IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 14, Issue 6 Ver. II (Nov. - Dec. 2017), PP 15-23
www.iosrjournals.org

The Use Of Mixed Eulerian-Lagrangian Displacement In Analysis Of Prestressed Concrete Bridge Girder

Binsar H. Hariandja[1]
[1](*Civil Engineering Department, Bandung Institute of Technology, Bandung, Indonesia*)

Abstract: *The paper discusses the use of mixed Eulerian-Lagrangian in the analysis of post-tensioned prestressed concrete bridge girder. The hose of the tendon is in stick condition with the surrounding concrete while the tendon in slip condition with the hose. The slip between the tendon with the hose is modeled by means of Eulerian displacement. The girder concrete is modeled by using brick elements while the tendon is modeled by using bar elements. The quadratic shape of tendon is modeled by piece-wise linear bar elements arrangement. If finite element mesh is construed to follow quadratic shape of tendon, then irregular shape of elements may results. To avoid this, the nodes on bar elements are connected to girder mid nodes with rigid bars. Bottom ends of bars are connected with bar nodes. Slip between tendon and rigid bar is modeled by Eulerian displacement. The modeled is applied to the analysis of post-tensioned prestressed concrete bridge girder. The results obtained are then compared to analysis result using conventional beam method, and the results agree well.*

Keywords:*mixed Eulerian-Lagrangian displacement,prestressed concrete, post-tensioned system, internal and external post-tension tendon*

Date of Submission: 13 -11-2017 Date of acceptance: 30-11-2017

I. Introduction

In post-tensioned prestressed concrete system, the tendon is freely slips relatively to hose, while the hose is rigidly connected to surrounding concrete. If conventional Lagrangian displacement is used, the slip between a tendon node and concrete node may separate the material points initialy occupied the same node. To over come this problematic case, Eulerian displacement is used to model slip between tendon and surrounding hose. The concept of mixed Eulerian-Lagrangian displacement is discussed in available reference [3]. The description of deformation is reffered to undeformed configuration [4]

The paper deals with the use of mixed Eulerian-Lagrangian displacement in the analysis of post-tensioned prestressed concrete bridge girder. The analysis is carried out by means of finite element method and the analysis is cast in matrix form. The analysis algorithm is then cast in a computer package program written in Fortran language. The program is then applied to the analysis of a post-tensioned prestressed concrete girder, and the results are compared to the results obtained by using conventional beam method.

II. Mixed Eulerian-Lagrangian Displacement Model

Mixed Eulerian-Lagrangian displacement field was first introduced by Hariandja [3]

2.1 Mixed Eulerian-Lagragian Model.

The following discussion deals with finite element formulation of system with finite displacement. The decomposition of displacement is desribed by showing displacement model in Figure 1. A typical material point in the structural system at time t occupies initial configuration V^t at location $\tilde{X}$. After loading, the material point eventually occupies current configuration v^t at location$\tilde{X}$ A configuration, which may be occupied by structural system at a particular time within loading process is chosen as referential configuration and denoted byv^r The displacement of a typical material point initially occupied location at$\tilde{X}$, is denoted by $\tilde{q}$ and this displacement decomposed into Eulerian displacement $\tilde{w}$ and Lagrangian displacement$\tilde{u}$,

$$\tilde{q} = \tilde{x} - \tilde{X} = (\tilde{x} - \tilde{x}^r) + (\tilde{x}^r - \tilde{X}) = \tilde{u} + \tilde{w} \qquad (1)$$

Deformation may be observed by inspecting elongation experienced by a typical line segment dS that mapped into ds, such that

