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Consistent Mass Formulation in Dynamic Analysis of Structural System

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Abstract: *The paper deals with frequency analysis of irregular framed structures. The analysis used finite element method cast in matrix formulation. In frequency analysis of framed structure with relative rigid floor system, the mass of structure is lumped at each floor. In the dynamic analysis proposed herein, the analysis is carried out by adopting consistent mass formulation, i.e., the inertial forces are applied material point wise. Using finite element formulation tends to increase structural degrees of freedom.To reduce structural degrees of freedom, static condensation and multi-point constraint algorithms are used. The natural frequency resulted out of proposed analysis is then compared to that obtained by assuming rigid floor. The difference between due two results differ significantly, especially for irregular type of structure.*

Keywords: *consistent mass formulation, dynamic analysis, finite element method, multi point constraint, natural frequency, static condensation.*

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I. Introduction

In the application of finite element method being a numerical analysis, the analysis is carried out using discrete model as a representation of real structrure. To preserve the accuracy of the solution, finer element meshing may be used. However, the use of finer elements tends to increase the number of degrees of freedom, and hence increase the numerical work involved. To decrease the number of degrees of freedom, and hence the number of dynamic modes, several assumptions are taken; for example, the floors are assumed to be rigid compared to columns that the nodal rotations at column and beam connection may be neglected, and the sidesways of floors are the remaining degrees of freedom. The mass of the structure is lumped in horizontal displacements of floors. In the proposed paper, consistent mass distribution is assumed and all degrees of freedom are assumed to be active. Then, some of the degrees of freedom are statically condensed by applying static condensation or multi-point constraint method. Therefore, assumptions that the floors are rigid and the mass is lumped at directions of horizontal displacements are not used. The results out of proposed method differ significantly from that given by conventional method. The proposed method especially suitable to be applied to irregular structural systems.

II. Dynamic Analysis of Regular and Irregular Structural System

Regular structural system is referred to the structure in which columns and beams are placed in regular fashion such as depicted in Fig. 1. The columns and beams are arranged in such a way that the strong displacements are horizontal sways of the floors as drawn in Fig. 1.

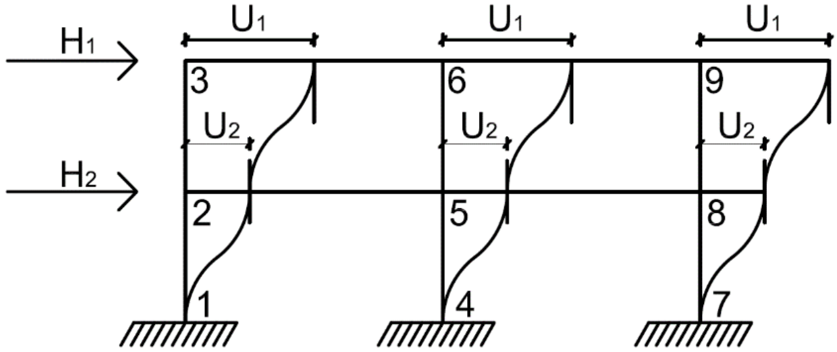

Figure 1: Regular Frame, Floors Are Rigid

By assuming that the floors are rigid relatively to the columns, the floors behave like rigid floors that undergo horizontal displacements and no end rotations. For plane frame, there are 6 nodes with 6 x 3 = 18 active degrees of freedom at nodes 2, 3, 5, 6, 8 and 9. Assuming that the floors are rigid, nodal rotations and axial deformations are ignored, ending with 2 degrees of freedom; i.e., horizontal sways at storeys. This simplified model is usually adopted in regular structural frame system. If the the structure is regular, the assumptions mentioned above would give fairly accurate results. But this is not the case in irregular structural system. In this kind of irregular structural system, assumptions considered in regular structural system no longer provide good results.

Now consider a system in Fig. 2 which actuallyis similar to the structure in Fig. 1 except that the mid column 45 is removed. With respect to lateral load, in addition to horizontal sways and nodal rotations, vertical sway occurs at node 5 as shown in Fig. 2.

