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Predicting Static Modulus of Elasticity of Laterite - Quarry Dust Blocks Using Osadebe's Regression Model

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Abstract: *The use of alternative materials other than river sand in the production of sandcrete blocks is gaining impetus in most major Nigerian cities. Manufacturers of these blocks are, however, faced with the problem of mix proportion required for the production of the desired property. Static modulus of elasticity of block is one key property required in the design of structures, which is usually neglected. In this work, a mathematical model is formulated using Osadebe's regression theory for predicting the static modulus of elasticity of laterite-quarry dust block. The model is tested for lack of fit using software and found adequate.*
Keywords: *Static modulus of elasticity, laterite, quarry dust, Osadebe regression model, mix proportion, Fisher Test.*

I. Introduction

Walls or masonry units are the vertical members of a building or structure which enclose the space within it and which may also divide that space. Sandcrete blocks are still the most popularly and commonly used material for the construction of walls unit in all developing countries including Nigeria. They are prismatic precast units made from a combination or mix of well defined proportion of sand and cement. Sandcrete blocks according to [1], are blocks made or moulded with sand, water and cement, which serve as a binder in the matrix. As a result of the high cost and negative environmental impact caused by sand mining, one of the major constituents used in the production of blocks, alternative materials are being sought for and utilized in block production. Such alternative materials currently being used to partially or wholly replaced river sand includes: quarry dust, laterite, recycled aggregate, etc. The use of these alternative materials in blocks production and concrete works have been reported by several researchers. For instance, [2] carried out a study on the strength and durability of concrete utilizing quarry dust as full replacement for natural sand. They found a 10% improvement in the properties investigated over the conventional concrete made with natural sand. Also,[3] in their work on effects of granite fines on some engineering properties of sandcrete blocks recommended 15% optimum replacement of sand with quarry dust in the production of sandcrete blocks. Also, the effect of partial replacement of sand with lateritic soil in sandcrete blocks was investigated by [4]. The study reveals that sand can be replaced up to 20% with laterite in sandcrete blocks. While most of the studies focus on some strength properties like compressive strength, properties like static modulus of elasticity (E_c) for blocks is scarcely documented. Knowledge of the value of static modulus of material is needed in structural design to avoid unrealistic assumptions.

There exist also the challenge of getting the appropriate mix proportion to attain a particular desired property as several methods of mix proportions have limitations and are usually not cost effective. Thus, researchers have worked on developing models for predicting properties of sand quarry dust and sand laterite block. A model for static modulus of elasticity for sand- quarry dust blocks was developed by [5]. Similarly, [6] developed a model for predicting the compressive strength and water absorption of sand - quarry dust blocks. Others works in this direction include those by [7] and [8]. This paper presents a model for predicting the static modulus of elasticity for laterite – quarry dust blocks. The model will help come up with the appropriate mix for laterite – quarry dusts blocks as well as help reduce the cost and effort expended in conducting trial mix.

II. Materials And Method

The materials used for this work are:

Cement

Unicem brand of Ordinary Portland cement, grade 32.5 obtained from a major dealer in Calabar conforming to BS 12 was used for all the tests.

Water

Potable pipe born water supplied by the Cross River State Water Board (CRSWB) Limited was used for both specimen preparations and curing.

Laterite
Laterite was obtained from a borrow pit site at Akim - Akim in Odukpani Local Government Area of Cross River State. The specific gravity for the laterite is 2.56.
Quarry dust
Quarry dust was obtained from the abundant deposits at Akamkpa quarry site in Akamkpa Local Government area of Cross River State; located at a few minutes' drive from Calabar Metropolis. The quarry dust had a specific gravity of 2.52.

III. Method

This study employs two methods: analytical and experimental. The analytical method deals with the arrangement of points within the experimental region and selection of a second degree polynomial equation to represent the response surface over the entire region. The response in this case is the static modulus of elasticity of the laterite-quarry dust blocks. The response function is assumed to be multi-varied.
The response y is expressed as a function of the actual proportions of the constituents of the mixture, Z_i by [9]. The sum of all the proportions as in all mixture experiments must add up to 1. That is:

$$Z_1 + Z_2 + \cdots + Z_q = \sum_{I=1}^{q} Z_i = 1 \quad (1)$$

He assumed that $y = F(Z)$, is continuous and differentiable with respect to its predictors, and can be expanded in the neighbourhood of a chosen point, *Z(0)* using Taylor's series.

