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Comparison of Newton Raphson and Hard Darcy methods for gravity main nonlinear water network

AbdulhamidSaad, HasanAhweedi, HusamEL-DinAbdulkaleq, AlwaleedKhaleel, Ahmed Abdulsalam.
[a] *Faculty of Engineering, Tobruk University, Tobruk, Libya*.

Abstract: *A water network of 24 pipes depending on mainly gravity and covers an area of 3.78 square kilometers was taken an as a case study to test and compare the analysis. The governing equation of this network are internal flow in pipe equations, which consist of the continuity equation, the modified Bernoulli's equation, and the head loss due to the length of the pipe. The three equations are nonlinear algebraic equations because of the square power of the discharge in the head loss equations, which need to be solved numerically. Hard Darcy method and Newton Raphson method are used to solve the system of nonlinear equations, and to compare the solution.So, twenty four nonlinear equations (nine Bernoulli's equations and fifteen continuity equations) in twenty four unknowns discharges were got by these two method by using MATLAB code. There are not differences in the resulted discharges between Hard Darcy and Newton Raphson methods. Also, it was found that Newton Raphson was faster than Hard Darcy Method when they compared by the number of iteration. The final solution of the discharges have tested by the basic of fluid mechanics that says the summation of head losses inside a loop must be equal zero which can be seen clearly in the plots of the two methods.*

Keywords: *comparison, duscharge, pipe, HardDrcy, Newton Raphson*

I. Introduction

Water pipe network systems are designed and operated to supply fresh water from the source (or treatment facility) to customers (Hund-Der & Yu-Chang, 2008). Nearly 80% to 85% of the cost of a total water supply system is contributed toward water transmission and the water distribution network (Abdulhamid, 2016). In this project, the distribution network of 24 pipes with nine looped network and gravity main is considered.

Analysis will take place bysetting up a system of a nonlinear equation as results of internal flow in pipe such as, the continuityequation, Bernoulli equation, and the major losses equations. This system cannot be solved analytically. Therefore, numerical method by using MATLAB software is used to solve the nonlinear systems of the network.

Nonlinear equations set can be formulated to describe the relationship between the nodal head and pipe flow rate. Hard Darcy method and Newton Raphson method was commonly used to solve the nonlinear equation set for obtaining the solution of the network (Hund-Der & Yu-Chang,2008).

The hydraulic and optimization analysis are linked through an iterative procedure. The analysis of the pipe network is to estimate the discharge in each pipe, velocities, and the total cost of the system. Also, proof of the solution in each method and the comparison between the two will be considered.

1.1 The modified Bernoulli equation

The Bernoulli equation is a relation between pressure, velocity,and elevation in steady, incompressible flow(Yunus A & John M,2006) as shown in the next equation.

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2 + h_L \quad [1]$$

Where $\frac{P}{\rho}$ is the flow energy, $\frac{V^2}{2}$ is kinetic energy , gz is potential energy and hL is head losses.

1.2 The major losses in pipe

The head loss due to viscous effects in the straight pipes, termed the **major loss** and denoted $h_{L\,major}$ (Munson et al., 2009).

$$h_L = f\,\frac{L}{D}\frac{V^2}{2g} \quad [2]$$

1.3 The minor losses in pipe

The fluid in a typical piping system passes through various fittings, valves,bends, elbows, tees, inlets, exits, enlargements, and contractions in additionto the pipes. These components interrupt the smooth flow of the fluid andcause additional losses because of the flow separation and mixing theyinduce. In a typical system with long pipes, these losses are minor comparedto the total head loss in the pipes (the **major losses**) and are called **minor losses** (Yunus A & John M,2006).

$$h_{L\,minor} = KL\frac{V^2}{2g} \qquad [3]$$

1.4 Volumetric flow rate (discharge)

The volume of the fluid flowing through a cross section per unit time is:

$$Q = VA_C \qquad [4]$$

1.5 Series and parallel network

For pipes in series, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.

$$h_{LT} = h_{L1} + h_{L2} + h_{L3} \qquad [5]$$

Since the same discharge passes through all the pipes, the continuityequation is

$$Q = Q_1 = Q_2 = Q_3 = ..Q_n \qquad [6]$$

For pipes in parallel, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

$$Q_A = Q_1 + Q_2 = Q_B \qquad [7]$$

$$h_{L1} = h_{L2} \qquad [8]$$

II. The Problem

Water supply networks consistof a of sources, pipe loops (M. Tabesh,2001) in this case study design a water network from node No1 which is the upstream to node No 13 which is the downstream by gravity main as shown figure (1). The dimensions of the network are listed inTables 1and2. The network covers an area of 3.78 kilometers square, consisted of nine loops (24 pipes, main lines and minor lines) what's more, the outside border of the network considered as the main lines, and the inner lines considered as minor line of the network. Furthermore, this network included of 16 nodes, the first node considered the upstream (with neglectedminor losses) (Swamee&Sharma,2008).

