

Finite Element Analysis of Funicular Shells with Rectangular plan ratio 1:0.7 under Uniformly Distributed Load using SAP2000

Siddesh T M¹, Harish B A², Dr. K Manjunatha³

¹PG Student, Department of Civil Engineering, UBDTCE, Davanagere-577004

²Assistant Professor, Department of Civil Engineering, GMT, Davanagere-577006

³Professor and chairman, Department of Civil Engineering, UBDTCE, Davanagere-577004

Abstract: A thin shell is a “Three-dimensional spatial structure made up of one or more curved surfaces whose thickness is small compared to their other dimensions”. Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. The shells are subjected to pure membrane state of stress, under appropriate loading and boundary condition the resulting bending and twisting moments are either zero or small which may be neglected. In this study doubly curved thin shells are analyzed using Finite Element software SAP 2000 with new version. Doubly curved shells which are in rectangular plan having 1mX0.7m are considered. The behavior of shells under uniformly distributed load varying from 1to5KN/m² is studied and compared with the slabs of same dimension and thickness. In this case study deflection curves, membrane stress and stress contour diagram are obtained. It is observed that with the increase in rise and thickness of funicular shell the deflection are reduced. The membrane stresses decreases with the increase in rise and thickness of concrete funicular shell.

Keywords – Edge beam, Funicular shell, Rise, Thickness, Stress contour.

I. Introduction

The shells of double curvature are stronger when compared to shells of single curvature, equivalent to cylindrical shells. Further, the arch distributes the load in all paths equally and resists the impact of loading at any point. These shells are used as floors and also as roofs. Funicular shells are a class of doubly curved shells, the form of which satisfies the desired state of stress in its body for the given loading and boundary conditions. The state of stress favored in an unreinforced concrete thin shell will be pure compression unaccompanied by shear and bending stresses. Under different conditions of loading, bending moments would strengthen and the shell will not behave only as a funicular element. Analytically, it's possible to compute the funicular surface of any ground plan for the given loading conditions.

Since ancient times Shell structures has been constructed. Well-known examples of shell structures are The Pantheon in Rome and the Hagia Sophia in Istanbul. The traditions of domes continued up to the 17th century after the end of the Roman times. Around AD 125's the shell structure Pantheon in Rome, was built and it is the oldest known concrete shell & is still standing, it has a 43m dia massive concrete dome. Modern concrete shell roof design was first developed in early 1920's by pair of German engineers. The style of shell structure extended to other nations after two decades, initially throughout the Europe & next to the United States of America & then to Australia. The Sydney Opera House of Sydney in Australia & The Hershey Sports Arena in Hershey in Pennsylvania are some of the famous concrete shell structures.

II. Literature Review

P. Sivakumar, K. Manjunatha, Harish B. A (2015) have done the work on Concrete funicular shells with square in ground plan having 80mm rise are analyzed for UDL (One-way action). Specimens of dimension 1.08m x 1.08m in plan having rectangular edge beam of dimension 50mm x 40mm are prepared using M20 grade cement concrete. By masonry mould method the pre-casting of the specimens is done and specimens are prepared with 25mm and 20mm thickness and moist cured for 28 days earlier than testing. Over the shell specimens uniformly distributed load is applied & the corresponding deflections and strains are noted. To relate experimental outcomes to theoretical results, the finite element technique is used to analyze similar model. Conclusions are made by way comparing the experimental & analytical results.

Harish B. A, N. Venkataramana, K. Manjunatha (2015) in this work studied doubly curved thin shells are analyzed using finite element application SAP 2000. Doubly curved shells which might be in square plan having 10mX10m and 15mX15m are considered and shells in rectangular plan having dimensions 10mX15m and 15mX20m are considered. The behavior of shells under self-weight, are living load various from

0-20KN/m (UDL) is obtained. On this case study deflection curves, membrane stress and stress contour diagram are acquired. It is determined that with the increase in rise and thickness of funicular shell the deflection are diminished. The membrane stresses decreases with the increase in rise and thickness of concrete funicular shell. The aim of this study is to develop shells of different sizes and investigation is done on the shells by finite element analysis under given UDL, to discover the behavior of shells in various circumstances making use of standard software, Structural Analysis Package (SAP 2000).