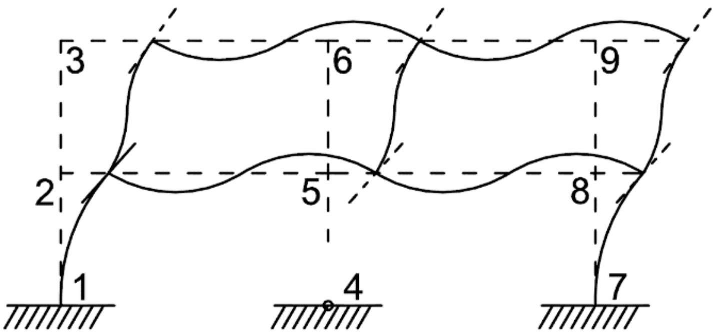

Figure 2: Irregular Frame

III Dynamic Analysis of Structural System

In this chapter, both regular and irregular structural plane frames are considered. The regular structure is depicted in previous Fig. 1, while irregular structure is shown in Fig. 2.

3.1. Analysis of Regular Structural System

Regular plane frame degrees of freedom is shown in Fig. 3. By assuming that floors are relatively rigid compared to adjoining columns, only horizontal sways of floors are considered as degrees of freedom, i.e., U_1 and U_2. Equilibrium of shear forces in directions of the two degrees of freedom leads to the following simultaneous equation

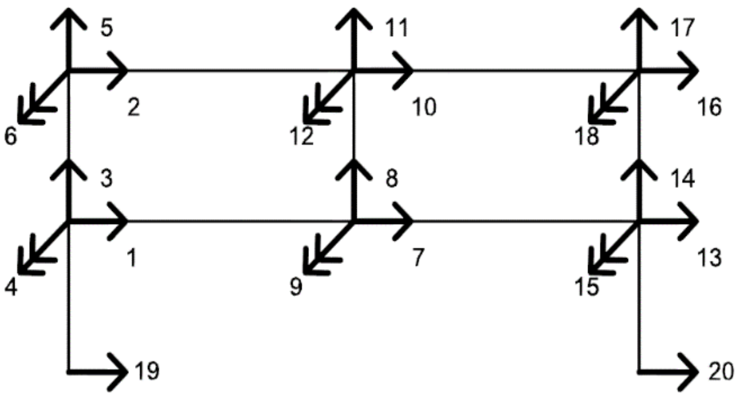

Figure 3: Degrees of Freedom, Regular Frame

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} \frac{72EI}{L^3} & -\frac{36EI}{L^3} \\ -\frac{36EI}{L^3} & \frac{36EI}{L^3} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = -\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}\ddot{U}_t \qquad (1)$$

in which { U_1, U_2 } is vector of horizontal displacement of storeys 1 and 2, { M_1, M_2 } mass vector, { $\ddot{U}_1, \ddot{U}_2$ } acceleration vector, $\{\ddot{U}_t\}$ ground acceleration, EI flexural rigidity of column and L height of floor.

For irregular frame shown in Fig. 2, the removal of column 45 but applying rigid floor assumptions leads to

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} \frac{60EI}{L^3} & -\frac{36EI}{L^3} \\ -\frac{36EI}{L^3} & \frac{36EI}{L^3} \end{bmatrix}\begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = -\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}\ddot{U}_t \tag{2}$$

3.2. Analysis of Irregular Structural System

In Section 3.1, the irregular structural system in Fig. 2 is obtained by deleting column 45 but upon which assumptions of rigid floor is still applied. In this section, irregular system is analyzed by applying consistent mass concept. All degrees of freedom are assumed to be active and included in equilibrium consideration. Therefore, there are 20 degrees of freedom as shown in Fig. 3. Degrees of freedom U_1 and U_2 are treated as master degrees of freedom, U_3 till U_{18} are condensed slave degrees of freedom, whereas U_{19} and U_{20} are restrained degrees of freedom. The displacement vector $\{U\}$ is decomposed to master displacement vector $\{U_m\}$, condensed displacement vector $\{U_s\}$, and restrained displacement $\{U_r\}$. Sub vectors $\{U_m\}$ and $\{U_s\}$ form free displacement vector $\{U_f\}$.

