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Application of Affine Theorem to Orthotropic Rectangular Reinforced Concrete Slab with Long Side Opening Symmetric About Vertical Axis

Dr.T.V.S. Vara Lakshmi[1], Prof. K. Rambabu[2], Prof. K. Srinivasa Rao[3]
[1](Department of Civil Engineering, University College of Engineering and Technology, Acharya Nagarjuna University, Guntur, Andhra Pradesh, India).
2(Department of Civil Engineering, College of Engineering, Andhra University, Visakhapatnam, Andhra Pradesh, India).
3(Department of Civil Engineering ,College of Engineering, Andhra University, Visakhapatnam, Andhra Pradesh, India).

Abstract: *An attempt has been made to apply affinity theorem to determine collapse load of two-way orthotropic slab with long side opening symmetric to vertical axis. Keeping in view the basic principles of yield line theory, all possible admissible yield line patterns for Continuous Slab (CS), Simply Supported Slab (SS), Two Short Sides Continuous Slab (TSC), and Two Long Sides Continuous Slab (TLC) are considered for the given configuration of the slab subjected to uniformly distributed load (udl). A computer program has been developed to solve the virtual work equations. Illustration of above methodology has been brought out with numerical examples. Relevant tables for given data and the governing admissible failure patterns of the slab for different sizes of openings are presented using affine theorem. In this paper, authors also present the transformation of orthotropic slab into an equivalent isotropic slab using affine theorem. A slab with longer side opening is used. Here aspect ratio of opening is quite different from that of slab.*

Keywords: *aspect ratio, long side opening, configuration, affinity theorem, orthotropic slab, uniformly distributed load, ultimate load and ultimate moment.*

I. Introduction

Openings in slabs are usually required for plumbing, fire protection pipes, heat and ventilation ducts and air conditioning. Larger openings that could amount to the elimination of a large area within a slab panel are sometimes required for stairs and elevators shafts. For newly constructed slabs, the locations and sizes of the required openings are usually predetermined in the early stages of design and are accommodated accordingly. Such two way slabs subjected to uniformly distributed load and supported on various edge conditions are being analyzed by using yield-line method as suggested by Johansen, K.W[1]. Many researchers (Goli, H.B. and Gesund,H.[2], Rambabu, K. and Gloi,H.B.[3], Islam,S. and Park,R.[4], Zaslavsky.Aron[5], Siva Rama Prasad, Ch and Goli,H.B.[6], Sudhakar, K.J. and Goli,H.B.[7], Veerendra Kumar and Milan Bandyopadhya[8]) adopted the yield-line analysis and virtual work method in deriving the virtual work equations of the rectangular reinforced concrete solid slabs subjected to uniformly distributed load and supported on various edge conditions. Johansen, K.W[1], also presented the analysis of orthogonal solid slabs implicitly to that of an equivalent Isotropic Slab by using "Affine Theorem" provided the ratio of negative to positive moments is same in orthogonal directions. Various design charts are presented for Two-way slabs with longer side opening subjected to uniformly distributed load for different edge conditions. Islam,S. and Park,R.[4] presented design charts for CS and SS slabs with equal openings, i.e. ratio of openings and aspect ratio are same.

II. Methodology

The method of determining collapse loads based on principle of virtual work has proved to be a powerful tool for a structural engineer, despite it gives an upper bound value. The work equations are formed by equating the energy absorbed by yield lines and the work done by the external load of the orthogonal rectangular slab with long side openings where a small virtual displacement is given to the slab. The same principle was also used by Islam,S. and Park,R.[4] in their paper. In other words, the work equation is given by

$$\iint W_{ult}\,\delta(x,y)\,dxdy = \sum\left(m_{ult,x}\theta_x y_0 + m_{ult,y}\theta_y x_0\right) \quad \text{--------------------(1)}$$

where W_{ult} is the ultimate load per unit area of slab, $\delta(x,y)$ is the virtual displacement in the direction of the loading at the element of area of dimensions *dx, dy*, $m_{ult,x}$ and $m_{ult,y}$ are the yield moments per unit width in the x and y directions, θ_x and θ_y are the components of the virtual rotation of the slab segments in the x and y directions and x_0 and y_0 are the projected length of the yield lines in x and y directions of slab. The equation (1) contains terms