P.Sivakumar, K.N.Lakshmikandhan, Linu Theresajose, K.Sivasubramanian, S. Saibabu, S. R. Balasubramanian (2014) investigated analytically about the funicular shells which spans between 1m and 3m with special span(s) to rise(r) ratio between 5 and 40. Both geometrical & material nonlinearities were considered under the finite element analysis & the results are compared with experimental outcome. The results obtained indicate that span to rise ratio in between 5 & 12.5 has better efficiency. The reduction in the thickness of shell is more helpful during lifting handling & placing of precast shells. In the present study about 30% of weight reduction is showed. Further, in one-way action the funicular shells performed better and the outcome concludes that to avoid two-way grid beams funicular shells can be used since favorable for one way slab action.

III. Objectives of the work

Concrete funicular shells of rectangular in ground plan, with doubly curved surfaces and various rises and thickness are analyzed by using finite element method. To study the behavior of funicular shells under UDL and comparing with the slab of same dimension. In this work an analytical investigation on doubly curved funicular shell with ground plan ratio 1:0.7 subjected to uniformly distributed load with distinct rise at L/10 and L/20 with thickness of 20mm, 40mm and 50mm is offered. The dimensions of the slabs and shells with their rise and thickness are shown in Table 1.

Table 1: Size of shells and slabs

SHELLS WITH VARYING RISES & THICKNESS				
Geometry of the shells	Designation of the shell	Plan dimensions in mm	Rise(R) in mm	Thickness in mm
Rectangle	FS I	1000x700	100	50
	FS II	1000x700	100	40
	FS III	1000x700	100	20
	FS IV	1000x700	50	50
	FS V	1000x700	50	40
	FSV VI	1000x700	50	20
SLABS WITH VARYING THICKNESS				
Geometry of the slab	Designation of the slab	Plan dimensions in mm	Thickness in mm	Rise (R) in mm
Rectangle	SLAB I	1000x700	50	0
	SLAB II	1000x700	40	0
	SLAB III	1000x700	20	0

IV. Finite Element Modeling And Analysis

The shells were modeled with dimensions as mentioned in “Table 1”, the edge beam thickness is taken as two to three times the thickness of shell, and all models are fixed supported with M20 grade concrete properties. The models were discretized and subjected uniformly distributed load varying from 1-5kN/m² .the 3D model is shown in Figure1, discretized model is shown in Figure 2 and model after applying load is shown in Figure 3.

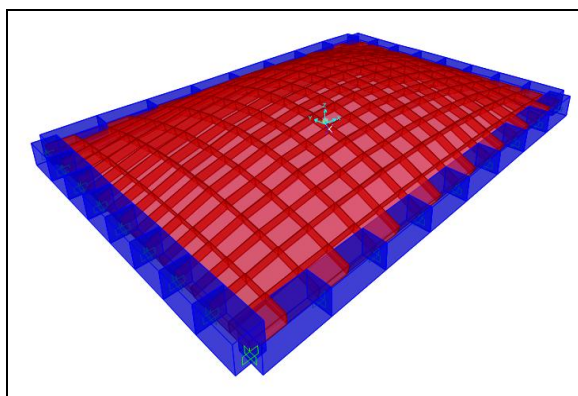


Fig 1: 3D Shell model.

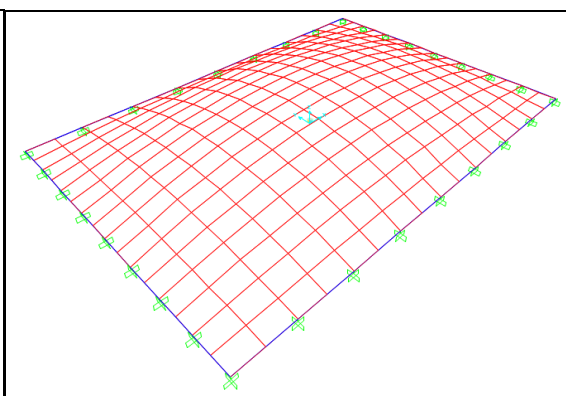


Fig 2: Discretized shell model.

