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Estimating Axial Forces in Columns using Tributary Area Method and Finite Element Method; a Comparison

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Abstract : *Axial forces in columns caused by gravity loadings are usually estimated for initial design using tributary area method. However, this method does not consider the axial deformations in columns nor the flexural rigidities of floors. Columns subjected to high axial loads are expected to deform axially which certainly would affect the distribution of forces to columns. Moreover, floors subjected to vertical loads are expected to bend or to change in curvature. While determining axial forces in columns using tributary area method are only governed by location of columns, actual distribution is governed by axial stiffnesses of columns and flexural rigidities (or slab thickness). This study aims at giving more insights on who forces are being transmitted to columns from slabs. Analytical solution for axial forces in columns is derived for the case of rigid slab. Afterwards, Finite element method is employed to perform parametric study to investigate the effect of column stiffness (i.e. area, length and material stiffness) on the load transmission. Also, rigid, semi-rigid and flexible floors are considered to determine the effect of flexural rigidity of floors on forces distribution. It was found that the use of tributary area method may lead to false results.*

Keywords – *Axial Forces, Tributary Area Method, Finite Element Method, SAP2000*

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I. Introduction

In usual design, axial forces in columns are determined using two methods. In the first one, designers estimate loads transmitted to beams from slabs and then calculate the reactions of beams that are directly framing to columns. Summing all the reactions transmitted to a certain column would result in the total force acting on that column. Alternatively, forces in columns can be estimated using tributary area method (TAM). This method is presented in many of classical structural analysis books [1], [2], [3]. Tributary area of a column is defined as the loaded area surrounding a column and that is directly contributes to the applied loads on that column [3]. It is usually considered as area bounded by a panel's centerlines [1]. The accuracy of TAM to determine the axial forces in columns under high loads is in question. TAM does not take into consideration axial rigidity of columns i.e. the area of each column, the material of which each column is made of, nor the height of each column. So, the aim of the current study is to develop more comprehensive insights into the distribution of forces from slabs to columns and to assess the accuracy of the commonly used TAM in the analysis of buildings.

II. Methodology of work

A closed form solution of axial forces in elastic columns supporting a simple rigid flat plate regular building is presented for comparison and validation of finite element results. Moreover, this solution was compared to results obtained using tributary area method and with finite element method (FEM) results. Afterwards, multi-bay building was considered to have more insights into the effect of changing column axial rigidity on distribution of forces from slab to columns.

III. Analysis and results

The analysis starts by considering a simple regular building which consists of rigid square flat plate supported by four elastic columns. This problem is presented in [4]. The aim here is to determine internal axial force in each column using classical mechanics. Each column is assumed to has its own length (L*i*) and is made of homogenous material with modulus of elasticity (E*i*) and has cross sectional area (A*i*). A distributed load of q (kN/m^2) intensity is assumed to act uniformly on the plate. The plate dimensions with the other properties are presented in Fig. 1. Four reactive forces (S_1, S_2, S_3 and S_4) resulting from the four columns act on the rigid plate and hence the problem is statically indeterminate to the first degree (see Fig. 2). Applying equilibrium equations and considering compatibility of deformations would yield equations necessary to determine the four reactive forces. The analysis is presented below:

Equilibrium equations:

