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Economic Sizing of the Elements of Hinged Elastic Portal Frames under Different Static Loads

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Abstract: *In this work hinged portal frames were examined under different static loads and the best height to span ratio (h/L) for each load was obtained. This was carried out by first obtaining the minimum depth of section for each frame element for each load case. These sections were used to compute the volume of the portal frame. The volume of the portal frame was taken to be proportional to the cost of the frame while the usefulness or benefit of the frame was taken as the ratio of the frames' cross sectional area to their perimeter. A graph of the cost/benefit against height to span ratio (h/L) were plotted for each load case and the values of h/L corresponding to minimum cost/benefit obtained. These values were found to depend on the ratio of load to grade of portal frame material (w/σ) and the thickness of the frame elements. A table showing the values of height to span ratios (h/L) corresponding to minimum cost/benefit at different values of w/σ for the selected load cases was presented.*

Keywords: *portal frames, frame cost*

I. Introduction

In its simplest form a structure is a system of connected components used to support a load [1]. Portal frames consist of vertical members called columns and a top member which may be horizontal, curved or pitched with monolithic joints at the junction of columns [2]. It is estimated that around 50% of the hot-rolled constructional steel used in the UK is fabricated into single-storey buildings [3]. Portal frames are mostly used in single storey industrial structures. BS 5950 Part 1[4] allows for a linear elastic analysis or a plastic analysis of portal frames. The elastic analysis produces heavier structures which are however stable with little need for stability bracing [5]. Under a plastic analysis the aim is for a hinge to be formed at the point where the highest moment occurs. Failure is deemed to have taken place when the plastic hinges form a mechanism [6, 7]. In the analysis and design of portal frames the engineer normally uses his experience in determining the member sizes. Work on an efficient method for selecting member sizes and rise/span (h/L) ratio was done by John Righiniotis [5], but this was based on a plastic analysis for steel structures and was not load specific. This work is based on an elastic analysis and it was tailored to meet the requirements for each design load characteristics.

II. Methods

The stress σ at a section of a loaded structural member is given by [8]

$$\sigma = \frac{M}{Z} \pm \frac{N}{A} \qquad (1)$$

Where M is the bending moment at the section, N is the axial force in the member, A is the cross-sectional area of the member and Z is the section modulus of the cross-section. For rectangular sections

$$Z = \frac{bd^2}{6} \qquad (2)$$

where b and d are the breadth and depth of the sections respectively.
By substituting equation (2) into equation (1) we have [9]

$$\sigma = \frac{M}{bd^2} \pm \frac{N}{bd} \qquad (3)$$

Equation (3) is an expression of the maximum and minimum stress at a section of a loaded rectangular section. By assuming that the stress σ is the maximum stress that can be resisted by the material of the structure (i.e the grade of the material), the depth d of the section can be expressed in terms of the stress σ, bending moment M and axial force N using the almighty formula as

$$d = \frac{N + \sqrt{N^2 + 24Mb\sigma}}{2b\sigma} \qquad (4)$$

$d = \frac{1}{2b}\left(\frac{N}{\sigma}\right) + \sqrt{\frac{1}{4b^2}\left(\frac{N^2}{\sigma^2}\right) + \frac{6}{b}\left(\frac{M}{\sigma}\right)}$ . . . . . . (4a)

Since the stress σ is the grade of the material, M the bending moment at the section, N the axial force at the section, d is therefore the minimum depth of section that can overcome these internal stresses. When d is expressed in the form of equation (4a) it would be seen that d depends on the ratio of the internal stress M and N to the grade of the material. But under an elastic analysis of structures, the internal stresses are proportional to the load w. hence d is dependent on the ratio of the load w to the grade of material (w/σ).

For portal frames consisting of two vertical columns and a horizontal beam, equation (4) can be expressed as

$d_i = \frac{N_i + \sqrt{N_i^2 + 24M_i b\sigma}}{2b\sigma}$ . . . . . . (5)

$i = 1,2,3$ i is the element number

The cost of a portal frame is proportional to its volume. For a portal frame made up of prismatic members the cost can be expressed as

$cost = K\sum_{i=1}^{3} L_i A_i$ ... . (6)

where Li is the length of the element i, Ai is the cross-sectional area of the element i and K is a constant of proportionality equivalent to the cost of a unit volume of the material of the portal frame.

For portal frames made up of rectangular elements of constant thickness b equation (6) reduces to

$C_c = \sum_{i=1}^{3} L_i d_i$ . . . . (7)

where C_c is a cost coefficient equal to cost/Kb.