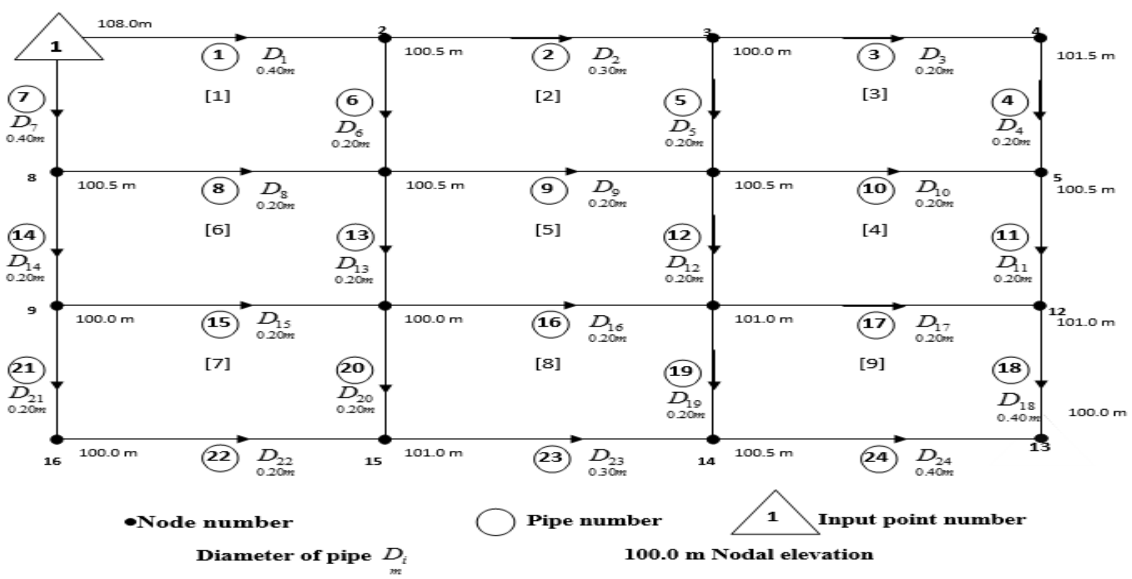

Fig 1: Gravity main looped network of 24 pipes

Table 1: The dimensions of the network

No. of pipe	1	2	3	4	5	6	7	8
Diameter (m)	0.4	0.3	0.2	0.2	0.2	0.2	0.4	0.2
Length (m)	800	800	800	800	600	600	600	600
No. of pipe	9	10	11	12	13	14	15	16
Diameter (m)	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Length (m)	600	600	600	600	600	600	600	600
No. of pipe	17	18	19	20	21	22	23	24
Diameter (m)	0.2	0.4	0.2	0.2	0.2	0.2	0.3	0.4
Length (m)	600	600	600	600	600	600	600	600

Table 2: The elevation of each node

No. of node	1	2	3	4	5	6	7	8
Height (m)	108	100.5	101	100.5	100.5	100.5	100.5	100.5
No. of node	9	10	11	12	13	14	15	16
Height (m)	100	100	101	101	100	100.5	101	100

III. Numerical Solution Of Nonlinear System Of Equations

One of the most common important steps in water resources engineering is pipe network analysis , the key methods for this analysis are Hard Darcy and Newton-Raphson (I.A. Oke;2007).

IV. The Solution By Using Newton Raphson

4.1 The assumption

All the discharges can be assumed for one value or different values as shown in table 3(Moosavian& Jaefarzadeh, 2014).. Therefore, in Newton Raphson not necessary to assume an initial guesses that satisfies the continuity equations as shown in table 5.1.