$$(ds)^2 - (dS)^2 = 2d\tilde{x} \cdot L \cdot d\tilde{x} \qquad (2)$$

following Euler description, and

$$(ds)^2 - (dS)^2 = 2d\tilde{X} \cdot E \cdot d\tilde{X} \qquad (3)$$

according to Lagrange description. The entity E is Green strain tensor given by

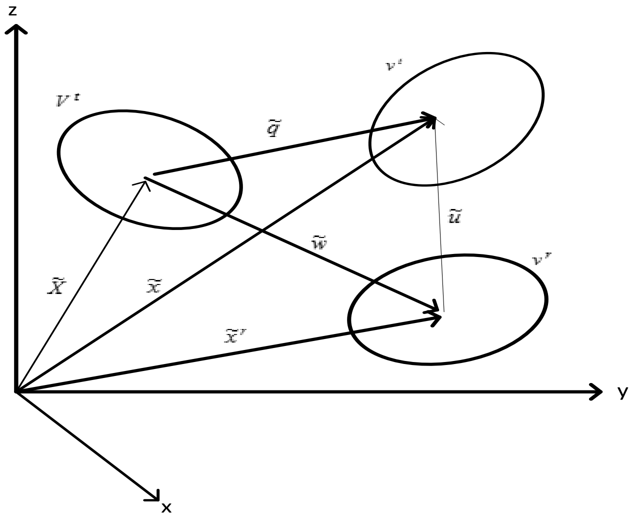

Figure1 Mixed Eulerian and Lagrangian displacement

$$E_{ij} = \frac{1}{2}\left[\frac{\partial x_k}{\partial X_i}\frac{\partial x_k}{\partial X_j} - \delta_{ij}\right] \qquad (4)$$

and L is Almansi strain tensor given by

$$L_{ij} = \frac{1}{2}\left[\delta_{ij} - \frac{\partial X_k}{\partial x_i}\frac{\partial X_k}{\partial x_j}\right] \qquad (5)$$

in which Einstein summation rule applies, i.e., summation is to be carried out for repeated index. The Green strain tensor gives Piola-Kirchoff stress tensor, while Almansi strain tensor gives Cauchy stress tensor (Gantmacher, 1975, Hariandja, 1985, Malvern, 1969). The displacement gradient may be expressed in term of Jacobian

$$\frac{\partial x_i}{\partial X_j} = J_{ij} = \frac{\partial x_i}{\partial x_k^r}\frac{\partial x_k^r}{\partial X_j} = \hat{J}_{ik}\bar{J}_{kj} \qquad (6)$$

in which $\hat{J}_{ik}$ is Lagrangian Jacobian and $\bar{J}_{kj}$ is Eulerian Jacobian given by

$$\hat{J}_{ik} = \frac{\partial x_i}{\partial x_k^r}; \quad \bar{J}_{kj} = \frac{\partial x_k^r}{\partial X_j} \quad (7)$$

Since the formulations are arranged in terms of referential parameter $\tilde{x}^r$ then partial derivatives with respect to $\tilde{X}$ need to be transformed into partial derivatives with respect to $\tilde{x}^r$ First, it is written that

$$\frac{\partial \cdot}{\partial x_i^r} = \frac{\partial X_k}{\partial x_i^r}\frac{\partial \cdot}{\partial X_k} = m_{ki}\frac{\partial \cdot}{\partial X_k} \qquad (8)$$

which, upon inversion gives

$$\frac{\partial \cdot}{\partial X_i} = \bar{m}_{ik}\frac{\partial \cdot}{\partial x_k^r} \qquad (9)$$

in which $\bar{m}_k$ is the element of inverted matrix of the matrix formed by m_{ki} Further, partial derivatives with respect to $\tilde{x}$ may be inverted to partial derivatives with respect to parametric coordinates $\tilde{\zeta}$ by writing

$$\frac{\partial \cdot}{\partial \xi_i} = \frac{\partial \cdot}{\partial x_k^r}\frac{\partial x_k^r}{\partial \xi_i} = n_{ki}\frac{\partial \cdot}{\partial x_k^r} \qquad (10)$$

which, upon inversion gives

$$\frac{\partial \cdot}{\partial x_i^r} = \bar{n}_{ik} \frac{\partial \cdot}{\partial \xi_k} \qquad (11)$$

in which $\bar{n}_{ik}$ is the element of inverted matrix of the matrix formed by n_{ki}. Therefore, the following may be obtained,

$$\frac{\partial \cdot}{\partial X_i} = \bar{m}_{ik} \frac{\partial \cdot}{\partial x_k^r} = \bar{m}_{ik}\bar{n}_{kj} \frac{\partial \cdot}{\partial \xi_j} = \bar{r}_{ij} \frac{\partial \cdot}{\partial \xi_j}; \quad \bar{r}_{ij} = \bar{m}_{ik}\bar{n}_{kj} \qquad (12)$$

2.2 Incrementation and Linearization Technique

It may be observed from the form of Equations 4 and 5 that the governing equilibrium equation is quadratic in terms of displacement components. Therefore, the problem would be geometrically nonlinear. The governing equilibrium equation may be expanded in terms of displacement components and the expression may be approximated by retaining linear terms. In this case, successive iteration scheme is applied.