The following dicussions deal with finite element formulation, static condensation algorithm and multi-point constraint scheme.

3.2.1 Consistent Mass Finite Element Formulation

The following is the finite element formulation of consistent mass scheme. The displacement of element in local coordinates reads

$$\begin{Bmatrix} u(x) \\ w(x) \end{Bmatrix} = \begin{bmatrix} N_1(x) & 0 & 0 & N_4(x) & 0 & 0 \\ 0 & N_2(x) & N_3(x) & 0 & N_5(x) & N_6(x) \end{bmatrix}\{\ddot{u}_e\} \tag{3}$$

in which [4]

$$\begin{aligned} N_1(x) &= 1 - x/L \\ N_2(x) &= 1 - 3(x/L)^2 + 2(x/L)^3 \\ N_3(x) &= L[(x/L) - 2(x/L)^2 + (x/L)^3] \\ N_4(x) &= x/L \\ N_5(x) &= 3(x/L)^2 - 2(x/L)^3 \\ N_6(x) &= L[-2(x/L)^2 + (x/L)^3] \end{aligned} \tag{4}$$

External work done by inertial force in direction of $\{u, w\}$ becomes

$$\delta W = \int \{\delta u\}^T [N]\, m dx + \iint \{\delta u\}^T [N]\, m da \tag{5}$$

The use of interpolating function in (12) in (13) provides

$$[m_e] = \int_0^l \begin{bmatrix} N_1N_1 & 0 & 0 & N_1N_4 & 0 & 0 \\ 0 & N_2N_2 & N_2N_3 & 0 & N_2N_5 & N_2N_6 \\ 0 & N_3N_2 & N_3N_3 & 0 & N_3N_5 & N_3N_6 \\ N_4N_1 & 0 & 0 & N_4N_4 & 0 & 0 \\ 0 & N_5N_2 & N_5N_3 & 0 & N_5N_5 & N_5N_6 \\ 0 & N_6N_2 & N_6N_3 & 0 & N_6N_5 & N_6N_6 \end{bmatrix}\{\ddot{u}_e\} mAdx \tag{6}$$

which results in consistent element mass matrix

$$\frac{\lfloor m_e \rfloor}{mA} = \begin{bmatrix} L/3 & 0 & 0 & L/6 & 0 & 0 \\ 0 & 13L/35 & 11L^2/210 & 0 & 9L/70 & 11L^2/210 \\ 0 & 11L^2/210 & L^3/105 & 0 & 13L^2/420 & L^3/140 \\ m/6 & 0 & 0 & L/3 & 0 & 0 \\ 0 & 9L/70 & 3L^2/420 & 0 & 13L/35 & 11L^2/210 \\ 0 & 11L^2/210 & L^3/140 & 0 & 11L^2/210 & L^3/105 \end{bmatrix} \tag{7}$$

in which m is the mass of element per unit volume, A element cross sectionand L the length of element. It seen that the mass consistent matrix is simetric and may be may be transformed from local to global coordinates,

$$\{m_e\} = [R_e]\{M_e\} \tag{8}$$

and assembled the mass matrix onto global mass matrix

$$[M] = \sum_{i=1}^{n} [T_i]^T [R_i]^T [m_i][R_i][T_i] \tag{9}$$

3.2.2 Static Condensation

The equilibrium equation is decomposed in the form

$$\begin{bmatrix} [K_{mm}] & [K_{ms}] & [K_{mr}] \\ [K_{sm}] & [K_{ss}] & [K_{sr}] \\ [K_{rm}] & [K_{rs}] & [K_{rr}] \end{bmatrix} \begin{Bmatrix} \{U_m\} \\ \{U_s\} \\ \{U_r\} \end{Bmatrix} = \begin{Bmatrix} \{P_m\} \\ \{P_s\} \\ \{P_r\} \end{Bmatrix} \tag{10}$$