Table 3: initial guesses of Newton Raphson method

Pipe discharges	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
The assumed values	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Pipe discharges	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
The assumed values	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Pipe discharges	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
The assumed values	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05

4.2 The equations of Newton Raphson method

❖ **The discharge equation of each node**

We know the summation of inflow and out flow at node should be equal zero, therefore:

$$F_1 = Q_1 + Q_7 - 0.15$$ [16]

$$F_2 = Q_4 - Q_3$$

[17]

$$F_3 = Q_{22} - Q_{21}$$

[18]

$$F_4 = Q_2 + Q_6 - Q_1$$

$$F_5 = Q_3 + Q_5 - Q_2$$

[19]

$$F_6 = Q_8 + Q_{14} - Q_7$$

[20]

$$F_7 = Q_{15} + Q_{21} - Q_{14}$$

$$F_8 = Q_9 + Q_{13} - Q_6 - Q_8$$ [21]

$$F_9 = Q_{16} + Q_{20} - Q_{15} - Q_{13}$$ [15]

$$F_{10} = Q_{18} - Q_{17} - Q_{11}$$

$$F_{11} = Q_{10} + Q_{12} - Q_5 - Q_9$$

$$F_{12} = Q_{23} - Q_{22} - Q_{20}$$

$$F_{13} = Q_{19} + Q_{17} - Q_{12} - Q_{16}$$

$$F_{14} = Q_{24} - Q_{19} - Q_{23}$$

$$F_{15} = Q_{11} - Q_{10} - Q_{4}$$

❖ **The head losses equation of each loop**

By using the basic of fluid mechanics, the sum of losses inside each loop should be equal zero, therefore:

$$F_{16} = h_{L1} + h_{L6} - h_{L8} - h_{L7} \quad [24]$$

$$F_{17} = h_{L2} + h_{L5} - h_{L9} - h_{L6} \quad [25]$$

$$F_{18} = h_{L3} + h_{L4} - h_{L10} - h_{L5} \quad [26]$$

$$F_{19} = h_{L8} + h_{L13} - h_{L15} - h_{L14} \quad [27]$$

$$F_{20} = h_{L9} + h_{L12} - h_{L13} - h_{L16} \quad [22]$$

$$F_{21} = h_{L10} + h_{L11} - h_{L17} - h_{L12} \quad [23]$$

$$F_{22} = h_{L15} + h_{L20} - h_{L22} - h_{L21} \quad [30]$$

$$F_{23} = h_{L16} + h_{L19} - h_{L23} - h_{L20} \quad [31]$$

$$F_{24} = h_{L17} + h_{L18} - h_{L24} - h_{L19} \quad [32]$$

Where $$h_L = \frac{8 f L Q^2}{\pi^2 g D^5} \quad [33]$$

4.3 Finding the jacobian

The equation (34) has the jacobian matrix which can be found as follow.

$$\left[\left(\frac{\partial f_{24*24}}{\partial Q} \right]^{-1}_{(Q_1, Q_2, Q_3 \ldots\ldots\ldots Q_{24})} \begin{bmatrix} \frac{\partial f_1}{\partial Q_1} & \frac{\partial f_1}{\partial Q_2} \cdots\cdots\cdots & \frac{\partial f_1}{\partial Q_{24}} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_{24}}{\partial Q_1} & \frac{\partial f_{24}}{\partial Q_2} \cdots\cdots\cdots & \frac{\partial f_{24}}{\partial Q_{24}} \end{bmatrix} \quad [34]$$

$$J^{-1}(1,1) = \frac{\partial f_1}{\partial Q_1} = \frac{\partial (Q_1 + Q_7 - 0.6)}{\partial Q_1} = 1$$

$$J^{-1}(24,24) = \frac{\partial f_{24}}{\partial Q_{24}} = \frac{\partial (h_{L17} + h_{L18} - h_{L24} - h_{L19})}{\partial Q_{24}} = -56$$