The following is incrementation process of terms. First, at time t the displacement is decomposed in Eulerian and Lagrangian displacement

$$\tilde{q}^t = \tilde{x}^t - \tilde{X}^t = (\tilde{x}^t - \tilde{x}^r) + (\tilde{x}^r - \tilde{X}^t) = \tilde{u}^t + \tilde{w}^t \qquad (13)$$

Strain component is given by

$$E_{ij}^t = \frac{1}{2}\left[J_{ki}^t J_{kj}^t - \delta_{ij}\right] \ (14)$$

For time $t + \Delta t$ displacement is given by

$$\tilde{q}^{t+\Delta t} = \tilde{q}^t + \tilde{q} = \tilde{q}^t + \tilde{u} + \tilde{w} \qquad (15)$$

in which $\tilde{q}$ is incremental displacement consisting Lagrangian incremental displacement $\tilde{u}$ and Eulerian incremental displacement $\tilde{w}$. Correspondingly, total Jacobian components are incremented

$$J_{ij}^{t+\Delta t} = J_{ij}^t + \Delta J_{ij} \qquad (16)$$

in which incremental Jacobian components are given by

$$\Delta J_{ij} = \hat{J}_{im}^t \Delta \bar{J}_{mj} + \Delta \hat{J}_{im} \bar{J}_{mj}^t \ (17)$$

which may further be written in terms of Lagrangian and Eulerian incremental Jacobians. Strain components may also be incremented by writing

$$E_{ij}^{t+\Delta t} = E_{ij}^t + \Delta E_{ij} = \frac{1}{2}(J_{ki}^t J_{kj}^t - \delta_{ij}) + \Delta E_{ij} \qquad (18)$$

which results in

$$\Delta E_{ij} = \frac{1}{2}(J_{ki}^t J_{kj} + J_{ki} J_{kj}^t) \qquad (19)$$

Eqn. (19) may be written in matrix form [1]

$$\{\Delta E\} = [B_1]\{\Delta J\}; \quad \{\Delta J\} = [B_2]\{\Delta \tilde{J}\}; \quad \{\Delta \tilde{J}\} = [B_3]\{\tilde{u}\} \qquad (20)$$

in which $\{\tilde{u}\}$ is incremental displacement vector containing Eulerian and Lagrangian incremental displacements. Therefore, the following relationship is established.

$$\{\Delta E\} = [B]\{\tilde{u}\}; \quad [B] = [B_1][B_2][B_3] \qquad (21)$$

In the following, equilibrium equation is written in incremental form. First, at time t, the equilibrium condition reads

$$[K^t]\{Q^t\} = \{P^t\} \ (22)$$

in which

$$[K^t] = \frac{1}{2}\iiint_v [E^t]^T [C][E^t] dv \ (23)$$

At time $t + \Delta t$,

$$[K^{t+\Delta t}]\{Q^{t+\Delta t}\} = \{P^{t+\Delta t}\} \qquad (24)$$

which may be expanded in the following form,

$$\left[K^{t+\Delta t}\right]\left\{\left\{Q^t\right\}+\left\{\Delta Q\right\}\right\}=\left\{\left\{P^t\right\}+\left\{\Delta P\right\}\right\} \qquad (25)$$

which, in view of equilibrium condition in Equation 22, provides linearized form

$$\left[K^{t+\Delta t}\right]\left\{\Delta Q\right\}=\left\{\Delta P\right\} \qquad (26)$$

where

$$\left[K^{t+\Delta t}\right]=\iiint_v \left[B\right]^T\left[C\right]\left[B\right]dv \quad (27)$$

In the formulation of global element stiffness, the following relationship may be used,

$$dv = dx_1dx_2dx_3 = \overline{m}dx_1^r dx_2^r dx_3^r = \bar{r}d\xi_1 d\xi_2 d\xi_3 \quad (28)$$

In the following chapter, incremental matrix stiffness of several types of elements, in this case, four node isoparametric membrane and bar elements, are developed.

III. Finite Element Formulation and Computer Programming

The following discussing deals with finite element formulation that may be found in several available references [2].

3.1 Finite Element Formulation

Due to the limitation on the space, only two types of elements are developed, i.e., four node isoparametric membrane and bar elements, considered in turn in the following.