Equation (7) was used to calculate the cost coefficient of a portal frame as the sum of the cost coefficient of the individual elements of the portal frame.

A portal frame is useful when internally it is spacious i.e. it has space to permit its use for different purposes. To satisfy this requirement the portal frame has to be less compact.

The compactness of a solid is expressed as the ratio of its surface area to its volume.

$Compactness = \frac{Surface\ \ Area}{Volume}$ . . . . . (8)

For a 2D structure, it has to be rewritten as

$Compactness = \frac{Perimeter}{Area}$ . . . . (9)

The less compact the frame is the more beneficial it would be for range of uses, hence

$Benefit \propto \frac{Area}{Perimeter}$ . . . . (10)

$Benefit = \frac{hL}{2(h+L)}$ . . . . . . (11)

Where h is the height of the portal frame, L is the span of the portal frame (the constant of proportionality has been made equal to unity).

III. Results and Discussion

The equations for the determination of the internal moments M and N of a loaded portal frame is dependent on the ratio of the second moment of area of the beam section I_2 to the second moment of area of the column section I_1[9]. If we designate this as m then

$m = \frac{I_2}{I_1} = \frac{d_2^3}{d_1^3}$ . . . . (12)

(where the thickness b of the section is the same for the beam and the column, d_1 is the depth of the column member while d_2 is the depth of the horizontal beam member)

While m can be calculated from equation (12) using equation (5) to obtain the required d, m must first be known before equation (5) can be evaluated. There is therefore need to estimate a suitable value of m that will produce a set of internal stress M and N which on substitution into equation (5) will yield the same value of m. For frame 1 (a portal frame with a uniformly distributed vertical load w) a graph of the estimated m, m_e against m_e plotted on the same graph with a graph of the calculated m, m_c against m_e is shown in Figure 1. The consensus value of m which is the value at the point where line m_e crosses line m_c is the value of m that can be used to evaluate m. These were evaluated for different values of the ratio of height to length (h/L) of the portal frame and presented in Table 1. A program was written to iteratively select the correct m depending on the value of w/σ. It was found that for some values of h/L there exists no value of m. For instance for w/a = 0.001, there is no value of m for h/L>0.55. For values of h/L > 0.55 the two lines (me and mc) do not meet but there exist a value of m = me for which the difference between me and mc is least that value is used. Figure 2 shows a case in which there is no consensus m. In such cases there is no proportioning of the portal frame that would result in each member of the frame being stress optimally at the same time.

By using the consensus values of m and plotting a graph of the cost coefficient per unit benefit (Cc/B) against h/L for different values of w/σ, we obtain cost-benefit curves with minima at certain values of h/L. These curves for certain values of w/σ are shown in Figure 3. A detailed results of the values of h/L, m, d_1 and d_2 corresponding to minimum Cc/B is given in Table 2.

The same analysis was carried out on Frame 2 (a portal frame with a uniformly distributed horizontal load w). For frame 2 there exist consensus values of m for h/L values within the range of 0.2 to 2 for a load over stress (w/σ) value of 0.002. These values vary with the load stress ratio (w/a) and with the h/L ratio. In cases where there is no consensus m (i.e. value of m when line mc crosses line me) the value of me that gave the minimum difference between me and mc is used. A graph of the cost coefficient per unit benefit (Cc/B) against h/L for different values of w/σ is presented in Figure 4. The values of h/L, d_1 and d_2 are presented in Table 2. Since the loading and internal stress distribution is not symmetrical the values of d_1 and d_3 obtained from equation (5) are different however the higher of the values was reported as d_1 since we intend to keep the frame symmetrical.

For frame 3 (portals frame with a concentrated horizontal load) there is a consensus value of m for most values of h/L for each value of P/σ. The graphs obtained are similar to the graph of figure 1.Using the consensus values of m, the Cc was evaluated for different values of h/L and the plot of Cc/B against h/L for different values of P/σ is presented in Figure 5. From the graph it would be seen that the cost coefficient per benefit established a minimum at h/L = 0.65 – 0.70 depending on the value of P/a. The values of h/L, m, d1 and d2 corresponding for minimum Cc/B for different values of P/a is presented in Table 2.

For frame 4 (portals frame with a vertical concentrated load P at the centre) there are consensus values of m for very limited values of h/L. For a value of p/a = 0.001, m exist only for h/L = 0.1 – 0.25. When there is no consensus value the graph obtained is similar to the one of figure 2 and the value of me that gave the minimum difference between me and mc is used.

The graph of cost coefficient per unit benefit against h/L shows that Cc/B decreased exponentially with increasing values of h/L. Hence the higher the h/L adopted the lower the cost-benefit. This is presented in figure 6.