4.4 The final matrix

The next matrix shows the calculation of the first iteration of each loop

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_3 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{14} \\ Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \\ Q_{20} \\ Q_{21} \\ Q_{22} \\ Q_{23} \\ Q_{24} \end{bmatrix}_{k+1} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_3 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_1 \\ Q_{14} \\ Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \\ Q_{20} \\ Q_{21} \\ Q_{22} \\ Q_{23} \\ Q_{24} \end{bmatrix}_{k} - \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
26 & 0 & 0 & 0 & 0 & 254 & -13 & -441 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 74 & 0 & 0 & 450 & -443 & 0 & 0 & -454 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 618 & 618 & -450 & 0 & 0 & 0 & 0 & -414 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 441 & 0 & 0 & 0 & 0 & 432 & -413 & -480 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 454 & 0 & 0 & 434 & -432 & 0 & 0 & -455 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 481 & 414 & -433 & 0 & 0 & 0 & 0 & -433 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 480 & 0 & 0 & 0 & 0 & 448 & 0 & -462 & -462 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 455 & 0 & 0 & 443 & -448 & 0 & 0 & -60 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 442 & 13 & -443 & 0 & 0 & 0 & 0 & -12
\end{bmatrix}^{-1} \cdot \begin{bmatrix} -0.05 \\ 0 \\ 0 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0.05 \\ 0 \\ 0 \\ -0.05 \\ 0 \\ -0.05 \\ 0 \\ -0.05 \\ 0.05 \\ 0.16 \\ -9 \\ 8 \\ -0.5 \\ -0.008 \\ 0.5 \\ 0.1 \\ 9 \\ -0.01 \end{bmatrix}$$

4.5The result of the pipe discharges of the first iteration

Table4: The pipe discharge for the first iteration

Pipe discharge	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Values	0.0787	0.0510	0.0212	0.0212	0.0299	0.0277	0.0713	0.0293
Discharges	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
Values	0.0227	0.0189	0.0401	0.0336	0.0343	0.0420	0.0176	0.0219
Pipe discharge	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
Values	0.0283	0.0683	0.0273	0.0300	0.0244	0.0244	0.0544	0.0817

4.6 The pipe discharges and velocities of the last iteration

The correct discharges and velocities can be got after 11 iteration (MATLAB code by using Newton Raphson method see App A), showed in table 5 and 6.

Table 5: The pipe dischargesfor the last iteration

Pipe discharges	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Values	0.0786	0.0510	0.0212	0.0212	0.0298	0.0277	0.0714	0.0293
Pipe discharges	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
Values	0.0227	0.0189	0.0401	0.0337	0.0343	0.0420	0.0176	0.0218
Pipe discharges	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
Values	0.0283	0.0683	0.0272	0.0300	0.0245	0.0245	0.0544	0.0817

In addition, by apply the equation [4] $Q = vA_C$ we get the following velocities:

Table 6: The pipe velocitiesfor the last iteration

Pipe velocities	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
Values	0.6259	0.7213	0.6739	0.6739	0.9491	0.8805	0.5678	0.9334
Pipe velocities	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
Values	0.7232	0.6012	1.2751	1.0711	1.0906	1.3378	0.5595	0.6954
Pipe velocities	v_{17}	v_{18}	v_{19}	v_{20}	v_{21}	v_{22}	v_{23}	v_{24}
Values	0.8994	0.5436	0.8672	0.9547	0.7783	0.7783	0.7702	0.6500

4.7 The accuracy of first iteration solution

In fluid mechanics basics, the algebraic sum of the head losses around a loop must be zero which is not shown in tables 7 and 8.

Table 7: The losses of each pipe for the first iteration

No. losses	h_{L1}	h_{L2}	h_{L3}	h_{L4}	h_{L5}	h_{L6}	h_{L7}	h_{L8}
Values	1.0055	1.7795	3.2227	3.2227	3.1226	3.6390	0.6589	3.8234
No. losses	h_{L9}	h_{L10}	h_{L11}	h_{L12}	h_{L13}	h_{L14}	h_{L15}	h_{L16}
Values	2.9242	1.7850	7.3278	4.5572	4.6284	7.5305	1.8169	2.8668
No. losses	h_{L17}	h_{L18}	h_{L19}	h_{L20}	h_{L21}	h_{L22}	h_{23}	h_{L24}
Values	3.6964	0.6398	3.6364	3.2845	2.5007	2.5007	1.3932	0.7747

Table 8: The summation of losses in each loop for the first iteration

Loop number	F_{16} (loop1)	F_{17} (loop2)	F_{18} (loop3)	F_{19} (loop4)	F_{20} (loop5)
Summation of head	0.1622	-1.6611	1.5378	-0.8956	-0.0138
Loop number	F_{21} (loop6)	F_{22} (loop7)	F_{23} (loop8)	F_{24} (loop9)	-
Summation of head	0.8592	0.1000	1.8255	-0.0748	-

4.8 The accuracy of last iteration solution

By using newton Raphson methods and using MATLAB code we got the sum of head loss around each loop is zero as shown in tables 9 and 10.