3.1.1 Four node isoparametric membrane

A four node isoparametric membrane element is depicted in Figure 2. Each node contains four degrees of freedom, i.e., Eulerian and Lagrangian displacement components in $\widetilde{x}^r$ coordinate. Therefore, the element has 16 degrees of freedom, arranged in the form

$$\begin{aligned}\left\{\widetilde{u}^t\right\}&=\left\{u_{11}^t \;\; w_{11}^t \;\; u_{21}^t \;\; w_{21}^t \;\; .. \;\; u_{14}^t \;\; w_{14}^t \;\; u_{24}^t \;\; w_{24}^t\right\}\\ \left\{\widetilde{u}\right\}&=\left\{u_{11} \;\; w_{11} \;\; u_{21} \;\; w_{21} \;\; .. \;\; u_{14} \;\; w_{14} \;\; u_{24} \;\; w_{24}\right\}\end{aligned} \qquad (29)$$

and displacement and nodal coordinates are interpolated by using shape functions,

$$\begin{aligned}\left\{u(\xi_1,\xi_2)\right\}&=\sum_{i=1}^{4}N_i(\xi_1,\xi_2)u_i\\ \left\{x^r(\xi_1,\xi_2)\right\}&=\sum_{i=1}^{4}N_i(\xi_1,\xi_2)x_i^r\end{aligned} \qquad (30)$$

in which

$$N_i(\xi_1,\xi_2)=\frac{1}{4}(1+\bar{\xi}_{1i}\xi_1)(1+\bar{\xi}_{2i}\xi_2);\quad \mathrm{i}=1,4 \qquad (31)$$

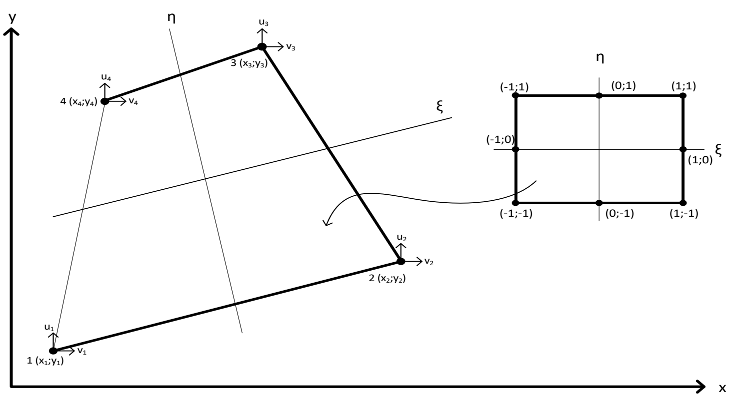

Figure 2 Four node isoparametric membrane element

First, Lagrangian and Eulerian Jacobian components are obtained by applying Equation 7,

$$\hat{J}_{ij}^t=\delta_{ij}+\overline{n}_{jk}\partial\hat{u}_i^t/\partial\xi_k;\quad \bar{J}_{ij}^t=\delta_{ij}+\bar{r}_{jk}\partial\hat{u}_i^t/\partial\xi_k \qquad (32)$$

The $[B_1]$ matrix then may be written in the form

$$[B_1]=\begin{bmatrix} J_{11}^t & 0 & J_{21}^t & 0 \\ 0 & J_{12}^t & 0 & J_{22}^t \\ J_{12}^t/2 & J_{11}^t/2 & J_{22}^t/2 & J_{21}^t/2 \end{bmatrix} \qquad (33)$$

$[B_2]$ matrix in the form

$$[B_2]=\begin{bmatrix} \bar{J}_{11}^t & \hat{J}_{11}^t & 0 & 0 & \bar{J}_{21}^t & \hat{J}_{21}^t & 0 & 0 \\ \bar{J}_{12}^t & 0 & 0 & \hat{J}_{11}^t & \bar{J}_{22}^t & 0 & 0 & \hat{J}_{21}^t \\ 0 & \hat{J}_{12}^t & \bar{J}_{11}^t & 0 & 0 & \hat{J}_{22}^t & \bar{J}_{21}^t & 0 \\ 0 & 0 & \bar{J}_{12}^t & \hat{J}_{12}^t & 0 & 0 & \bar{J}_{22}^t & \hat{J}_{22}^t \end{bmatrix} \qquad (34)$$

and $[B_3]$ matrix in the form

$$[B_3]=\begin{bmatrix} \cdots\cdots & a_i & 0 & 0 & 0 & \cdots\cdots \\ \cdots\cdots & 0 & b_i & 0 & 0 & \cdots\cdots \\ \cdots\cdots & c_1 & 0 & 0 & 0 & \cdots\cdots \\ \cdots\cdots & 0 & d_i & 0 & 0 & \cdots\cdots \\ \cdots\cdots & 0 & 0 & a_i & 0 & \cdots\cdots \\ \cdots\cdots & 0 & 0 & 0 & b_i & \cdots\cdots \\ \cdots\cdots & 0 & 0 & c_1 & 0 & \cdots\cdots \\ \cdots\cdots & 0 & 0 & 0 & d_i & \cdots\cdots \end{bmatrix} \qquad (35)$$