$$\sum F_y = 0, \quad S_1 + S_2 + S_3 + S_4 = W \qquad (1)$$

$$\sum M_{line\,1} = 0, \quad a\,S_1 = aS_4 \qquad (2)$$

$$\sum M_{line\,2} = 0, \quad a\,S_2 = aS_3 \qquad (3)$$

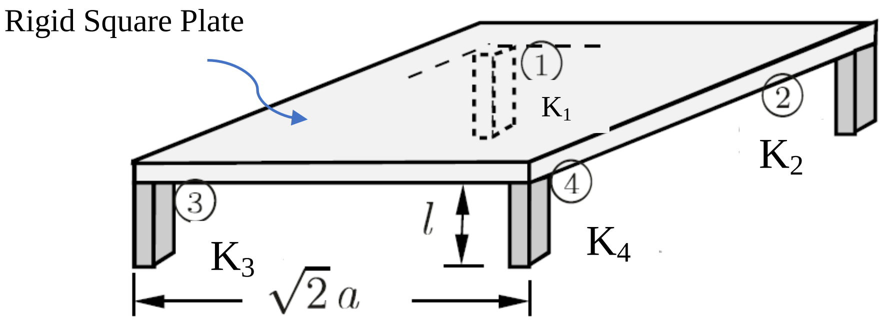

Figure 1: Rigid square plate supported by four elastic columns [4].

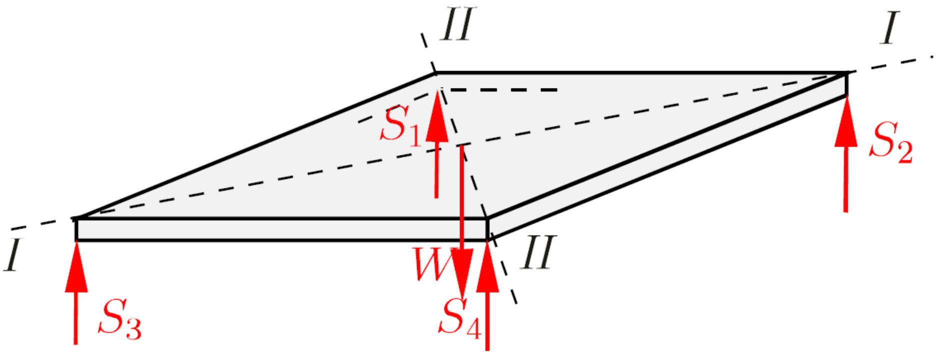

Figure 2: Free body diagram of the rigid plate [4].

Compatibility equation:

Since the plate is rigid, then the displacement (*f*) at the center of the plate (refer to Fig. 3) can be expressed in terms of deformations at edges as follows:

$$f = \frac{1}{2}(u_1 + u_4) = \frac{1}{2}(u_2 + u_3) \qquad (4)$$

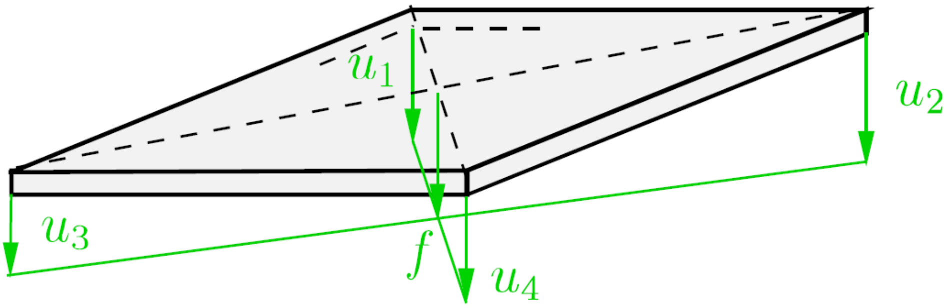

Figure 3: Corner displacements of the rigid plate [4].

Assuming linear elastic behavior of the material, then the axial deformation (u_i) in an axially loaded column is:

$$u_i = \frac{S_i\, L_i}{A_i\, E_i} \qquad (5)$$

This can be written in the following form:

$$u_i = \frac{S_i}{k_i} \qquad (6)$$

Where ki is the stiffness of elastic column i and equals to

$$k_i = \frac{A_i\, E_i}{L_i} \qquad (7)$$

Substituting in eq. (4), yields:

$$\frac{1}{2}\left(\frac{S_1}{k_1} + \frac{S_4}{k_4}\right) = \frac{1}{2}\left(\frac{S_2}{k_2} + \frac{S_3}{k_3}\right) \qquad (8)$$

Solving all the above equations gives:

$$S_4 = S_1 = \frac{W}{2 + 2\left(\frac{k_1 k_4}{k_2 k_3}\right)\left(\frac{k_4 + k_1}{k_2 + k_3}\right)} \qquad (9)$$

Where *W* is resultant of the distributed load *q* acting on the area A_p of the rigid plate.