Table 9: The losses of each pipe for the last iteration

No. losses	h_{L1}	h_{L2}	h_{L3}	h_{L4}	h_{L5}	h_{L6}	h_{L7}	h_{L8}
Values	1.0248	1.9004	2.8160	2.8160	3.9089	3.4130	0.6454	3.7925
No. losses	h_{L9}	h_{L10}	h_{L11}	h_{L12}	h_{L13}	h_{L14}	h_{L15}	h_{L16}
Values	2.3963	1.7232	6.6913	4.8680	5.0304	7.3063	1.5165	2.2339
No. losses	h_{L17}	h_{L18}	h_{L19}	h_{L20}	h_{L21}	h_{L22}	h_{23}	h_{L24}
Values	3.5464	0.5970	3.3206	3.9505	2.7335	2.7335	1.6040	0.8228

Table 10: The summation of losses in each loop for the last iteration

Loop number	F_{16} (loop1)	F_{17} (loop2)	F_{18} (loop3)	F_{19} (loop4)	F_{20} (loop5)
Summation of head	0	0	0	0	0
Loop number	F_{21} (loop6)	F_{22} (loop7)	F_{23} (loop8)	F_{24} (loop9)	-
Summation of head	0	0	0	0	-

V. The Solution By Using Hard Darcy

The overall procedure for the looped network analysis can be summarized in the following steps:

1. Number all the node and pipe links, Also number the loops, for clarity, pipe numbers are circled and the loop numbers are put in square brackets.
2. Adopt a sign convention that a pipe discharge is positive if it flows from a lower node number the higher node number, otherwise negative.
3. Apply nodal continuity equation at all nodes to obtain pipe discharge .starting from nodes having two pipes with unknown discharge, assume an arbitrary discharge (say $0.1m^3/s$) in one of the pipes and apply continuityto obtain discharge in the other pipe. Repeat the procedure until all the pipe flows are known .if there exist more than two pipes having unknown discharges, assume arbitrary discharges in all the pipe except one and apply continuity equation to get discharge in the other pipe. The total number of primary loops in the network.
4. Assume friction factors $f_i = 0.02$ in all pipes links and compute corresponding K_i
5. Assume loop pipe flow sign convention to apply loop discharge corrections; generally, clockwise flows positive and counterclockwise flows negative are considered.
6. Calculate ΔQ_k for the existing pipe flows and apply pipe corrections algebraically.
7. Apply the similar procedure in all the loops of a pipe network.

Repeat steps 6 and 7 until the discharge corrections in all the loops are relatively very small (Swamee& Sharma,2008).

5.1 The assumption

The initial discharges should satisfy continuity equation at each node as table 11 (Moosavian& Jaefarzadeh,2014). Also, the number of assumed discharge should be equaled to the number of loops which is nine.

Table 11: The assumed initial guesses for the first iteration

Pipe discharge	Q_1	Q_2	Q_3	Q_8	Q_9	Q_{10}	Q_{15}	Q_{16}	Q_{17}
The assumed value	0.10	0.03	0.02	0.02	0.02	0.01	0.02	0.02	0.02

Then the rest of the discharge of the first iteration are listed in table 12.

Table 5.12: The discharge obtained from continuity equation

Pipe discharge	Q_4	Q_5	Q_6	Q_7	Q_{11}	Q_{12}	Q_{13}	Q_{14}
The assumed values	0.02	0.03	0.07	0.05	0.01	0.04	0.07	0.03
Pipe discharge	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}	-
The assumed values	0.03	0.04	0.07	0.01	0.01	0.08	0.12	-

5.2 The equation of Hard Darcy method

❖ **The discharge equation of each node**

We know the summation of inflow and out flow at node should be equal zero, therefore:

$F_1 = Q_1 + Q_7 - 0.15$ [35]

$F_2 = Q_4 - Q_3$ [36]

$F_3 = Q_{22} - Q_{21}$ [36]

$F_4 = Q_2 + Q_6 - Q_1$ [37]

$F_5 = Q_3 + Q_5 - Q_2$ [38]