IV. Conclusion

Table 2 presents a summary of the economical values of h/L ratio for use in frames under different kinds of load. As seen in the discussion above, the ratio of load to grade of frame material (material of the portal frame) and the height to width ratio (h/L) of a portal frame affect the cost of the frame. For portal frames supporting mostly a uniformly distributed vertical load (frame 1) the economical h/L ratio for various w/σ ratio can be seen from table 2 to be 0.7. For frames supporting mostly a horizontal uniformly distributed load (frame 2) the ratio h/L = 0.60 – 0.65 proved to be the most economical for values of the ratio w/σ ranging from 0.001 to 0.01. Frames that support most a horizontal concentrated force (Frame 3) should be designed with a h/L ratio of 0.60 – 0.70 depending on the P/a ratio as shown in Table 2. Frames designed primarily to support a vertical concentrated load (frame 4) should be assigned the maximum possible value of h/L as the higher the value of h/L the lower the frame's cost.

References

[1]. Hibbeler, R. C.(2006). *Structural Analysis*. Sixth Edition, Pearson Prentice Hall, New Jersey

[2]. Raju K. (1990), Advanced Reinforced Concrete Design, CBS Publishers and Distributors Shahdara, Delhi

[3]. Graham R, Alan P.,(2007). Single Storey Buildings: *Steel Designer's Manual* Sixth Edition, Blackwell Science Ltd, United Kingdom

[4]. British Standards Institution (2000), *Structural use of steelwork in building*. Part 1: *Code of practice for design in simple and continuous construction*. BS 5959, BSI, London

[5]. John R.,(2007). Single Storey Buildings: *Steel Designer's Manual* Sixth Edition, Blackwell Science Ltd, United Kingdom

[6]. Struart S. J. M. (1996). *Plastic Methods for Steel and Concrete Structures*, 2nd Edition, Macmillian Press Ltd London

[7]. MacGinley T. J., Ang T. C.(1992). *Structural Steelwork: Design to Limit State Theory* Butterworth-Heinemann, Oxford. Great Britain.

[8]. Nash, W.,(1998). *Schaum's Outline of Theory and Problems of Strength of Materials*. Fourth Edition, McGraw-Hill Companies, New York

[9]. Reynolds, C. E.,Steedman J. C. (2001). *Reinforced Concrete Designer's Handbook*, 10th Edition) E&FN Spon, Taylor & Francis Group, London