or

$$\begin{bmatrix} [K_{ff}] & [K_{fr}] \\ [K_{rf}] & [K_{rr}] \end{bmatrix} \begin{Bmatrix} \{U_f\} \\ \{U_r\} \end{Bmatrix} = \begin{Bmatrix} \{P_f\} \\ \{P_r\} \end{Bmatrix} \tag{11}$$

First, to find structural force vector due to earthquake excitation, the equation is further arranged as follows. Due to the fact that acceleration is second derivative of displacement with respect to time, then ground acceleration also obeys kinematically admissible criterion of displacement field, then can be written that.

$$\begin{bmatrix} [K_{ff}] & [K_{fr}] \\ [K_{rf}] & [K_{rr}] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}_f\} \\ \{\ddot{U}_r\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{12}$$

The ground acceleration creates foundation acceleration

$$\{\ddot{U}_r\} = \begin{Bmatrix} P_{19} \\ P_{20} \end{Bmatrix} \ddot{U}_t = \{P_r\}\ddot{U}_t \tag{13}$$

that together with Eqn. 5 give

$$\{\ddot{U}_f\} = -\{[K_{ff}]^{-1}[K_{fr}]\{P_r\}\}\ddot{U}_t = \{P_f\}\ddot{U}_t \tag{14}$$

and the structural acceleration becomes

$$\{\ddot{U}\} = \begin{Bmatrix} \{P_f\} \\ \{P_r\} \end{Bmatrix} \ddot{U}_t = \begin{Bmatrix} [K_{ff}]^{-1}[K_{fs}] \\ [I] \end{Bmatrix} \{\ddot{U}_r\} \tag{15}$$

This acceleration is then utilized to form intertia force in elements as follows. First, acceleration on element ends are computed by

$$\{\ddot{U}_e\} = [T_e]\{\ddot{U}\} \tag{16}$$

in global system coordinates, and

$$\{\ddot{u}_e\} = [R_e]\{\ddot{U}_e\} \tag{17}$$

The global dynamic equilibrium of the system then

$$\begin{bmatrix} [K_{mm}] & [K_{ms}] & [K_{mr}] \\ [K_{sm}] & [K_{ss}] & [K_{sr}] \\ [K_{rm}] & [K_{rs}] & [K_{rr}] \end{bmatrix} \begin{Bmatrix} \{U_m\} \\ \{U_s\} \\ \{U_r\} \end{Bmatrix} + \begin{bmatrix} [M_{mm}] & [M_{ms}] & [M_{mr}] \\ [M_{sm}] & [M_{ss}] & [M_{sr}] \\ [M_{rm}] & [M_{rs}] & [M_{rr}] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}_m\} \\ \{\ddot{U}_s\} \\ \{\ddot{U}_r\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \end{Bmatrix} \tag{18}$$

in which sub-matrices related to mass matrices in (18) are formed based on computation of equivalent intertial force in (8). The equilibrium equation may also be partitioned in the form

$$\begin{bmatrix} [K_{mm}] & [K_{ms}] \\ [K_{sm}] & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \{U_m\} \\ \{U_s\} \end{Bmatrix} + \begin{bmatrix} [M_{mm}] & [M_{ms}] \\ [M_{sm}] & [M_{ss}] \end{bmatrix} \begin{Bmatrix} \{\ddot{U}_m\} \\ \{\ddot{U}_s\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{19}$$