$$Z(0) = \left(Z_1^{(0)}, Z_2^{(0)}, \dots, Z_q^{(0)}\right)^T \quad (2)$$

Thus,

$$y(Z) = F\left(Z^{(0)}\right) + \sum_{i=1}^{q} \frac{\partial f\left(Z^{(0)}\right)}{\partial Z_i}\left(Z_i - Z^{(0)}\right) + \frac{1}{2!}\sum_{i=1}^{q-1}\sum_{j=1}^{q} \frac{\partial^2 f(Z^0)}{\partial Z_i \partial Z_j}\left(Z_i - Z_i^{(0)}\right)\left(Z_j - Z_j^{(0)}\right) + \frac{1}{2!}\sum_{i=1}^{q} \frac{\partial^2 f\left(Z^{(0)}\right)}{\partial Z_i^2}\left(Z_i - Z^{(0)}\right) \dots \quad (3)$$

For convenience, the point Z^0 can be taken as the origin without loss in generality of the formulation.Thus

$$Z_1^{(0)} = 0,\ Z_2^{(0)} = 0, \dots, Z_q^{(0)} = 0 \quad (4)$$

Let

$$b_0 = F(0), \quad b_i = \frac{\partial F(0)}{\partial Z_i}, \quad b_{ij} = \frac{\partial^2 F(0)}{\partial Z_i \partial Z_j}, \quad b_{ii} = \frac{\partial^2 F(0)}{\partial Z_i^2} \quad (5)$$

Substituting Equation (2.41 5) into Equation (2.39 3) gives:

$$y(Z) = b_0 + \sum_{i=1}^{q} b_i Z_i + \sum_{i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i=1}^{q} b_{ii} Z_i^2 \quad (6)$$

The number of terms in Equation (6) is $C_n^{(q+n)}$
Multiplying Equation (1) by b_0 gives the expression:

$$b_0 = b_0 Z_1 + b_0 Z_2 + \cdots \dots \dots + b_0 Z_q \quad (7)$$

Multiplying Equation (1) successively by $Z_1, Z_2 \dots Z_q$ and rearranging, gives respectively:

$$Z_1^{\ 2} = Z_1 - Z_1 Z_2 - \dots \dots \dots \dots . - Z_1 Z_q$$
$$Z_2^{\ 2} = Z_2 - Z_1 Z_2 - \quad \dots \dots \dots . - Z_2 Z_q$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$Z_q^{\ 2} = Z_1 - Z_1 Z_q - \dots \dots \dots \dots . - Z_{(q-1)} Z_q \quad (8)$$

Substituting Equations (7) and (8) into Equation (6) and simplifying yields Equation (2.459)

$$y(Z) = \sum_{i=1}^{q} \beta_i Z_i + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i Z_j \quad (9)$$

Where

$$\beta_i = b_0 + b_i \dots \dots + b_{ii} \quad (10)$$
$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \quad (11)$$

Equation (9) is Osadebe's regression model equation. It is defined if the unknown constant coefficients β_i and β_{ij} are uniquely determined.
If the number of constituents, *q*, is 4, and the degree of the polynomial *n*, is 2 then Osadebe's regression equation is given as:

$$y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{23} Z_2 Z_3 + \beta_{24} Z_2 Z_4 + \beta_{34} Z_3 Z_4 \quad (12)$$

The number of coefficients, N is the same as that for the Scheffe's {4, 2} model as provided by Equation 12. That is:

$$N = C_n^{(q+n-1)} = N = C_2^{(4+2-1)} = 10$$

The response function is generally expressed as:

$$y(Z) = \sum_{i=1}^{q} \beta_i Z_i + \sum_{i \leq j \leq q}^{q} \beta_{ij} Z_i Z_j \quad (13)$$

IV. Determination of the coefficients of the Osadebe's regression equation

The least number of experimental runs or independent responses necessary to determine the coefficients of Osadebe's regression coefficients is N.

Let $y^{(k)}$ be the response at point k and the vector corresponding to the set of component proportions (predictors) at point k be $Z^{(k)}$. That is:

$$Z^{(k)} = \{ Z_1^{(k)}, Z_2^{(k)}, \ldots, Z_q^{(k)} \} \quad (14)$$

Substituting the vector of Equation (3) into Equation (12) gives:

$$y^{(k)} = \sum_{i=1}^{q} \beta_i Z_i^{(k)} + \sum_{i \leq j \leq q}^{q} \beta_{ij} Z_i^{(k)} Z_j^{(k)} \qquad k = 1,2, \ldots, N \quad (15)$$

Substituting the predictor vectors at each of the N observation points successively into Equation (9) gives a set of N linear algebraic equations which can be written in matrix form as:

$$\boldsymbol{Z\beta} = \boldsymbol{y} \quad (16)$$

Where

$\boldsymbol{\beta}$ is a vector whose elements are the estimates of the regression coefficients.