As a special case, consider columns; one, two, three and four to have k, 2k, 3k and 4k stiffnesses, respectively. Substituting in equation (9) would give values of all reactions (or internal forces in columns) as follows:

$$S_1 = S_4 = \frac{3}{10} W \text{ and } S_2 = S_3 = \frac{1}{5} W$$

Using tributary area method, the forces in each column would be P/4. As such, the results confirm that the use of tributary area in such case to determine axial forces in columns is illusive.

The same regular building is modeled in SAP2000 [5] to determine axial forces in the four elastic columns as shown in Fig. 4. It is worth mentioning that the slab bending modifiers are set to be 1000 so that the plate becomes rigid. Parametric study has been done to determine the effect of axial load, rigidity of the slab and rigidity of column on internal axial forces in columns. Table 1 presents values of forces for the case of equal rigid of column and different rigidities.

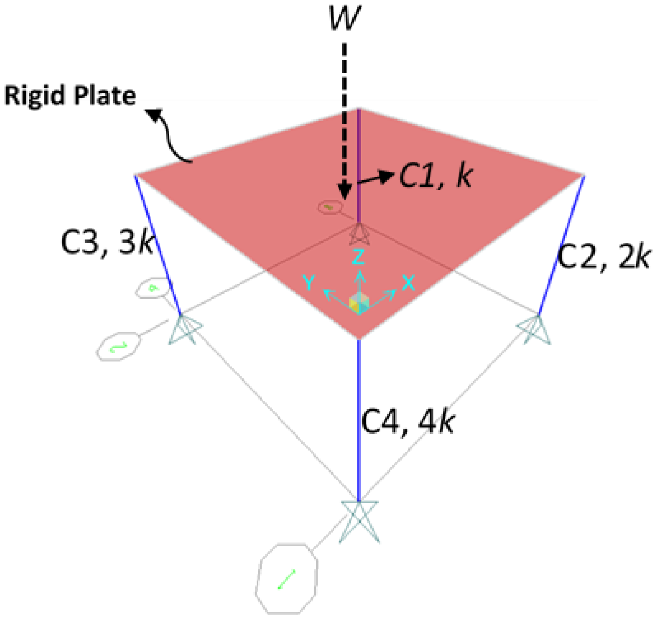

Figure 4: Regular building modelled in SAP2000.

At first, the axial forces predicted using FEM, for the case of rigid slab, were all verified by the analytical formula derived before. There is a perfect match between SAP2000 results and those predicted by the formula derived for the case of rigid slab. As it can be seen from the table, that the use of TAM in case of rigid slab would result in misleading values for axial forces in columns where maximum percent of error equals 25. On the other hand, the forces predicted using TAM goes in line with those predicted by FEM method for the case of flexible slab. In facts, axial forces predicted in all columns are always equal in the case of flexible slab irrespective of column's rigidity. It is also well shown that increasing the applied load by increasing the number of storeys does not affect the distribution of forces to columns for the case of rigid slab, semi-rigid slab or flexible slab. For the case of semi-rigid slab, axial forces in columns predicted by TAM are close enough to those predicted using SAP2000. It should be noted that TAM results and FEM results are not compared with analytical solution for the case of flexible and semi-rigid slab as the analytical formula is only valid for the case of rigid slab.

The effect of changing slab rigidity on the distribution of forces has been considered carefully in this study. Figures 5 and 6 present the change in axial force in columns C1 and C2 with the change in slab thickness. Increasing slab thickness would certainly increase its flexural rigidity. As shown in the two figures 5 and 6, forces in columns using TAM are close to the ones obtained by FEM for small thicknesses (flexible slab). However, as slab rigidity increases, the difference between TAM and FEM method results increases.