The solution to (18) is obtained by first performing static condentation process, i.e., the sub-solution

$$\{U_s\} = -[K_{ss}]^{-1}\{[K_{sm}]\{U_m\} + [M_{sm}]\{\ddot{U}_m\} + [M_{ss}]\{\ddot{U}_s\}\} \tag{20}$$

which in turn is used to find

$$[K'_{mm}]\{U_m\} + [M'_{mm}]\{\ddot{U}_m\} = \{0\} \tag{21}$$

in which

$$\begin{aligned}[K'_{mm}] &= [K_{mm}] - [K_{ms}][K_{ss}]^{-1}[K_{sm}] \\ [M'_{mm}] &= [M_{mm}] - [M_{ms}][K_{ss}]^{-1}[M_{sm}]\end{aligned} \tag{22}$$

Solution $\{U_m\}$ for (21) then is inserted to (20) to computed for $\{U_s\}$ to complete the solution. Observe that the above scheme deals with lesser number of final degrees of freedom.

One may raise a question as how to select condensed degrees of freedom $\{U_s\}$ and retained degrees of freedom $\{U_m\}$. Generally, the degrees of freedom that define the shape of the structure may be chosen as retained degrees of freedom. Other retained degrees of freedom may also be chosen by performing sensitivity analysis.

3.2.3. Multi-point Constraint Algorithm

To begin, consider a special relationship among master degrees of freedom vector $\{U_m\}_{(q-p)x1}$ and slave degrees of freedom $\{U_s\}_{(px1)}$ written in a matrix form

$$\begin{bmatrix}[A_m] & \vdots & [A_s]\end{bmatrix}\begin{Bmatrix}\{U_m\} \\ \cdots \\ \{U_s\}\end{Bmatrix} = \{B\} \tag{23}$$

The size of coefficient matirx $[A]$ in (23) is $(p \times q)$, then the constrained displacement vector $\{U_C\}$ is partitioned in 3 sub-vectors; i.e., master displacement vector $\{U_m\}$ with size $(q-p)\times 1$, constrained displacement vector $\{U_s\}$ with size $(p \times 1)$. The third sub-vector is $\{U_f\}$ containing free displacement components. The partition of displacement vector is

$$\{U\} = \begin{Bmatrix}\{U_f\} \\ \cdots \\ \{U_m\} \\ \cdots \\ \{U_s\}\end{Bmatrix} \tag{24}$$

which consistently partition stiffness matrix in the form

$$\begin{bmatrix} [K_{ff}] & \vdots & [K_{fm}] & \vdots & [K_{fs}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{mf}] & \vdots & [K_{mm}] & \vdots & [K_{ms}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{sf}] & \vdots & [K_{sm}] & \vdots & [K_{ss}] \end{bmatrix}\begin{Bmatrix}\{U_f\} \\ \cdots \\ \{U_m\} \\ \cdots \\ \{U_s\}\end{Bmatrix} = \begin{Bmatrix}\{P_f\} \\ \cdots \\ \{P_m\} \\ \cdots \\ \{P_s\}\end{Bmatrix} \tag{25}$$

Prior to the substitution of (23) into (25), we observe that constraint in (23) is identically with the application of constraint vector on upper portion of lower partition (25) with the application of reaction force vector $\{R_s\}$ such that

$$\lfloor K_{sf} \rfloor \{U_f\} + [K_{sm}]\{U_m\} + [K_{ss}]\{U_s\} = \{P_s\} + \{R_s\} \qquad (26)$$

Displacement vector in (25) is denoted with bar on top to state that the vector is not yet modified by constraint,

$$\begin{bmatrix} [K_{ff}] & \vdots & [K_{fm}] & \vdots & [K_{fs}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{mf}] & \vdots & [K_{mm}] & \vdots & [K_{ms}] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{sf}] & \vdots & [K_{sm}] & \vdots & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \{\overline{U}_f\} \\ \cdots \\ \{\overline{U}_m\} \\ \cdots \\ \{\overline{U}_s\} \end{Bmatrix} = \begin{Bmatrix} \{P_f\} \\ \cdots \\ \{P_m\} \\ \cdots \\ \{P_s\} \end{Bmatrix} \qquad (27)$$