$\boldsymbol{Z}$ is an $N \times N$ matrix whose elements are the mixture component proportions and functions of the component proportions.

$\boldsymbol{y}$ is a vector of the observations or responses at the various N observation points.

The solution to Equation (16) is given as:

$$\boldsymbol{\beta} = \boldsymbol{Z}^{-1}\boldsymbol{y} \quad (17)$$

V. Experimental method

The pseudo proportion units were converted to actual mix proportion as indicated in table 1. The actual mix proportions; water (Z1), cement (Z2), quarry dust (Z3), and laterite (Z4), were measured by weight and used to produce machine vibrated laterite- quarry dust hollow blocks of size 450mm x 150mm x 225mm. The blocks were cured for 28 days after 24 hours of demoulding by sprinkling with water in the morning and evening. They were tested for compressive strength using the universal compression testing machine, in accordance with BS EN 12390-4. The crushing load and net cross sectional area of the blocks were recorded. The compressive strength was obtained from the following relation:

$$fc = P/A \quad (18)$$

Where fc = the compressive strength, P = crushing load, and A = cross-sectional area of the specimen

The Static modulus of elasticity for the block was computed as a function of compressive strength and density using the relation established by [10] represented as:

$$Ec = 1.7\rho^2 fc^{0.33} * 10^{-6} \quad (19)$$

Where,

Ec = Static modulus of Elasticity, ρ = density and

fc = compressive strength

Table 1 below shows the design matrix in the pseudo and real ratios along with the experimental test results for the compressive strength and static modulus of elasticity.

Table 1: Experimental test results

S/N	Pseudo components units				Actual mix ratio				Average response (y)	
	Water (X1)	Cement (X2)	Quarry dust (X3)	Laterite (X4)	Water (X1)	Cement (X2)	Quarry dust (X3)	Laterite (X4)	Compressive strength, f_c (Nmm^{-2})	Static modulus of Elasticity *(GPa)*
1	0	1	0	0	0.63	1	3.0	3.0	1.87	6.9476
2	0.25	0.25	0.25	0.25	0.72	1	5.6	2.4	2.56	8.287
3	1	0	0	0	0.54	1	5.4	0.6	2.50	8.8566

4	0.5	0	0.5	0	0.67	1	7.2	0.8	2.24	7.9930
5	0	0.5	0	0.5	0.77	1	4.0	4.0	2.37	8.2285
6	0.5	0	0	0.5	0.72	1	5.2	2.8	2.37	8.0286
7	0	0	1	0	0.80	1	9.0	1.0	1.81	7.9795
8	0.125	0.125	0.125	0.625	0.81	1	5.3	3.7	2.42	8.6075
9	0.5	0.5	0	0	0.58	1	4.2	1.8	2.56	8.3193
10	0	0	0.5	0.5	0.85	1	7.0	3.0	2.09	8.1875
11	0	1	0	0	0.63	1	3.0	3.0	1.89	7.0700
12	0	0	0	1	0.90	1	5.0	5.0	2.20	7.6358
13	1	0	0	0	0.54	1	5.4	0.6	2.45	8.4865
14	0	0.5	0.5	0	0.72	1	6.0	2.0	2.54	8.8676
15	0	0	0	1	0.90	1	5.0	5.0	2.20	8.2145
16	0.625	0.125	0.125	0.125	0.63	1	5.5	1.5	2.56	8.9084
17	0	0	1	0	0.80	1	9.0	1.0	1.90	8.0343
18	0.25	0.25	0.25	0.25	0.72	1	5.6	2.4	2.49	8.1386
19	0.125	0.625	0.125	0.125	0.67	1	4.3	2.7	2.50	8.9417
20	0.125	0.125	0.625	0.125	0.76	1	7.3	1.7	2.30	8.4656

A total of 15 mixes were considered. Out of this number,10 mixes were selected and used for the formulation of the model, while the remaining were used for validation of the model as shown in table 2.
The table also contained the average experimental values for compressive strength and static modulus of elasticity. Cells having two run order numbers indicate the replicate mixes and the response in this case is the average response for the replicate mixes.

