.+Q_{xz}\frac{\partial\delta(w-\theta)}{\partial x}+Q_{yz}\frac{\partial\delta(w-\theta)}{\partial y}\right]dA$$
(7)

The nonlocal stress resultants N_i, Q_j and M_i are obtained from the following expression

$$N_i=\int_{-h/2}^{h/2}\sigma_i dz$$
$$Q_j=K_s\int_{-h/2}^{h/2}\sigma_i dz$$
$$M_i=\int_{-h/2}^{h/2}z\sigma_i dz\qquad(i=xx,yy,xy;\ j=xz,yz)$$
(8)

We can define the varation of the work done by applied loads in the integral form

$$\delta V=\int_A\left(-\left(N_{xx}^m+N^T+N^H\right)\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x}--\left(N_{yy}^m+N^T+N^H\right)\frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y}-k_w\delta w+k_G\left(\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x}+\frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y}\right)\right)dA$$

(9)

where, k_w and k_G are Winkler and Pasternak parameter of elastic foundation, N^T and N^H are the external thermal and hygro forced due to the changes of temperature and moisture

$$N^T=\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\alpha(z)\Delta T dz$$
(10)

$$N^H=\int_{-h/2}^{h/2}\frac{E(z)}{1-\nu}\beta(z)\Delta H dz$$
(11)

The governing equations of equilibrium for buckling behaviour are obtained by substituting Eqs. (7) and (9) in Eq. (5) when the coefficients of δw and $\delta\theta$ are equal to zero

$$\delta w:\frac{\partial Q_{xz}}{\partial x}+\frac{\partial Q_{yz}}{\partial y}-k_w w+k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\left(N_{xx}^m+N^T+N^H\right)\frac{\partial^2 w}{\partial x^2}-\left(N_{yy}^m+N^T+N^H\right)\frac{\partial^2 w}{\partial y^2}=0$$
$$\delta\theta:\ \frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}-\frac{\partial Q_{xz}}{\partial x}-\frac{\partial Q_{yz}}{\partial y}=0$$
(12)

2.3 The nonlocal elasticity theory for FG nanoplates

Making certain assumptions presented by Eringen [1,2] we will assume the nonlocal differential constitutive equation as

$$\left[1-(e_0a)^2\nabla^2\right]t_{ij}(x)=C_{ijkl}\varepsilon_{kl}\qquad i,j,k,l=x,y,z$$
(13)

where ∇^2 is the Laplacian operator which is defined by $\nabla^2=(\partial^2/\partial x^2+\partial^2/\partial y^2)$,and e_0a is the nonlocal parameter that takes into account the small scale effects into the differential constitutive equation.

Based on Eqs. (11) the stress-strain constitutive equations of a rectangular FG nanoplates can be written as

$$[1-(e_0\ell)^2\nabla^2]\begin{Bmatrix} t_{xx} \\ t_{yy} \\ t_{xy} \\ t_{xz} \\ t_{yz} \end{Bmatrix} = \frac{E(z)}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & (1-\nu)/2 & 0 & 0 \\ 0 & 0 & 0 & (1-\nu)/2 & 0 \\ 0 & 0 & 0 & 0 & (1-\nu)/2 \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

(14)

where ν denote Poisson's ratios.

2.4 Equations of equilibrium of single-layered FG nanoplates

Equations of equilibrium of nonlocal elasticity theory and new first order shear deformation theory for investigation the buckling behaviour of single-layered FG nanoplates with uneven porosities and hygrothermal in-plane loadings can be written in the terms of displacements as follows

$$K_S F\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 \theta}{\partial x^2}-\frac{\partial^2 \theta}{\partial y^2}\right)+$$
$$(1-(e_0\ell)^2\nabla^2)\left[-k_w w+k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\left(N_{xx}^m+N^T+N^H\right)\frac{\partial^2 w}{\partial x^2}-\left(N_{yy}^m+N^T+N^H\right)\frac{\partial^2 w}{\partial y^2}\right]=0$$

(15)

$$D\left(\frac{\partial^4 \theta}{\partial x^4}+\frac{\partial^4 \theta}{\partial x^2 \partial y^2}+\frac{\partial^4 \theta}{\partial y^4}\right)+K_s F\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 \theta}{\partial x^2}-\frac{\partial^2 \theta}{\partial y^2}\right)=0$$

(16)

where

$$D=\int_{-h/2}^{h/2}\frac{E(z-z_0)^2}{1-\nu^2}dz,\quad F=\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}dz$$

III. Solution by Galerkin's method

In this section, by using Galerkin's method, the governing equations of equilibrium for buckling study are solved for simply supported (S) and clamped (C) boundary conditions.

The analytical solution for displacement field of Eqs. (15) and (16) can be introduced as:

$$w(x,y)=W_{mn}X(x)Y(y)$$
$$\theta(x,y)=\theta_{mn}X(x)Y(y)$$

(17)

In this study the single-layered FG nanoplate is assumed to have simply supported (S) and clamped (C) boundary condition or have combinations of them, and the functions $X(x)$ and $Y(y)$ that satisfy above boundary conditions can be written as:

SSSS:

$$X(x)=\sin(\alpha x),\quad Y(y)=\sin(\beta y)$$

(18)

CCCC:

$$X(x)=\sin^2(\alpha x),\ Y(y)=\sin^2(\beta y)$$

(19)

SCSC:

$$X(x)=\sin^2(\alpha x),\ Y(y)=\sin(\beta y)$$

where $\alpha=m\pi/L_x$, $\beta=n\pi/L_y$

Substituting Eq. (17) into Eqs. (15) and (16) and implementing the Galerkin's method the equations of equilibrium in terms of parameters W_{mn}, θ_{mn}, can be obtained from

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ \theta_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$
(20)

IV. Numerical results and discussions

In this research the material properties of single-layered FG nanoplate (Al and Al_2O_3) are given as:

$E_c = 380 GPa,\ \nu_c = 0.3,\ E_m = 70 GPa,\ \nu_m = 0.3$

In the present investigation the following dimensionless parameters are used:

$$\hat{N} = \frac{NL_x^{\ 2}}{E_m h^3},\ k_{wN} = \frac{k_w L_x^4}{D},\ k_{GN} = \frac{k_G L_x^2}{D}$$

where

$N_{xx}^m = N,\ N_{yy}^m = kN$

In Fig. 2 the nondimensional critical buckling load is plotted as a function of the uneven porosity volume fraction for SSSS boundary conditions. From Fig. 2 it can be seen that the value increase of the uneven volume porosity fraction has a linear decreasing effect on the nondimensional critical buckling load.