Further, displacement vector $\{\overline{U}\}$ in (27) is related to $\{U\}$ with transformation process

$$\begin{Bmatrix} \{\overline{U}_f\} \\ \cdots \\ \{\overline{U}_m\} \\ \cdots \\ \{\overline{U}_s\} \end{Bmatrix} = \begin{bmatrix} [I] & \vdots & [0] & \vdots & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & \vdots & [I] & \vdots & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & \vdots & [A_m] & \vdots & [A_s] \end{bmatrix} \begin{Bmatrix} \{U_f\} \\ \cdots \\ \{U_m\} \\ \cdots \\ \{U_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ \cdots \\ \{0\} \\ \cdots \\ \{0\} \end{Bmatrix} \equiv [R]\{U\} - \{B'\} \quad (28)$$

in which lower partition provides

$$\{\overline{U}_s\} = [A_m]\{U_m\} + [A_s]\{U_s\} - \{B\} \qquad (29)$$

Referring to the form in (29), fulfillment of (23) is identic with the application of the following constraint,

$$\{\overline{U}_s\} = \{0\} \qquad (30)$$

on (27). This can be done by applying modification on (27),

$$\begin{bmatrix} [K_{ff}] & \vdots & [K_{fm}] & \vdots & [O] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{mf}] & \vdots & [K_{mm}] & \vdots & [O] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [O] & \vdots & [O] & \vdots & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \{\overline{U}_f\} \\ \cdots \\ \{\overline{U}_m\} \\ \cdots \\ \{\overline{U}_s\} \end{Bmatrix} = \begin{Bmatrix} \{P_f\} - [K_{fs}]\{B\} \\ \cdots \\ \{P_m\} - [K_{ms}]\{B\} \\ \cdots \\ \{O\} \end{Bmatrix} \quad (31)$$

or

$$[K_m^{'}]\{\overline{U}\} = \{P'\} \qquad (32)$$

Now a virtual but kinematically admissible displacement field is assumed,

$$\begin{Bmatrix} \{\delta\overline{U}_f\} \\ \cdots \\ \{\delta\overline{U}_m\} \\ \cdots \\ \{\delta\overline{U}_s\} \end{Bmatrix} = \begin{bmatrix} [I] & \vdots & [0] & \vdots & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & \vdots & [I] & \vdots & [0] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [0] & \vdots & [A_m] & \vdots & [A_s] \end{bmatrix} \begin{Bmatrix} \{\delta U_f\} \\ \cdots \\ \{\delta U_m\} \\ \cdots \\ \{\delta U_s\} \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ \cdots \\ \{0\} \\ \cdots \\ \{0\} \end{Bmatrix} \qquad (33)$$

Insertion of (28) in(31) and with pre-multiplication of obtained equation with (33) result in

$$\{\delta U\}^T [R]^T [K_m][R]\{U\} = \{\delta U\}^T [R]^T \{\{P'\} + \{B'\}\} \qquad (34)$$

Due to the fact that $\{\delta U\}$ is arbitraly but kinematically admissible, (34) gives

$$\begin{bmatrix} [K_{ff}] & \vdots & [K_{fm}] & \vdots & [O] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [K_{mf}] & \vdots & [K_{mm}]+[A_m]^T[K_{ss}][A_m] & \vdots & [A_m]^T[K_{ss}][A_s] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ [O] & \vdots & [A_s]^T[K_{ss}][A_m] & \vdots & [A_s]^T[K_{ss}][A_s] \end{bmatrix} \begin{Bmatrix} \{U_f\} \\ \cdots \\ \{U_m\} \\ \cdots \\ \{U_s\} \end{Bmatrix} = \begin{Bmatrix} \{P_f\} - [K_{fs}]\{B\} \\ \cdots \\ \{P_m\} + [A_m]^T[K_{ss}]\{B\} - [K_{ms}]\{B\} \\ \cdots \\ [A_s]^T[K_{ss}]\{B\} \end{Bmatrix} \qquad (35)$$