C_1, C_2, C_3 and C_4 which define the positions of the node points of the yield lines. The values of C_1, C_2, C_3 and C_4 to be used in the equation are those which give the minimum load to cause failure. A computer program has been written to find the minimum values of C_1, C_2, C_3 and C_4 (in terms of r_1, r_2, r_3 and r_4) which in turn will give the minimum load carrying capacity of the slab. For definitions of various parameters refer notations. Johansen[1] has proved that the yield line theory is an upper bound method, so care has been taken to examine all the possible yield line patterns for each boundary (4 edge's) condition of slab to ensure that the most critical collapse mode is considered otherwise the load carrying capacity of the slab will be overestimated.

III. Formulation Of Virtual Work Equations

There are several possible yield line patterns associated with different edge conditions of the slab. For four edge condition of slab, the possible admissible failure yield line patterns are only ten for CS, SS, TSC and TLC edge conditions. These admissible failure yield line patterns are obtained basing on the yield line principle (Johansen K.W[1]). For the given configuration of the slab, these ten failure patterns and corresponding equations have been investigated depending upon the edge condition of the slab using a computer program.

The orthogonal reinforced rectangular slab having long side opening with the given configuration and the yield criteria are shown in notations. The slab is subjected to uniformly distributed load (W_{ult}) and supported on different boundary conditions. Note that the slab is not carrying any load over the area of the opening.

The virtual work equations are derived for the predicted possible admissible failure yield line patterns using the virtual work equation for continuous edge (CS) condition of slab. To get the equations for other edge conditions of the slab, modification should be carried out in the numerators of the equations of each failure patterns. For SS slab I_1=I_2=0, for TSC slab I_2=0, for TLC slab I_1=0.

IV. Virtual Work Equations For Continuous Slab (CS)

Ten possible failure patterns are predicted for Continuous Slab (CS), Simply Supported Slab (SS), Two Long Sides Continuous Slab (TSC) and Two Long Sides Continuous Slab (TLC) and four edge conditions of the slab. Some of the governing failure pattern for different edge conditions and for different data is presented in Table-1. Considering the failure Pattern-1of a continuous slab. Three unknown dimensions C_1, C_2, & C_3 are necessary to define the yield line propagation completely. The remaining failure patterns are as shown in Fig.2.