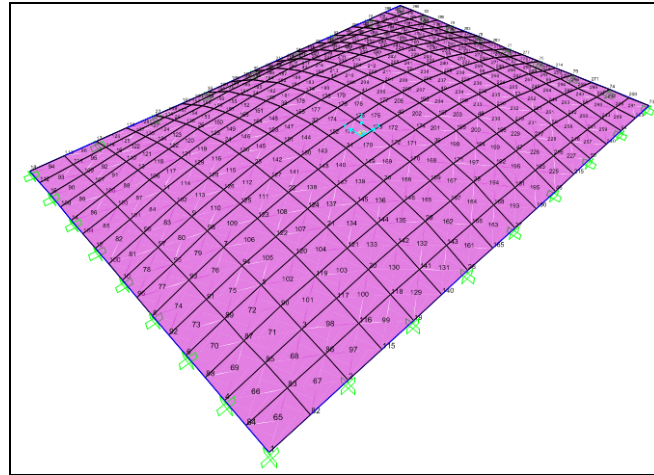


Fig 3: Model after loading.

After applying load the corresponding deflections and stresses at nodes were noted and graph is plotted with corresponding load and distance. The stress contour and deflected model is shown in Figure 4 and Figure 5 respectively and the graph of stress and deflection values for shell FS I is shown in Figure 6 and Figure 7 respectively. Similarly analysis is carried out for every shell and the values are obtained.