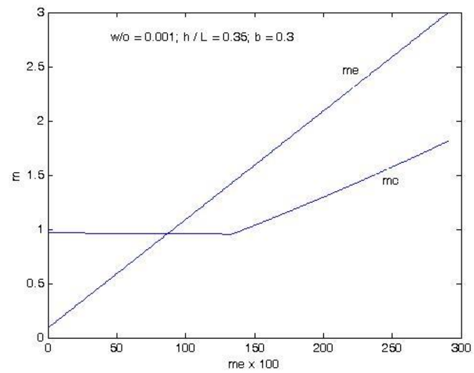


Figure 1: Graph of me and mc against me for Frame 1

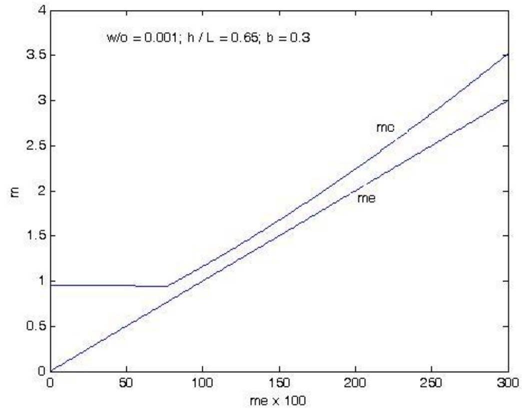


Figure 1: Graph of me and mc against me for h/L>0.75 in Frame 1

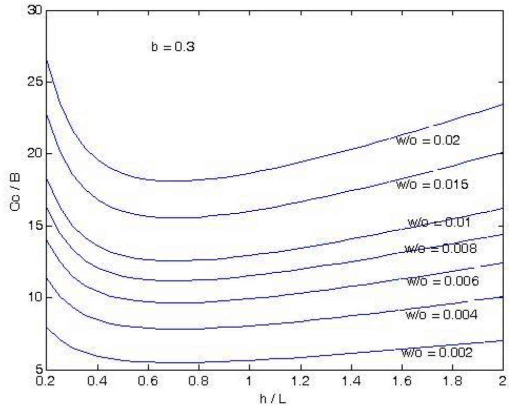


Figure 3: Graph of Cc/B against h/L for Frame 1

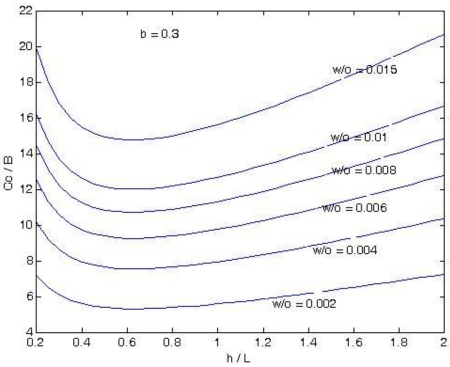


Figure 4: Graph of me and mc against me for Frame 2

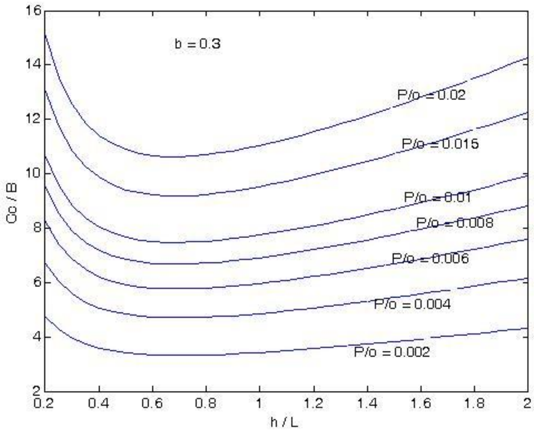


Figure 5: Graph of Cc/B against h/L for Frame 2

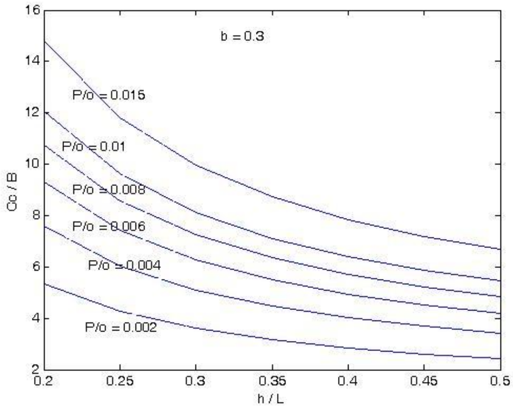


Figure 6: Graph of Cc/B against h/L for Frame 3

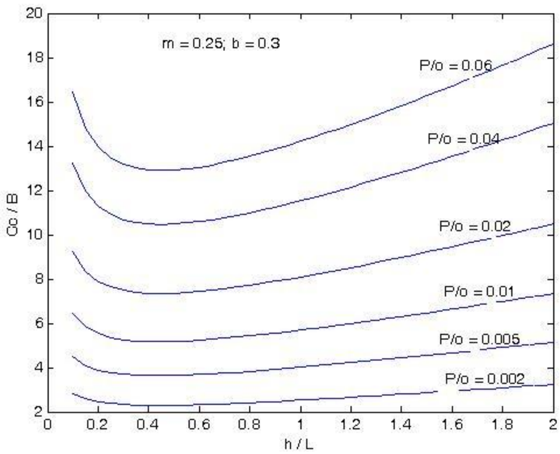
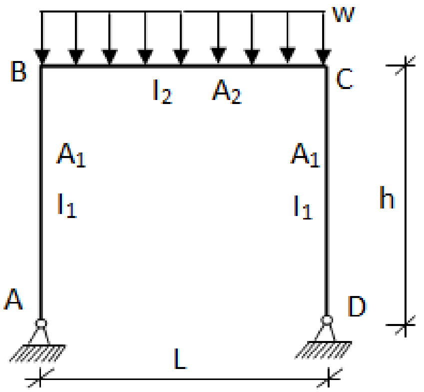
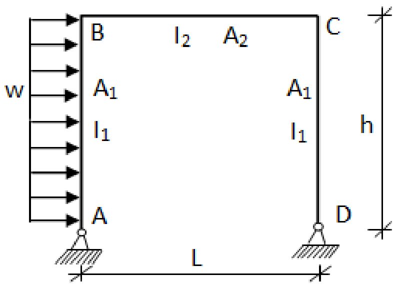
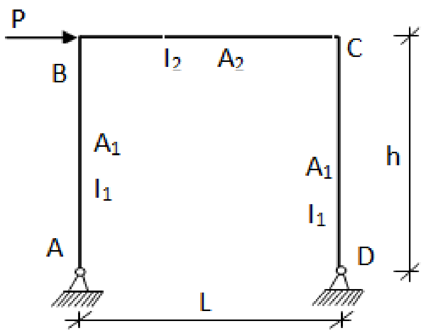
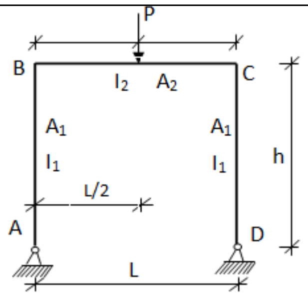