Table 2: Actual and fractional mix for Osadebe

Run Order	Components in actual ratios				Component proportions				Responses	
	Water (X_1)	Cement (X_2)	Quarry dust (X_3)	Laterite X_4	Water (Z_1)	Cement (Z_2)	Quarry dust (Z_3)	Laterite (Z_4)	y_c (N/mm	Ec (GPa)
3,13	0.54	1	5.4	0.6	0.071618	0.132626	0.716180	0.079576	2.48	8.6716
9	0.585	1	4.2	1.8	0.077126	0.131839	0.553724	0.237310	2.56	8.3193
4	0.67	1	7.2	0.8	0.069286	0.103413	0.744571	0.082730	2.24	7.9930
6	0.72	1	5.2	2.8	0.074074	0.102881	0.534979	0.288066	2.37	8.0286
1,11	0.63	1	3	3	0.082569	0.131062	0.393185	0.393185	1.88	7.0088
14	0.72	1	6	2	0.074074	0.102881	0.617284	0.205761	2.54	8.8676
5	0.77	1	4	4	0.078813	0.102354	0.409417	0.409417	2.37	8.2285
7,17	0.8	1	9	1	0.067797	0.084746	0.762712	0.084746	1.86	8.0069
10	0.85	1	7	3	0.071730	0.084388	0.590717	0.253165	2.09	8.1875
12,15	0.63	1	5.5	1.5	0.073001	0.115875	0.637312	0.173812	2.20	7.9252
MIXES FOR MODEL VALIDATION										
2,18	0.72	1	5.6	2.4	0.074074	0.102881	0.576132	0.246914	2.53	8.2128
16	0.63	1	5.5	1.5	0.073001	0.115875	0.637312	0.173812	2.56	8.9084
19	0.674	1	4.3	2.7	0.077703	0.115287	0.495734	0.311275	2.50	8.9417
20	0.76	1	7.3	1.7	0.070632	0.092937	0.678439	0.157993	2.30	8.4656
8	0.81	1	5.3	3.7	0.074931	0.092507	0.490287	0.342276	2.42	8.6075

VI. Results And Discussion

The test results of the compressive strength of the laterite-quarry dust blocks based on 28-day strength and corresponding static modulus of elasticity are presented as part of Table 1.

Formulation of Model equation for static modulus of elasticity

The elements of the Z matrix with reference to equation 17 are as provided in Table 3

Table 3: Elements of the **Z** matrix for the Osadebe's model

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.071618	0.132626	0.71618	0.079576	0.009498	0.051291	0.005699	0.094984	0.010554	0.05699
0.077126	0.131839	0.553724	0.23731	0.010168	0.042707	0.018303	0.073003	0.031287	0.131405
0.069286	0.103413	0.744571	0.08273	0.007165	0.051589	0.005732	0.076998	0.008555	0.061598
0.074074	0.102881	0.534979	0.288066	0.007621	0.039628	0.021338	0.055039	0.029636	0.154109
0.082569	0.131062	0.393185	0.393185	0.010822	0.032465	0.032465	0.051531	0.051531	0.154594
0.074074	0.102881	0.617284	0.205761	0.007621	0.045725	0.015242	0.063507	0.021169	0.127013
0.078813	0.102354	0.409417	0.409417	0.008067	0.032267	0.032267	0.041905	0.041905	0.167622
0.067797	0.084746	0.762712	0.084746	0.005745	0.051709	0.005745	0.064637	0.007182	0.064637
0.07173	0.084388	0.590717	0.253165	0.006053	0.042372	0.018159	0.04985	0.021364	0.149549
0.07563	0.084034	0.420168	0.420168	0.006355	0.031777	0.031777	0.035308	0.035308	0.176541

Substituting the numerical average replicate values of static modulus of elasticity observed at the ten design points of the simplex into Equation (17) and solving simultaneously gives the following values of the coefficients:

β_1 = - 72079.9872 β_2 = 527.8603 β_3 = - 406.1002 β_4 = - 736.4554

β_{12} = 80706.0764, β_{13} = *83672.7022* β_{14} = 87627.9165 β_{23} = - 1058.1584

β_{24} = - 982.6645 β_{34} = 58.9751

The resulting second degree regression equation for Osadebe's model is given below:

$$\hat{y} = -72079.9872Z_1 + 527.8603Z_2 - 406.1002Z_3 - 736.4554Z_4 + 80706.0764Z_1Z_2 + 83672.7022Z_1Z_3 + 87627.9165Z_1Z_4 - 1058.1584Z_2Z_3 - 982.6645Z_2Z_4 + 58.9751Z_3Z_4 \quad (20)$$

Table 4 presents the experimental and predicted values of static modulus of elasticity of the laterite quarry dust blocks.