$F_6 = Q_8 + Q_{14} - Q_7$ [39]

$F_7 = Q_{15} + Q_{21} - Q_{14}$ [40]

$F_8 = Q_9 + Q_{13} - Q_6 - Q_8$ [41]

$F_9 = Q_{16} + Q_{20} - Q_{15} - Q_{13}$ [42]

$F_{10} = Q_{18} - Q_{17} - Q_{11}$ [43]

$F_{11} = Q_{10} + Q_{12} - Q_5 - Q_9$ [44]

$F_{12} = Q_{23} - Q_{22} - Q_{20}$ [45]

$F_{13} = Q_{19} + Q_{17} - Q_{12} - Q_{16}$ [46]

$F_{14} = Q_{24} - Q_{19} - Q_{23}$ [47]

$F_{15} = Q_{11} - Q_{10} - Q_4$ [48]

❖ **The loss equation**

The algebraic sum of the head loss in a loop must be equal to zero

$\sum_{loop,k} k_i Q_i |Q_i| = 0$ for all loops $k = 1,2,3,\ldots,k_L$ [49]

Where $K_i = \dfrac{8 f_i L_i}{\pi^2 g D_i^5}$

5.3 The first iteration of Hard Darcy method

Table 13 to table 21 show the calculation of the first iteration of each loop.

Where:

$$\Delta Q_k = -\frac{\sum_{loop,k} K_i Q_i |Q_i|}{2\sum_{loop,k} K_i |Q_i|} \quad [50]$$

Pipe	Discharge (m ^{3}/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
1	0.1	129.1045	1.2910	25.8209	0.0747
6	0.07	3098.5	15.1827	433.7910	0.0447
7	-0.05	96.8283	-0.2421	9.6828	-0.0753
8	-0.02	3098.5	-1.2394	123.9403	-0.0453
Total			14.9923	593.2350	
ΔQ			-0.0253		

Table 13: loop 1

Table 14: loop 2

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
2	0.03	544.0452	0.4896	32.6427	0.0434
5	0.03	3098.5	0.3099	61.9701	0.0234
6	-0.0447	3098.5	-6.1988	277.1798	-0.0313
9	-0.02	3098.5	-1.2394	123.9403	-0.0066
Total			-6.6388	495.7330	
ΔQ			0.0134		

Table 15: loop 3

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
3	0.02	4131.3	1.6525	165.2537	0.0176
4	0.02	4131.3	1.6525	165.2537	0.0176
5	-0.0234	3098.5	-1.6954	144.9593	-0.0258
10	-0.01	3098.5	-0.3099	61.9701	-0.0124
Total			1.2998	537.4369	
ΔQ			-0.0024		

Table 16: loop 4

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
10	0.0124	3098.5	0.4778	76.9577	0.0109
11	0.01	3098.5	2.7887	185.9104	0.0285
12	-0.04	3098.5	-1.2394	123.9403	-0.0215
17	-0.02	3098.5	-1.2394	123.9403	-0.0215
Total			0.7877	510.7487	
ΔQ			-0.0015		

Table 17 : loop 5

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
9	0.0066	3098.5	0.1353	40.9511	0.0269
12	0.0215	3098.5	1.4379	133.4976	0.0418
13	-0.07	3098.5	-15.1827	433.7910	-0.0497
16	-0.02	3098.5	-1.2394	123.9403	0.00028
Total			-14.8489	732.1800	
ΔQ			0.0203		

Table 18: loop 6

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
8	0.0453	3098.5	6.3506	280.5514	0.0342
13	0.0497	3098.5	7.6596	308.1134	0.0386
14	-0.03	3098.5	-2.7887	185.9104	-0.0411
15	-0.02	3098.5	-1.2394	123.9403	-0.0311
Total			9.9822	898.5156	
ΔQ			-0.0111		

Table 19: loop7

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
15	0.0311	3098.5	2.9988	192.7867	0.0077
20	0.07	3098.5	15.1827	433.7910	0.0466
21	-0.01	3098.5	-0.3099	61.9701	-0.0334
22	-0.01	3098.5	-0.3099	61.9701	-0.0334
Total			17.5617	750.5180	
ΔQ			-0.0234		