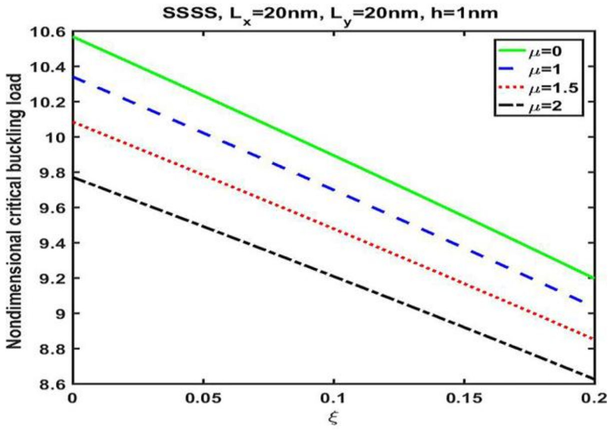

Figure 2: Variation of nondimensional critical buckling load versus uneven porosities for SSSS boundary conditions

From Fig. 3 it can be noticed that in the case of SCSCS boundary conditions the behaviour of a FG single-layered nanoplate with the change of value of uneven porosity volume fraction and nonolocal parameter is the same as in the case of SSSS boundary conditions. It can easily be seen that in the case of SCSC boundary conditions buckling, the value of nondimensional critical buckling load is higher than the case of SSSS boundary conditions.

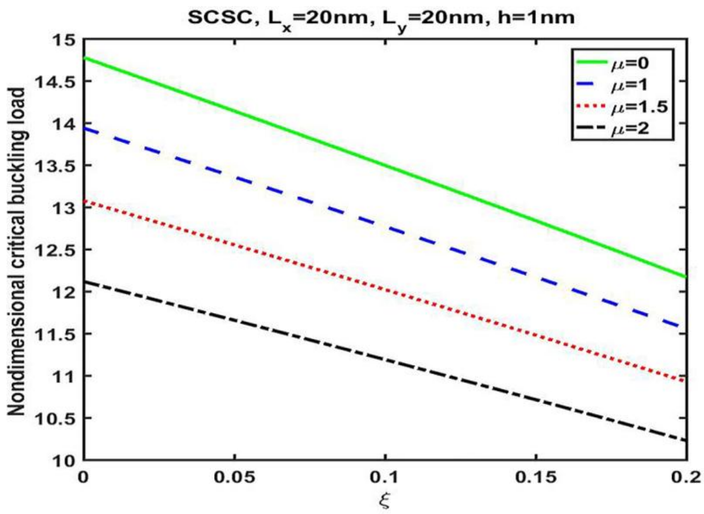

Figure 3: Variation of nondimensional critical buckling load versus uneven porosities for SCSC boundary conditions

Fig. 4 demonstrates the effects of the uneven porosities on value of nondimensional critical buckling load for CCCC boundary conditions. It can be concluded that the value of the nondimensional critical buckling load is reduced when the value of the uneven porosities volume fraction rises.

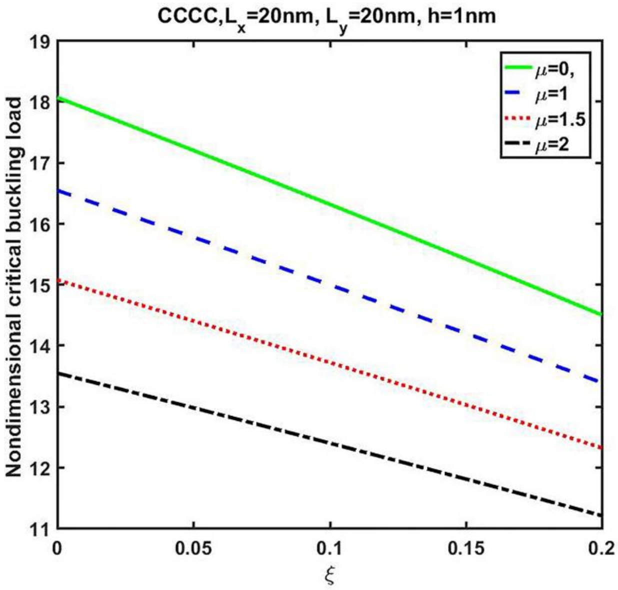

Figure 4: Variation of nondimensional critical buckling load versus uneven porosities for CCCC boundary conditions

V. Conclusion

We present in this paper the buckling behaviour of the FG uneven porous single-layered nanoplates subjected to in- plane mechanical and hygrothermal and various boundary conditions. The Galerkin's method has been used to solve the equations of equilibrium for SSSS, SCSC and CCCC boundary conditions. Numerical results are presented to investigate the effects of uneven porosity volume fraction on nondimensional critical buckling load for three observed boundary conditions. It is noticed that increasing the value of uneven porosity volume fraction will decrease the value of nondimensional critical buckling load for the case of SSSS, SCSC and CCCC boundary conditions.

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