in which

$$\begin{aligned} a_i &= \overline{m}_{11}\partial N_i/\partial\xi_1 + \overline{m}_{12}\partial N_i/\partial\xi_2 \\ b_i &= \bar{r}_{11}\partial N_i/\partial\xi_1 + \bar{r}_{12}\partial N_i/\partial\xi_2 \\ c_i &= \overline{m}_{21}\partial N_i/\partial\xi_1 + \overline{m}_{22}\partial N_i/\partial\xi_2 \\ d_i &= \bar{r}_{21}\partial N_i/\partial\xi_1 + \bar{r}_{22}\partial N_i/\partial\xi_2 \end{aligned} \qquad (36)$$

for node i. Therefore, $[B]$ matrix may be constructed by inserting Equations 33, 34 and 35 in Equation 21. The result reads

$$[B]=\begin{bmatrix} \cdots\cdots & p_{11i} & p_{12i} & p_{13i} & p_{14i} & \cdots\cdots \\ \cdots\cdots & p_{21i} & p_{22i} & p_{23i} & p_{24i} & \cdots\cdots \\ \cdots\cdots & p_{31i} & p_{32i} & p_{33i} & p_{34i} & \cdots\cdots \end{bmatrix} \qquad (37)$$

in which

$$\begin{aligned} & p_{11i} = a_i(J_{11}^t\bar{J}_{11}^t) + c_i(J_{21}^t\bar{J}_{11}^t); \quad p_{12i} = b_i(J_{11}^t\hat{J}_{11}^t + J_{21}^t\hat{J}_{12}^t); \\ & p_{13i} = a_i(J_{11}^t\bar{J}_{21}^t) + c_i(J_{21}^t\bar{J}_{21}^t); \quad p_{14i} = b_i(J_{11}^t\hat{J}_{21}^t + J_{21}^t\hat{J}_{22}^t); \\ & p_{21i} = a_i(J_{12}^t\bar{J}_{12}^t) + c_i(J_{22}^t\bar{J}_{12}^t); \quad p_{22i} = d_i(J_{12}^t\hat{J}_{11}^t + J_{22}^t\hat{J}_{12}^t); \\ & p_{23i} = a_i(J_{12}^t\bar{J}_{22}^t) + c_i(J_{22}^t\bar{J}_{12}^t); \quad p_{24i} = d_i(J_{12}^t\hat{J}_{21}^t + J_{22}^t\hat{J}_{22}^t); \\ & p_{31i} = \frac{a_i}{2}(J_{12}^t\bar{J}_{11}^t + J_{11}^t\bar{J}_{12}^t) + \frac{c_i}{2}(J_{22}^t\bar{J}_{11}^t + J_{21}^t\bar{J}_{12}^t); \\ & p_{32i} = \frac{b_i}{2}(J_{12}^t\hat{J}_{11}^t + J_{22}^t\hat{J}_{12}^t) + \frac{d_i}{2}(J_{22}^t\hat{J}_{11}^t + J_{21}^t\hat{J}_{12}^t); \\ & p_{33i} = \frac{a_i}{2}(J_{12}^t\bar{J}_{21}^t + J_{11}^t\bar{J}_{22}^t) + \frac{c_i}{2}(J_{22}^t\bar{J}_{21}^t + J_{21}^t\bar{J}_{12}^t); \\ & p_{34i} = \frac{b_i}{2}(J_{12}^t\hat{J}_{21}^t + J_{21}^t\hat{J}_{22}^t) + \frac{d_i}{2}(J_{11}^t\hat{J}_{21}^t + J_{21}^t\hat{J}_{22}^t) \end{aligned} \qquad (38)$$

for node i. For flat plane membrane, stress-strain relationship is controlled by constitutive equation

$$[C]=\frac{Et}{(1-\nu^2)}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & (1-\nu)/2\end{bmatrix}(39)$$

The obtained matrices may be inserted in Equation 27 to construct element stiffness matrix. The element stiffness matrix is computed by using Gauss numerical integration technique.