Table 20: loop 8

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
16	0.00028	3098.5	0.000243	1.7373	0.0172
19	0.04	3098.5	1.2394	123.9403	0.0369
20	-0.0334	3098.5	-6.7287	288.7840	-0.0297
23	-0.08	408.0339	-2.6114	65.2854	-0.0631
Total			-8.1005	479.7470	
ΔQ			0.0169		

Table 21: loop 9

Pipe	Discharge (m^3/s)	K (s^2/m^5)	$KQ\|Q\|$ (m)	$2K\|Q\|$ (s/m^2)	Corrected Flow $Q = Q + \Delta Q$ (m^3/s)
17	0.0215	3098.5	1.4379	133.4976	0.0305
18	0.03	96.8283	0.2421	9.6828	0.0590
19	-0.0369	3098.5	-4.2155	228.5766	-0.0279
24	-0.12	96.8283	-0.9683	19.3657	-0.0910
Total			-3.5038	391.1228	
ΔQ			0.0090		

5.4 The pipe discharges for the first iteration

The discharges of the first iteration are shown in table 22 by (MATLAB code).

Table 22: The pipe discharge of the first iteration

Pipe discharge	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Values	0.0747	0.0367	0.0231	0.0231	0.0336	0.0380	0.0753	0.0329
Pipe discharge	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
Values	0.0289	0.0262	0.0193	0.0463	0.0420	0.0424	0.0089	0.0112
Pipe discharge	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
Values	0.0308	0.0501	0.0267	0.0397	0.0335	0.0335	0.0732	0.0999

5.5 The pipe discharges and velocities of the last iteration

The correct discharges and velocities can be got after many number of iteration (MATLAB code by using Hard Darcy method), showed in tables23 and 24.

Table 23: The pipe discharges for the last iteration

Pipe discharge	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8
Values	0.0786	0.0507	0.0213	0.0213	0.0294	0.0279	0.0714	0.0296
Pipe discharge	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}
Values	0.0232	0.0187	0.0401	0.0339	0.0343	0.0418	0.0174	0.0226
Pipe discharge	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_{21}	Q_{22}	Q_{23}	Q_{24}
Values	0.0285	0.0686	0.0274	0.0297	0.0243	0.0243	0.0540	0.0814

In addition, by apply the equation [4] $Q = vA_C$ we get the following velocities:

Table 24: The pipe velocities for the last iteration

Velocities	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
Values	0.6257	0.7179	0.6796	0.6796	0.9357	0.8875	0.5680	0.9420
Velocities	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
Values	0.7389	0.5966	1.2762	1.0779	1.0907	1.3299	0.5554	0.7199
Velocities	v_{17}	v_{18}	v_{19}	v_{20}	v_{21}	v_{22}	v_{23}	v_{24}
Values	0.9071	0.5458	0.8729	0.9970	0.7745	0.7745	0.7638	0.6478

5.6 The accuracy of first iteration solution

The solution that showed above only for the first iteration, which is not correct. The next test, shows that ΔQ are not equal to zero which is not correct as shown in table 25

Table 25: The correction factor in each loop of the first iteration

No. loop	1	2	3	4	5	6	7	8	9
ΔQ	-0.0253	0.0067	0.0031	0.0093	0.0156	-0.0124	-0.0235	0.0068	0.0201

5.7 The accuracy of last iteration solution

Hard Darcy method by using MATLAB code was run to get the next results as a proof of the accuracy of the solution of the discharges as shown in table 26

Table 26: The correction factor in each loop of the last iteration

No. loop	1	2	3	4	5	6	7	8	9
ΔQ	0	0	0	0	0	0	0	0	0

VI. Flow Rate Comparison

The differences between the discharges obtained by Newton Raphson and Hard Darcy method are approximately zero as shown in fig (2)

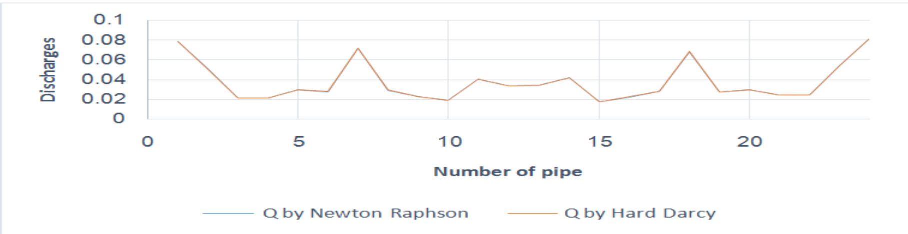

Fig 2 : Flow rates obtained by Newton Raphson and Hard Darcy methods with number pi

VII. The Number Of Iteration With The Summation Of Head Losses In Each Loop For Newton Raphson Method

The correct flow rates by Newton Raphson method were got after 3 iteration as shown fig (3).