Observing the form, (35) resembles the combination of basic equilibrium equation and multi-point constraint equation. So the solution of this equation satisfies the two equations. It is interesting to see that the new equilibrium equation involves modified stiffness matrix which is symmetric. The obtained solution then may be used in (26) to compute for reaction force vector,

$$\{R_s\} = [K_{sf}]\{U_f\} + [K_{sm}]\{U_m\} + [K_{ss}]\{U_s\} - \{P_s\} \qquad (36)$$

IV. Computer Programming

A computer program package for structural analysis was written in Fortran leaguage. The program was equipped with several features; i.e., multi-point constraint [3] and static condensation [1]. The degrees of freedom were catogarized in master, slave and restraint sub-vector $\{U_m\}$, $\{U_s\}$ and $\{U_r\}$. The grouping of degrees of freedom was done by denoting activity of node with 1, 2, 3, and 0 for master, slave, restraint inactive degrees of freedom such that the degrees of freedom are grouped in the same sub-vector. Unfortunately, this scheme will alter the size of half-bandwidth of the global stages matrix. And other method is not to arrange degrees of freedom sequently according to category, but to perform static condensation and multi-point constraint by row-wise. In this method, partitioning process as in (18) is not carried out formally. The program was then applied to case study in the following chapter.

V. Case Study

In the study, three cases are investigated; i.e., (1) regular plane frame as in Fig. 1 with the assumptions that the floors are rigid and structural mass is lumped, (2) irregular plane frame as in Fig. 2 with the assumptions that the floors are rigid and structural mass is lumped and (3)irregular plane frame as in Fig. 2 with no assumptions that the floors are rigid and structural mass is lumped. See Table 1 as explanation. In all cases, only horizontal sways of the floors are retained as degrees of freedom, leading to 2 dynamic modes. Only natural frequencies are computed for all cases and then compared one to another.

The beam span is 600 cm and the dimension is b x d = 30 cm x 60 cm. The clolumn height is 400 cm and the dimension is b x d = 30 cm x 30 cm. The components are made of concrete with elastic modulus E = 20,000 MPa, Poisson's ratio ν = 0.0 and mass m = 0.024 kgm/m^3.

Table 1: Study Cases

Analysis		Remark
I		2-storey regular frame, with assumed rigid floor and lumped mass
II	1	2-storey irregular frame, with assumed rigid floor and lumped mass
	2	2-storey irregular frame, with no assumed rigid floor and lumped mass

Based on the computer run for dynamic analysis of the three cases in Table 1, the computed natural frequencies are tabulated and compared in Table 2. It is shown that the natural frequencies of Cases I and II.1 are the same. However, the natural frequency of Case II.2 is different significantly to those of Cases I and II.1. The difference is due to the computation of structural mass and stiffness matrices that are computed in different manner. Mass matrices in Cases I and II.1 are computed based on lumped mass procedure, whereas the matrix in Case II.2 is computed based on consistent mass procedure.

Table 2: Comparison of Natural Frequencies

Analysis	Natural Frequency (rad/sec)	
	mode 1	mode 2
I	0.646	1.644
II.1	0.674	1.939
II.2	0.5366	1.197

Table 2 demonstrates that stiffness of irregular frame is less that of regular one, hence the angular frequency is smaller and natural period is larger that of regular one. The natural frequency of Case II.2 is more accurate since its discrete model represents the actual structural system more accurately. Therefore, modal analysis of Case II.2 will be more closely to the actual result compared to those of Cases I and II.1.

VI. Conclusions

Based on the comparison among cases in Chapter V, it is concluded simplification in structural dynamic analysis based on the assumptions that the floors are rigid and the mass is lumped in retained nodes, provides results with accuracy that depends on irregularity of the system. For regular system, the assumptions still provide relatively accurate results, but this is not the case with irregular system. The proposed method using consistent mass formulation may be applied to regular as well as irregular systems with good accuracy of results.

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