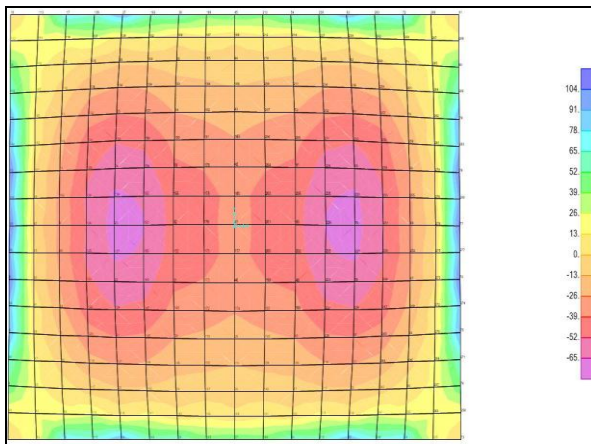


Fig 4: Stress contour for FS I

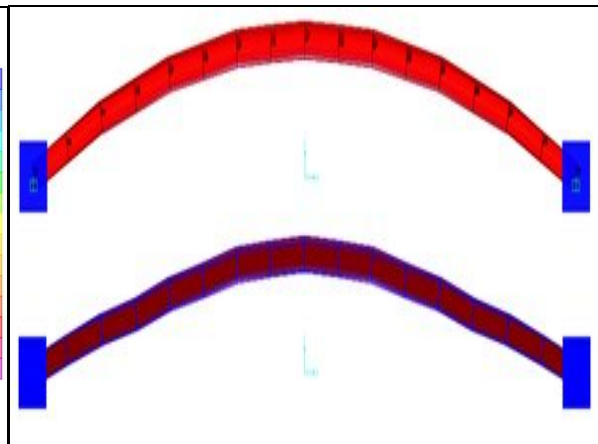


Fig 5: Deflected shape of FS I

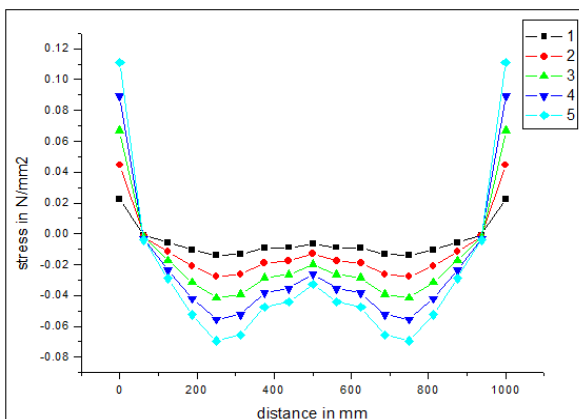


Fig 6: Graph showing Stress v/s distance for FS I

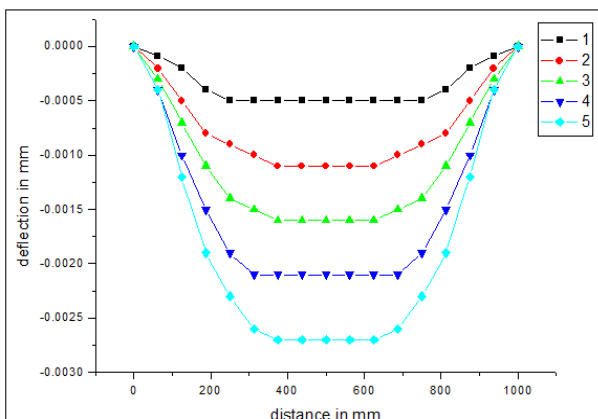


Fig 7: Graph showing Deflection v/s distance for FS I

The maximum deflection is obtained at the center of the shell and maximum stress is obtained at the edge of shell, values are tabulated in Table 2 with corresponding loadings.

Table 2: Stress and Deflection of FS I

Load in KN/m ²	Stress in N/mm ²	Deflection in mm
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1	-0.0066	0.0005
2	-0.0131	0.0011
3	-0.0196	0.0016
4	-0.0262	0.0021
5	-0.0328	0.0027

The values of maximum deflection and membrane stress for FS II, FS III and FS IV are shown in Table 3 and for FS V and FS VI shown in Table 4.

Table 3: Stress and Deflection of FS II, FS III and FS VI

FS II			FS III			FS VI		
Load in KN/m ²	Stress in N/mm ²	Deflection in mm	Load in KN/m ²	Stress in N/mm ²	Deflection in mm	Load In KN/m ²	Stress in N/mm ²	Deflection in mm
1	-0.0066	0.0007	1	-0.0069	0.0013	1	-0.0155	0.0012
2	-0.0132	0.0014	2	-0.0138	0.0027	2	-0.0311	0.0024
3	-0.0199	0.0021	3	-0.0208	0.004	3	-0.0467	0.0037
4	-0.0265	0.0028	4	-0.0277	0.0053	4	-0.0622	0.0049
5	-0.0332	0.0035	5	-0.0346	0.0067	5	-0.0777	0.0061

Table 4: Stress and Deflection of FS V and FS VI

FS V			FS VI		
Load in KN/m ²	Stress in N/mm ²	Deflection in mm	Load in KN/m ²	Stress in N/mm ²	Deflection in mm
1	-0.0195	0.0019	1	-0.0256	0.005
2	-0.0390	0.0038	2	-0.0512	0.0101
3	-0.0586	0.0056	3	-0.0769	0.0151
4	-0.0781	0.0075	4	-0.1025	0.0201
5	-0.0977	0.0094	5	-0.1282	0.0251

Slabs are analyzed to compare their results with the shells of the same dimension and thickness. Slabs of the dimension 1000mm X 700mm with the thickness of 20mm, 40mm and 50mm are analyzed under UDL. Results of the analysis are shown below.

Under UDL condition Slabs are analyzed and the corresponding deflections are shown in Table 5 and the maximum stresses are shown in Table 6.

Table 5: Deflection of Slab I, Slab II and Slab III in mm

Load KN/m ²	Slab I	Slab II	Slab III
1	0.0022	0.0042	0.0338
2	0.0043	0.0085	0.0677
3	0.0065	0.0127	0.1015
4	0.0087	0.017	0.1354
5	0.0108	0.0212	0.1692

Table 6: Stress of Slab I, Slab II and Slab III in N/mm²

Load KN/m ²	Slab I	Slab II	Slab III
1	-0.0202	-0.0316	-0.1263
2	-0.0404	-0.0633	-0.2526
3	-0.0606	-0.0949	-0.3789
4	-0.0808	-0.1266	-0.5052
5	-0.1010	-0.1583	-0.6315

The analysis results of each shell are compared between one another shells and slabs of same dimensions under uniformly distributed load. The maximum deflection of shells and slabs under UDL are tabulated in Table 7 and variations of values are shown in Figure 8.

Table 7: Maximum deflection of shells and Slab.