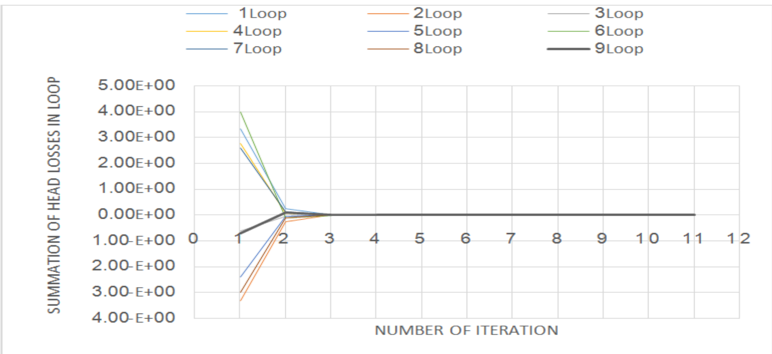

Fig 3: The relationship between the numbers of iteration with the summation of head losses equations in each loop for Newton Raphson method

VIII. The Number Of Iteration With The Summation Of Head Losses In Each Loop For Hard Darcy Method

The correct flow rates by Hard Darcy method were got after 20 iteration as shown fig (4).

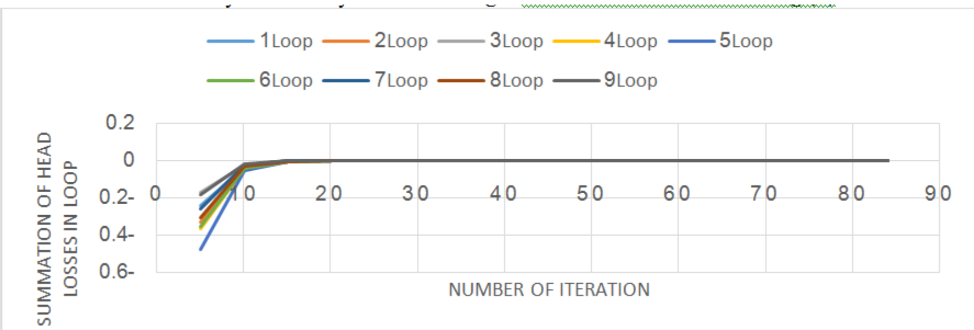

Fig 4: The relationship between the numbers of iteration with the summation of head losses equations in each loop for Hard Darcy method

IX. Comparison Between The Summation Of Head Losses Equations By Newton Raphson And Hard Darcy

The next table shows the summation of the head loss equation in each loop that must be approximately zero, which can be seen that newton Raphson is faster than hard Darcy to converge to the solution.

Table 27: Thesummation head losses equations by Newton Raphson and Hard Darcy methods

Loop number	The summation of head losses (Newton Raphson) after 11 iteration. (m)	The summation of head losses (Hard Darcy) after 84 iteration.(m)
1	-5.5511e-016	1.1102e-016
2	0	-4.4409e-016
3	-8.8818e-016	0
4	-1.7764e-015	-8.8818e-016
5	8.8818e-016	-1.3323e-015
6	0	-8.8818e-016
7	0	-4.4409e-016
8	0	-8.8818e-016
9	-1.8127e-006	-4.4409e-016

X. Conclusion

A nonlinear systems network were simulated by Newton Raphson and Hard Darcy methods using MATLAB software. The nonlinearity is showed in the square power of the discharge in head losses equations. The discharges resulted of each pipe were found the same in each method. Also, the final solution was validated by using the basic of fluid mechanics which that the summation of losses inside a loop must be equal to zero. Thus numerically, in Newton Raphson, which summation has a high accuracy and approximately zero compared to Hard Darcy method. Also, the solution in Newton Raphson method can be got at less number of iterations (faster) compared to Hard Darcy method. In addition, initial guesses (the assumption) is more complicated in Hard Darcy because the value of each discharge must satisfy the continuity equations which need more calculations. However, the initial guesses can be chosen randomly in Newton Raphson method without satisfying the continuity equations.

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