Table 1 Parametric Study on the effect of slab rigidity, storey number and column stiffness on distribution of forces from slab to columns

flexural rigidity of Slab	***effect of load (no. of storeys)***	***Effect of stiffness***	***C1***		***C2***		***C3***		***C4***	
			TAM	***SAP2000***	***TAM***	***SAP2000***	***TAM***	***SAP2000***	***TAM***	***SAP2000***
Rigid Slab	One-Storey building	same stiffness	125	125	125	125	125	125	125	125
		different stiffness	125	100	125	150	125	150	125	100
	Ten- Storey building	same stiffness	1250	1250	1250	1250	1250	1250	1250	1250
		different stiffness	1250	1000	1250	1500	1250	1500	1250	1000
	Twenty-Storey Building	same stiffness	2500	2500	2500	2500	2500	2500	2500	2500
		different stiffness	2500	2000	2500	3000	2500	2999.5	2500	2001
Semi-rigid Slab	One-Storey building	same stiffness	125	125	125	125	125	125	125	125
		different stiffness	125	122.85	125	127.15	125	127.15	125	122.85
	Ten- Storey building	same stiffness	125	125	125	125	125	125	125	125
		different stiffness	1250	1092.5	1250	1407.42	1250	1407.42	1250	1092.5
	Twenty-Storey Building	same stiffness	2500	2500	2500	2500	2500	2500	2500	2500
		different stiffness	2500	2095	2500	2906	2500	2906	2500	2095
Flexible Slab	One-Storey building	same stiffness	125	125	125	125	125	125	125	125
		different stiffness	125	125	125	125	125	125	125	125
	Ten- Storey building	same stiffness	1250	1250	1250	1250	1250	1250	1250	1250
		different stiffness	1250	1250	1250	1250	1250	1250	1250	1250
	Twenty-Storey Building	same stiffness	2500	2500	2500	2500	2500	2500	2500	2500
		different stiffness	2500	2500	2500	2500	2500	2500	2500	2500

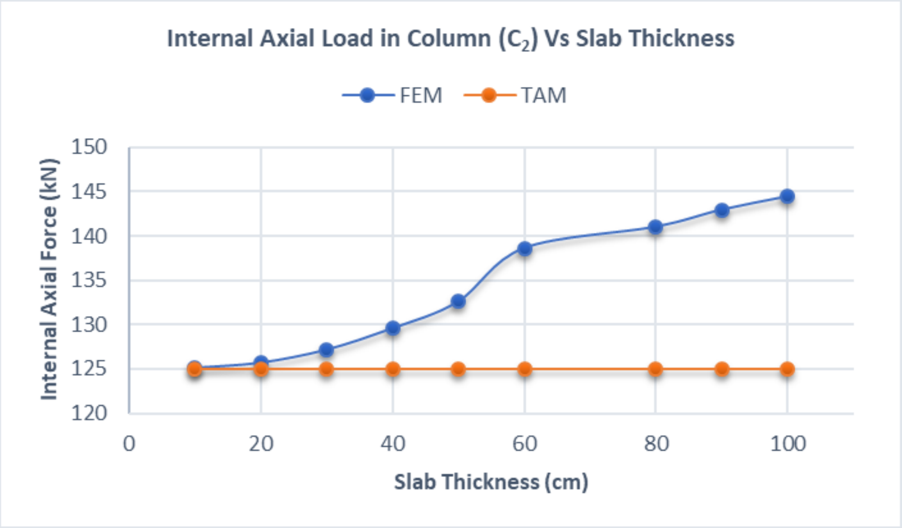

Figure 5: Effect of changing slab thickness on Internal axial forces in column C2