Thickness in mm	R100 Deflection in mm	R50 Deflection in mm	Slab Deflection in mm
20	0.0067	0.0251	0.1692
40	0.0035	0.0094	0.0212
50	0.0027	0.0061	0.0108

Table 8: Membrane stress of shells and slab

Thickness in mm	R100 Stress in N/mm ²	R50 Stress in N/mm ²	Slab Stress in N/mm ²
20	-0.0346	-0.1282	-0.6315
40	-0.0332	-0.0977	-0.1583
50	-0.0327	-0.0778	-0.1010

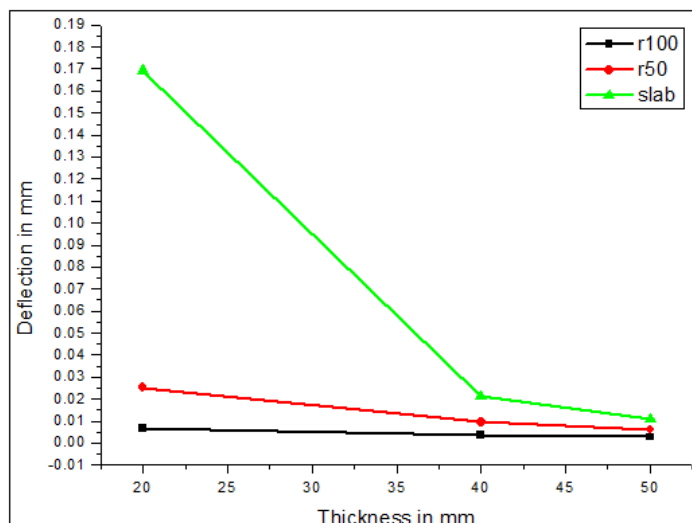


Fig 8: Deflection of shell and slab at different thickness

The maximum membrane stresses of shells and slabs under UDL are tabulated in Table 8 and the variations are shown in Figure 9.

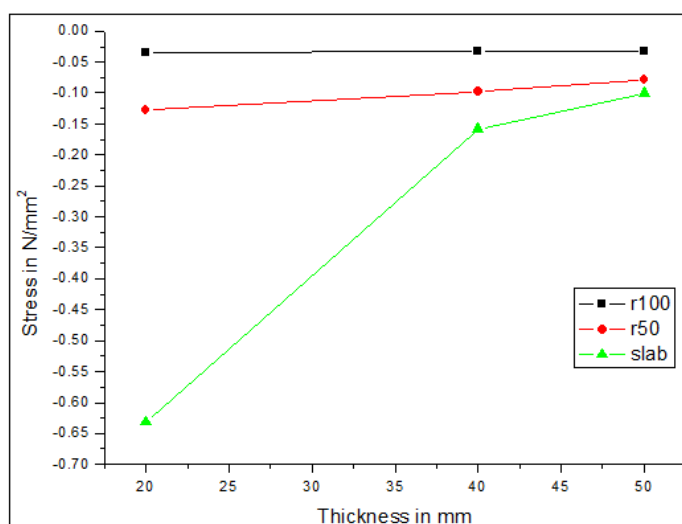


Fig 9: Membrane stress of shell and slab at different thickness

From comparison of shells with slabs it is clear that Slab deflects more when compared to shells, membrane stress also more in case of slabs. The rate of increase in deflection is high in case of slab when compared to shells.

V. Conclusion

The following conclusions are obtained from the outcomes of the investigation:

1. There is decrease in deflection with increase in rise and thickness of shell.
2. There is decrease in membrane stresses with increase in rise and thickness of shell.
3. In case of uniformly distributed load there is a maximum tension at edges of the shells and compression at the region around the center of the shell.
4. There is 22% more deflection in FS II when compared to FS I due to the 10mm lesser thickness.
5. In case of FS III it deflects 60% more than FS I and 48% more than FS II.
6. Rate of deflection is more in case of FS III when compared to FS I and FS II due to the lesser thickness.
7. There is 35% more deflection in FS V when compared to FS IV due to the 10mm lesser thickness.
8. In case of FS VI it deflects 76% more than FS IV and 63% more than FS V.
9. There is 1.5% more stress in FS II when compared to FS I.
10. In case of FS III 5.5% more stress than FS I and 4% more than FS II.
11. There is 20% more stress in FS V when compared to FS IV.

12. In case of FS VI 39% more stress than FS IV and 24% more than FS V.
13. When compared with slabs, FS I deflects 75% lesser and FS IV deflects 44% lesser than slab I under uniformly distributed load.
14. FS II deflects 83% lesser and FS V deflects 56% lesser than slab II.
15. FS III deflects 96% lesser and FS VI deflects 85% lesser than slab III.

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