Table 4: Experimental and model predicted Static modulus of elasticity results

Run Order	Actual mix ratios				Experimental result (GPa)	Model predicted result (Osadebe) GPa
	Water	Cement	Quarry dust	Laterite		
1	0.63	1	3	3	6.9476	6.95
2	0.72	1	5.6	2.4	8.2870	8.55
3	0.54	1	5.4	0.6	8.8566	8.49
4	0.67	1	7.2	0.8	7.9930	7.99
5	0.77	1	4	4	8.2285	8.23
6	0.72	1	5.2	2.8	8.0286	8.03
7	0.8	1	9	1	7.9795	7.98
8	0.81	1	5.3	3.7	8.6075	8.18
9	0.585	1	4.2	1.8	8.3193	8.32
10	0.85	1	7	3	8.1875	8.19
11	0.63	1	3	3	7.0700	6.95
12	0.9	1	5	5	7.6358	7.64
13	0.54	1	5.4	0.6	8.4865	8.49
14	0.72	1	6	2	8.8676	8.87
15	0.9	1	5	5	8.2145	7.64
16	0.63	1	5.5	1.5	8.9084	8.41
17	0.8	1	9	1	8.0343	7.98
18	0.72	1	5.6	2.4	8.1386	8.55
19	0.674	1	4.3	2.7	8.9417	8.34
20	0.76	1	7.3	1.7	8.4656	8.41

Test for adequacy of the model

The model was tested for adequacy against the controlled experimental results. The hypotheses for the model are as follows:

Null Hypothesis (H_o): there is no significant difference between the experimental and the theoretical estimated results at a 95% confidence level

Alternative Hypothesis (H_1): There is a significant difference between the experimental and theoretically expected result at a 95% confidence level. The Fisher Test was used to test for the adequacy of the model. The analysis of variance for the Fisher test using [11] at the check point is as shown in Table 45 below. The calculated F from the table is 3.72 which is less than the critical (tabulated) F value of 5.05, justifying the adequacy of the model equation. Again the *p-value* of 0.088 which is greater than 0.05 further indicates the adequacy of the model.

Table 5: Analysis of Variance table for Static modulus of Elasticity (Osadebe's model)

Run order	*y(observed) (GPa)*	*y(predicted) (GPa)*		*y(observed) (GPa)*	*Y(predicted) (GPa)*
2	8.287	8.548	Mean	8.625167	8.359667
18	8.139	8.548	Variance	0.118078	0.031698
16	8.908	8.13	Observations	6	6
19	8.942	8.341	df	5	5

20	8.868	8.41	F	3.725114	
8	8.607	8.181	P(F<=f) one-tail	0.087631	
			F Critical one-tail	5.050329	

Normal probability plot

Figure 1shows normal probability plot. The points in figure lie very close to the reference line with a *p-value* of 0.145 which is greater than 0.05. The data therefore follow a normal distribution, thereby justifying the assumption required for use of analysis of variance

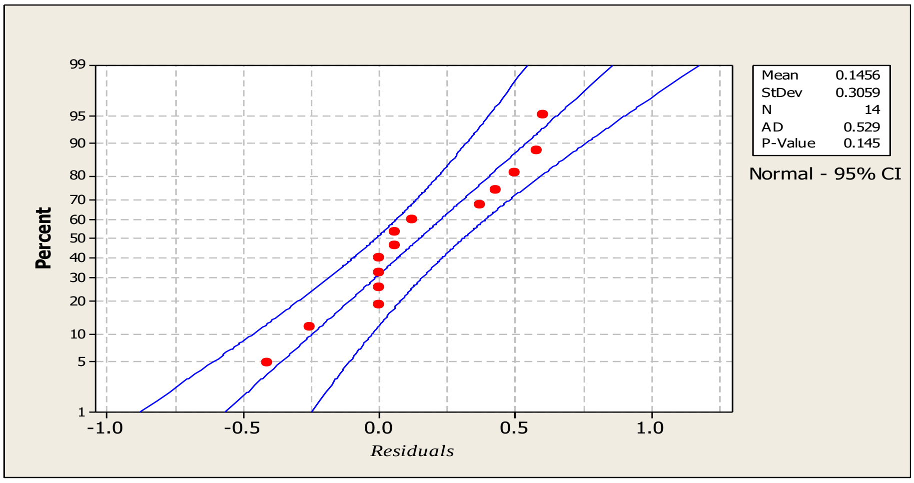

Figure 1: Normal probability plot for Static modulus of elasticity residuals Osadebe's model)

VII. Conclusion

A mathematical model for predicting the static modulus of elasticity of laterite-quarry dust block based on actual proportions using Osadebe's theory was formulated. The model was tested for lack of fit and was found to be adequate. There was no significant difference between the experimented and predicted values.

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