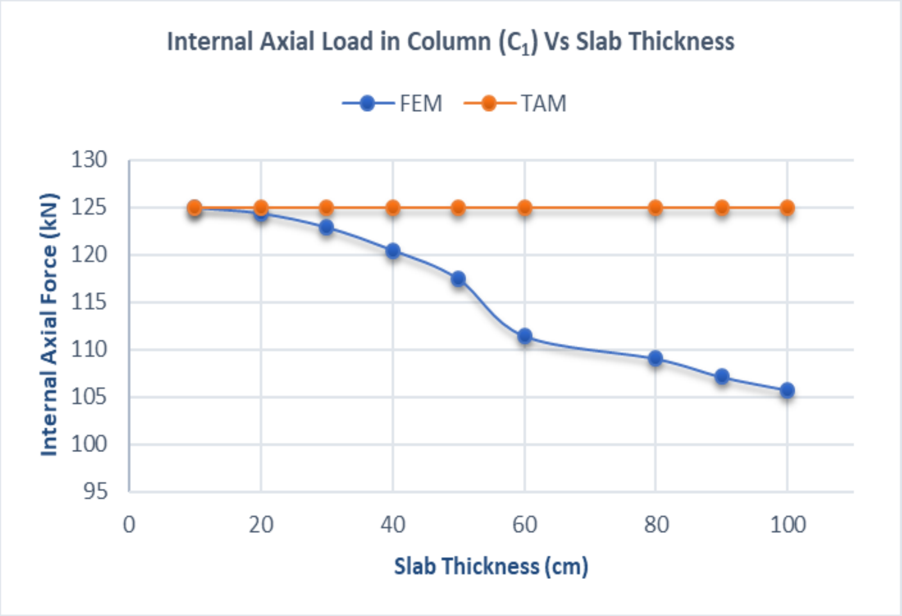

Figure 6: Effect of changing slab thickness on Internal axial forces in column C1.

As a further step, multi-bay building (see Fig. 7) is considered for further analysis. However, because of the complexity of this building, no analytical solution was derived. Instead, only a comparison between tributary area and SAP2000 results were made. The building is three equal bays of 15 m length in each direction with 3 m storey height. Table 2 shows the values forces in three selected columns using FEM and TAM. The effect of flexural rigidity (by changing slab thickness) on the forces in column is presented in Fig. 8. Forces estimated using TAM are far away from those predicted using FEM with large percent of errors up to 55%. Generally, as the slab thickness increases results estimated using the two methods are getting closer to each other for a certain column.

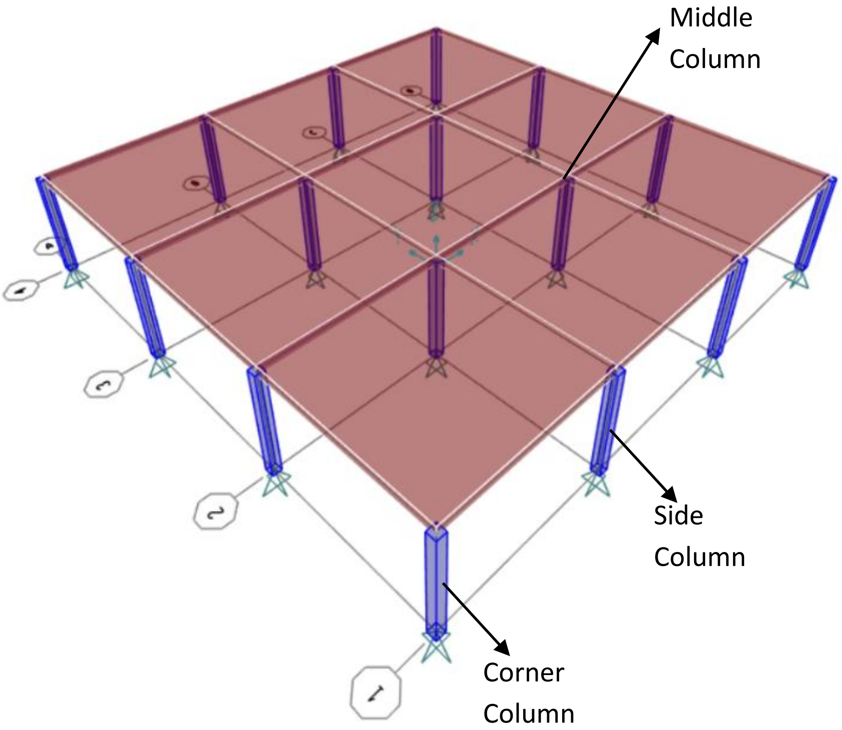

Figure 7: Multi-bay building considered for further investigation

Table 2 Results on parametric study on multi-bay building to investigate forces distribution from slabs to columns