3.1.2 Bar element

Bar element is depicted in Figure 3. The element has two nodes and each node has two degrees of freedom arranged in the form

$$\begin{aligned}\{\tilde{u}^t\}&=\{u_{11}^t \;\; w_{11}^t \;\; u_{21}^t \;\; w_{21}^t\}\\ \{\tilde{u}\}&=\{u_{11} \;\; w_{11} \;\; u_{21} \;\; w_{21}\}\end{aligned} \qquad (40)$$

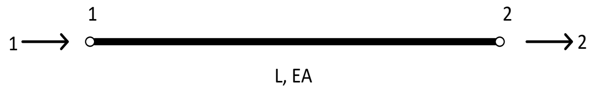

Figure 3 Bar element

The displacement is found by interpolating nodal displacement vector with shape functions,

$$\{u(\xi_1)\}=\sum_{i=1}^{4}N_i(\xi_1)u_i \qquad (41)$$

in which

$$N_i(\xi_1)=\frac{1}{4}(1+\bar{\xi}_{1i}\xi_1);\;\; \mathrm{i}=1,2;\;\; \xi_1=2x_1^r/L \quad (42)$$

and $\bar{n}_{11}=2/L;\;\; \bar{r}_{11}=1-\partial\bar{u}/\partial x_1^r=1-(L/2)\partial\bar{u}/\partial\xi_1$, and Jacobian components may be computed and used to construct element stiffness matrix. The result is

$$[k]=\frac{J_{11}^tJ_{11}^tEA}{L}\begin{bmatrix}\bar{J}_{11}^t\bar{J}_{11}^t & \hat{J}_{11}^t\bar{J}_{11}^t & -\bar{J}_{11}^t\bar{J}_{11}^t & -\hat{J}_{11}^t\bar{J}_{11}^t\\ \hat{J}_{11}^t\bar{J}_{11}^t & \hat{J}_{11}^t\hat{J}_{11}^t & -\hat{J}_{11}^t\bar{J}_{11}^t & -\hat{J}_{11}^t\hat{J}_{11}^t\\ -\bar{J}_{11}^t\bar{J}_{11}^t & -\hat{J}_{11}^t\bar{J}_{11}^t & \bar{J}_{11}^t\bar{J}_{11}^t & \hat{J}_{11}^t\bar{J}_{11}^t\\ -\hat{J}_{11}^t\bar{J}_{11}^t & -\hat{J}_{11}^t\hat{J}_{11}^t & \hat{J}_{11}^t\bar{J}_{11}^t & \hat{J}_{11}^t\hat{J}_{11}^t\end{bmatrix} \qquad (43)$$

3.2 Computer Programming

Finite formulation described in previous discussion is cast in a computer package program writen in Fortran language. Incrementation technique is also embedded in the program. Contsructed program is then validated to prove its accuracy and correctness, before applied in case study as discuss in followed.

IV Case Study

For a case study, a prestressed concrete I girder of brigde is analyzed [5]. The I beam has a span of 2400 cm, divided into 24 rows and 2 columns of 3 dimension brick isoparametric elements being shown in Fig. 4. Girder dimension is b x h = 55.00 cm x 132.00 cm. The prestressing force is 10,800.00 kN. The girder is analyzed by using beam theory to computed stresses and displacements due to own weight, superimposed, and live load.

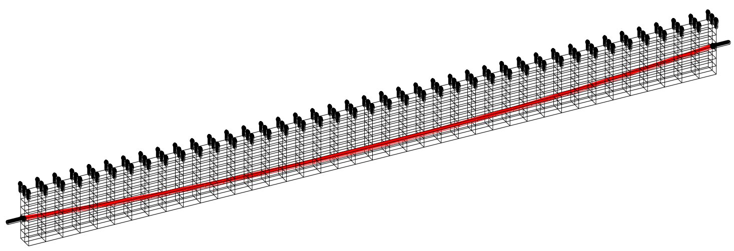

Figure 4: Element Meshing of Prestressed Concrete Girder

To compute stresses and displacements due to prestressing force, uniform uplift load

$$w_F = \frac{8F \cdot e}{L^2} = 66.00\ kg/cm \qquad (44)$$

is used. The results are used as comparison to the results obtained by means of computer program. The comparison is shown in Table 1.

Table 1 : Comparison Between Manual and Computer Output

No.	Quantity	Manual	Program	Error Percentage
1	Displacement in Z direction of mid span	-6.00	-6.04	0.64%
2	Stress in upper fiber	-295.53	-293.29	0.76%
3	Stress in mid-height fiber	-148.76	-149.66	0.36%
4	Stress in lower fiber	-2.00	-4.65	-133.03%

By observing the table, it is concluded that the results out of program agree well with manual results. The element meshing is used to analyze the same girder with parabolic tendon with mid span eccentricity e = -44.00 cm and F = 10,800.00 kN. For tendon, rupture stress $0.70 \times f_{pu}$=126,000.00 MPa of high tension tendon is used.If element meshing is taken to follow parabolic shape of tendon, then it is ended with rather irregular shape of elements and this may eventually cause error due to shape factor of element dimention. To avoid this problem, cable is run below rigid bars connecting mid section nodes with the cable. The rigidity of the connecting bars may be simulated by setting large number of axial rigidity of bars, or giving the same number of displacements between upper and lower nodes of the bars. The cable may slip freely beneath the bars. The slip between cable and bar is modeled by Eulerian displacement and the displacements of rigid bar lower end is modeled by Lagrange displacements. See Fig. 5 as explaination.