Figure 7: Graph of Cc/B against h/L for Frame 4

Table 1: Values of Consensus m for different values of h/L

h / L	m	d1	d2
0.20	0.9840	0.7845	0.7801
0.25	0.9730	0.6194	0.6136
0.30	0.9650	0.5096	0.5036
0.35	0.9600	0.4314	0.4254
0.40	0.9560	0.3729	0.3672
0.45	0.9520	0.3275	0.3222
0.50	0.9490	0.2914	0.2864
0.55	1.0830	0.2573	0.2643

Table 2: Values of h/L, m, d_1 and d_2 corresponding to minimum Cc/B for different values of w/σ

FRAME 1	**w/□**	**h / L**	**m**	**d_1**	**d_2**
	0.001	0.70	0.9542	0.2375	0.2338
	0.002	0.70	0.9359	0.3387	0.3313
	0.003	0.70	0.9221	0.4176	0.4064
	0.004	0.70	0.9106	0.4848	0.4699
	0.005	0.70	0.9006	0.5447	0.5260
	0.006	0.70	0.8916	0.5993	0.5768
	0.007	0.70	0.8835	0.6499	0.6236
	0.008	0.70	0.8760	0.6973	0.6672
	0.009	0.70	0.8690	0.7422	0.7083
	0.010	0.70	0.8624	0.7850	0.7472
	0.015	0.70	0.8343	0.9755	0.9183
	0.020	0.70	0.8113	1.1402	1.0634
	0.030	0.70	0.7742	1.4253	1.3088
	0.040	0.70	0.7443	1.6744	1.5175
	0.060	0.70	0.6970	2.1106	1.8713
	0.080	0.70	0.6596	2.4968	2.1734
	0.10	0.70	0.6285	2.8513	2.4424
	0.15	0.70	0.5676	3.6523	3.0241
	0.20	0.70	0.5215	4.3781	3.5240
FRAME 2	**w/□**	**h / L**	**m**	**d_1**	**d_2**
	0.001	0.65	0.5454	0.3484	0.2847
	0.002	0.65	0.5451	0.4941	0.4036
	0.004	0.65	0.5448	0.7013	0.5727
	0.006	0.65	0.5445	0.8613	0.7033
	0.008	0.65	0.5443	0.9969	0.8139
	0.01	0.60	1.1152	0.9118	0.9118
	0.02	0.60	0.5467	1.5891	1.2994
	0.03	0.60	0.5469	1.9576	1.6009
	0.04	0.60	0.5471	2.2716	1.8578
	0.08	0.60	0.5476	3.2614	2.6682
	0.10	0.60	0.5478	3.6686	3.0017
FRAME 3	**P/□**	**h / L**	**m**	**d_1**	**d_2**
	0.001	0.70	0.9888	0.2012	0.2004
	0.002	0.70	0.9842	0.2852	0.2837
	0.004	0.70	0.9778	0.4047	0.4017
	0.006	0.70	0.9728	0.4969	0.4924
	0.008	0.70	0.9687	0.5751	0.5690
	0.01	0.70	0.9651	0.6441	0.6366
	0.15	0.70	0.9574	0.7923	0.7809
	0.20	0.70	0.9509	0.9181	0.9028
	0.30	0.70	0.9403	1.1310	1.1080
	0.40	0.65	0.9387	1.3090	1.2817
	0.50	0.65	0.9318	1.4694	1.4352
	0.60	0.65	0.9255	1.6156	1.5744
	0.70	0.65	0.9198	1.7509	1.7027
	0.80	0.65	0.9145	1.8776	1.8225
	0.90	0.65	0.9096	1.9974	1.9352
	0.10	0.65	0.9049	2.1113	2.0421
	0.20	0.65	0.8683	3.0534	2.9130
	0.30	0.65	0.8596	3.7771	3.5914
	0.40	0.60	0.8398	4.4200	4.1701
	0.60	0.60	0.8078	5.5356	5.1554
FRAME 4	**P/□**	**h / L**	**d_1**	**d_2**	**d_3**
	Adopt the highest value of h/L possible for all values of w/σ				