Slab Thickness (cm)	FEM Axial Forces (kN)			TAM Axial Forces (kN)			Corner Column Error %	Middle Column Error %	Side Column Error %
	Corner Column	Middle Column	Side Column	Corner Column	Middle Column	Side Column			
10	98	570	228	125	500	250	27.5	12.3	9.5
15	96	580	223	125	500	250	29.1	13.9	11.7
25	92	591	220	125	500	250	35.3	15.5	13.4
35	89	590	222	125	500	250	40.3	15.3	12.2
45	86	581	228	125	500	250	43.8	14.0	9.6
60	85	559	239	125	500	250	45.9	10.7	4.3
70	86	541	248	125	500	250	43.9	7.7	0.8
rigid	281	281	281	125	500	250	55.6	77.8	11.1

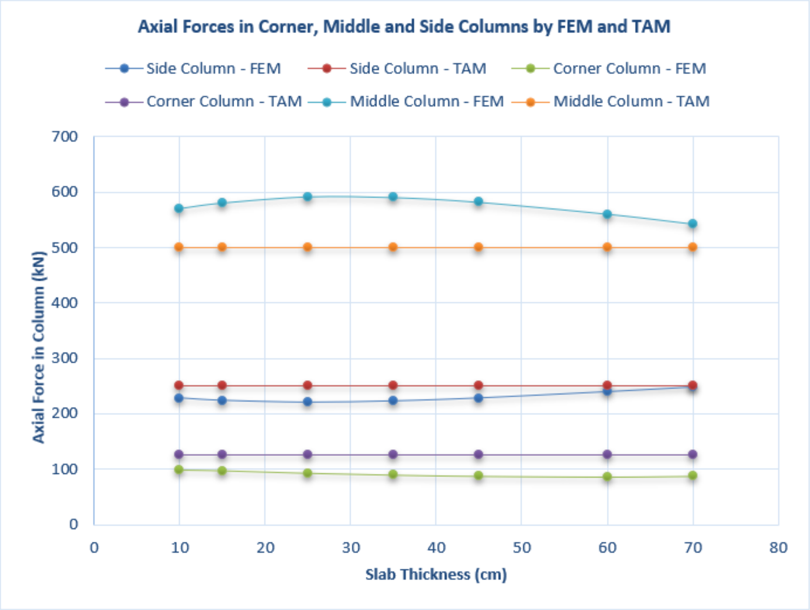

Figure 8: Comparison between axial forces resulted from FEM and TAM.

IV. Conclusion

The distribution of forces from slabs to columns has be thoroughly investigated using analytical, finite element and tributary area methods. The aim was to give more insights on the suitability and hence the accuracy of the commonly used tributary area method to evaluated axial forces transmitted from slabs to columns. Based on the extensive analysis presented above, the following are the main observations to consider:

- If all columns are having the same rigidity, then the use of TAM gives reliable and accurate results.
- For the case of rigid slab supported by elastic column, the average percent of errors is about 20% and use of TAM for predication of axial forces in columns may give false and illusive results. So, its use for such case is not recommended.
- For the case of rigid slab, increasing number of floors or alternately changing the applied loads on slabs has no effects on changing the distribution of forces from slabs to columns.
- For the case of flexible slabs with small applied loads, forces tend to be distributed according to TAM regardless of the magnitude of the applied loads.
- For the case of semi-rigid slabs, forces could be predicted using TAM with acceptable accuracy. However, as applied loads increases, the accuracy of this method decreases. So, the use of TAM for building of seven floors or more is not recommended.
- FEM method is more reliable and accurate than TAM to predict axial forces in columns and should be used to evaluate axial forces in columns.
- Authors of structural analysis books introducing TAM as a method to find axial forces in columns should include the limitations of this method.

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