The beam is then analyze using the computer program. The results are compared with the results by manual. The comparison is carried out for stresses and displacements at mid span cross section, as shown in Table 2.The table clearly shows that the results out of computer program agree well with manual results.

Table 2: Comparison Between Computer and Manual Results

No.	Quantity	Manual	Program	Error Percentage
1	Displacement in Z direction of mid span	-6.00	-5.90	1.67%
2	Stress in upper fiber	-295.53	-294.76	0.26%
3	Stress in mid-height fiber	-148.76	-148.56	0.13%
4	Stress in lower fiber	-2.00	-2.40	20.00%

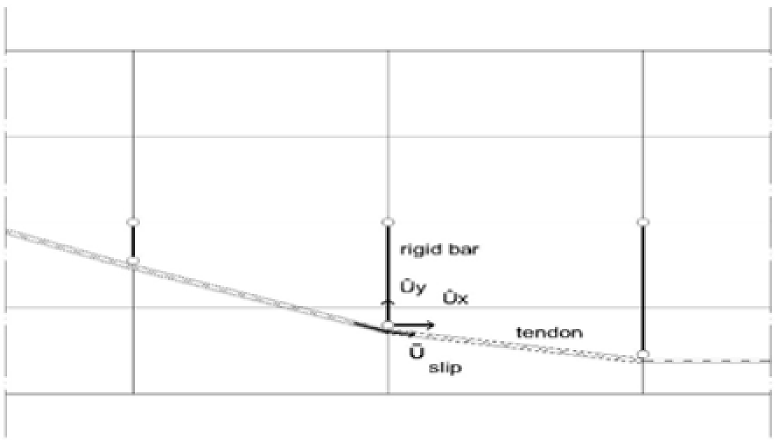

Figure 5: Tendon With Connecting Rigid Bar to Beam Centeroid

The girder analyzed previously using cross section dimension b x h = 55 cm x 132 cm, which results in the use of internal prestressing tendon. In the following, the cross section dimension is changed to I shape with the height 100 cm, and upper and lower flange thicknesses 30 cm. girder CTC is 400 cmand the loads are

$$\begin{aligned} q_g &= 0.0024 \times 7200 = 172.8\ N/cm \\ q_s &= 0.072 \times 400 = 288.0\ N/cm \\ q_l &= 0.090 \times 400 = 360.0\ N/cm \\ P_l &= 49.00 \times 400 = 196{,}000.0 N \end{aligned} \tag{45}$$

The moments at mid span are

$$\begin{aligned} M_g &= \tfrac{1}{8} \times 172.8 \times 2400^2 = 1254.2\ kN-m \\ M_s &= \tfrac{1}{8} \times 288.0 \times 2400^2 = 2073.6\ kN-m \\ M_l &= \tfrac{1}{8} \times 360.0 \times 2400^2 = 2592.0\ kN-m \\ M_p &= \tfrac{1}{4} \times 196{,}000 \times 2400 = 1176.0\ kN-m \\ \hline M_t &= 7085.8\ kN-m \end{aligned}$$ (46

Based on allowable compression stress 315.00 kg/cm^2 then by Magnel scheme, the prestressing force is computed $F_0 = 10{,}800\ kN$ and eccenctricity $e = -60.00\ cm$. Since the eccentricity falls outside cross section, external prestressing tendon is used.

By manual, the displacements at mid span are computed as

$$\begin{aligned} \delta_g &= -\frac{5}{384} \times \frac{q_g L^4}{EI} = -4.690\ cm;\ \ \delta_s = -\frac{5}{384} \times \frac{q_s L^4}{EI} = -7.820\ cm \\ \delta_l &= -\frac{5}{384} \times \frac{q_l L^4}{EI} = -9.770\ cm;\ \ \delta_p = -\frac{1}{48} \times \frac{P L^3}{EI} = -3.550\ cm \end{aligned} \tag{47}$$

so that total displacement due to loads becomes

$$\delta_t = -25.82\ cm \tag{48}$$

uplift equivalent load is

$$w_F = \frac{8F \cdot e}{L^2} = 82.50\ kg/cm \tag{49}$$

resulting in camber

$$\delta_F = \frac{5}{384} \times \frac{w_F L^4}{EI} = 22.39\ cm \tag{50}$$

so that the total displacement at mid span becomes

$$\delta_T = -3.43\ cm \tag{51}$$

For check up of stresses, two stages of loading are considered, i.e., transfer and service stages. For transfer stage, the stresses are

$$\begin{aligned} f_{ct} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_t + \frac{M_g}{I_{zz}} y_t = -15.00 + 37.312 - 7.815 = +14.436 MPa \\ f_{cb} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_b + \frac{M_g}{I_{zz}} y_b = -15.00 - 37.312 + 7.855 = -44.496\ MPa \end{aligned} \tag{52}$$

in which the stress at both upper and lower fibers violate allowable stresses this problem is may be overcome by stressing the tendon when the girder is ready set at place and superimpose load already active. So,

$$M_d = M_g + M_s = 331.78\ t-m \tag{53}$$

The stresses become

$$\begin{aligned} f_{ct} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_t + \frac{M_d}{I_{zz}} y_t = -15.00 + 37.312 - 20.840 = +1.471\ Mpa \\ f_{cb} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_b + \frac{M_d}{I_{zz}} y_b = -15.00 - 37.312 + 20.840 = -31.471\ MPa \end{aligned} \tag{54}$$

The total stresses at service stages are

$$\begin{aligned} f_{ct} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_t + \frac{M_t}{I_{zz}} y_t = -15.00 + 37.312 - 44.509 = -221.97\ kg/cm^2 \\ f_{cb} &= \frac{F_0}{A} + \frac{F_0 e}{I_{zz}} y_b + \frac{M_t}{I_{zz}} y_b = -15.00 - 37.312 + 44.509 = -78.03\ kg/cm^2 \end{aligned} \tag{55}$$

The comparison is depicted in Table 3, which demostrates that the results out of program agree well with manual results.

Tabel 3: Comparison Between Computer and Manual Results

No.	Quantity	Manual	Program	Error Percentage
1	Displacement in Z direction of mid span	-3.43	-4.17	21.57%
2	Stress in upper fiber	-221.97	-224.55	1.16%
3	Stress in mid-height fiber	-150.00	-150.49	0.33%
4	Stress in lower fiber	-78.03	-74.90	4.01%

V Discussions of the Results

Several observation out of the comparison of results are drawn as follows. First, a finite element model is established for the analysis of prestressed concrete bridge girder. Curved shape of tendon is connected to the regular rectangular shape of element meshing with rigid bars. The rigidity of the bars are represented by giving the same number between lower and upper bar displacements.

The newly developed model is applied to two cases, i.e., internal cable case and external cable cases. The results out of the pakage program agree well with the results obtained by manual computation. The slip between the tendon and the lower end of rigid bar is modeled by means of Eulerian displacement.

VI Conclusions

Based on the results obtained in this paper, several conclusions are drawn as follows. First, a newly developed finite element model has been established for the analysis of post-tensioned prestressed concrete brigde girder.Secondly, the model is incorporated in an algorithm analysis of the girder and the algorithm is cased in a computer package program written in Fortran language. The model may be applied to both internal and external tendon systems. The results obtained by the application of the program agree well with the results obatained by the use of beam theory.

Acknowledgements

The paper is written partly based on the thesis of Mr. Firli Syafriza, carried out at Civil Engineering Department, Bandung Institute of Technology, in which the author was the academical advisor. Typing and drawing of the paper were carried out by Mr. Donni Canra, to which the author extend his cordial and sincere appreciation.

References

[1] Argyris, J.M., dan Kelsey, S., *Energy Theorems and Structural Analysis*, Butterworth, London (1960).
[2] Cook, R. D., Malcus, D.S., Plesha, M.E., *Concept and Application of Finite Element*, edisi ketiga, John Wiley & Sons, Inc., New York (1989).
[3] Hariandja, B., *Adaptive Finite Element Analysis Nonlinear Frictional Contact with Mixed Eulerian-Lagrangian Coordinates*, Ph.D dissertation, University of Illinois at Urbana-Champaign (1985).
[4] Malvern, L.E, *Introduction to the Mechanics of a Continuous Medium*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1977)
[5] Syafriza, F., *Penerapan Perpindahan Euler dan Lagrage Dalam Analisis Gelagar Beton Prategang Jembatan*, master thesis, Civil Engineering Department, Bandung Institute of Technology, Bandung, Indonesia (2017)

Binsar H. Hariandja The Use Of Mixed Eulerian-Lagrangian Displacement In Analysis Of Prestressed Concrete Bridge Girder." IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) , vol. 14, no. 6, 2017, pp. 15-23.