Table 1: Governing failure patterns for different data for four edge conditions

Edge Cond. Failure Pattern	CS	TLC	TSC	SS	Edge Cond. Failure Pattern	CS	TLC	TSC	SS
1	α=0.2 β=0.3 r=1.8	α=0.2 β=0.1 r=1.4	α=0.4 β=0.2 r=1.3	α=0.3 β=0.1 r=1.4	6	α=0.2 β=0.1 r=1.3	α=0.4 β=0.1 r=1.2	α=0.3 β=0.1 r=1.2	α=0.4 β=0.1 r=1.0
2	α=0.1 β=0.5 $r^{\#}$=1.7	α=0.1 β=0.6 r=1.8	α=0.2 β=0.4 r^{*}=1.9	α=0.2 β=0.3 r=1.9	7	α=0.3 β=0.5 r=1.1	α=0.1 β=0.4 r=1.2	α=0.3 β=0.5 r=1.0	α=0.1 β=0.4 r=1.0
3	α=0.6 β=0.5 r=1.9	α=0.3 β=0.4 r=1.5	α=0.6 β=0.6 r=2.0	α=0.3 β=0.4 r=1.6	8			α=0.1 β=0.6 r=1.4	α=0.1 β=0.6 r=1.5
4	α=0.5 β=0.5 r=1.4	α=0.5 β=0.1 r=1.3	α=0.5 β=0.3 r=1.6	α=0.5 β=0.2 r=1.6	9	α=0.5 β=0.6 r=1.2	α=0.6 β=0.6 r=1.1	α=0.4 β=0.5 r=1.2	α=0.4 β=0.5 r=1.0
5	α=0.5 β=0.3 $r^{\#}$=1.7	α=0.6 β=0.2 r=1.9			10	α=0.6 β=0.2 r=1.2	α=0.6 β=0.3 r=1.5	α=0.6 β=0.2 r=1.3	α=0.5 β=0.3 r=1.1
	K'_x=0.5 K'_y=0.5 I_1=1.3 I_2=0.7	K'_x=1.8 K'_y=0.3 I_1=0.0 I_2=0.9	K'_x=0.4 K'_y=1.6 I_1=2.0 I_2=0.0	K'_x=2.0 K'_y=2.0 I_1=0.0 I_2=0.0		K'_x=0.5 K'_y=0.5 I_1=1.3 I_2=0.7	K'_x=1.8 K'_y=0.3 I_1=0.0 I_2=0.9	K'_x=0.4 K'_y=1.6 I_1=2.0 I_2=0.0	K'_x=2.0 K'_y=2.0 I_1=0.0 I_2=0.0
Orthogonal coefficients for CS pattern-2				# K^1_x=0.6,K'_y=1.0,I_1=1.0,I_2=1.4					
Orthogonal coefficients for TLC pattern-2				* K^1_x =1.2,K'_y=0.6,I_1=0.0,I_2=1.2					

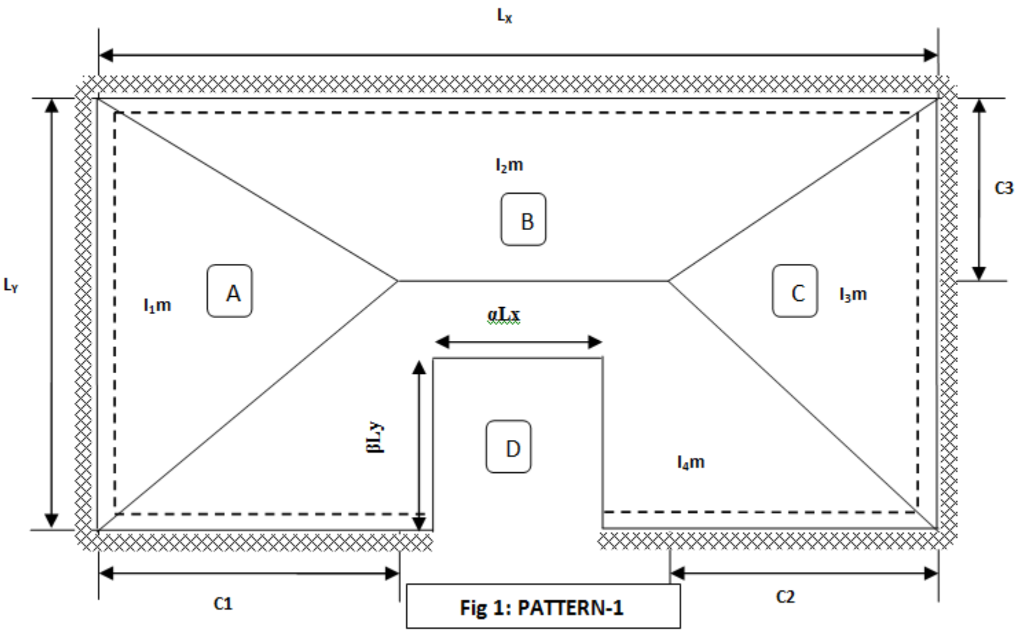


Fig 1: PATTERN-1

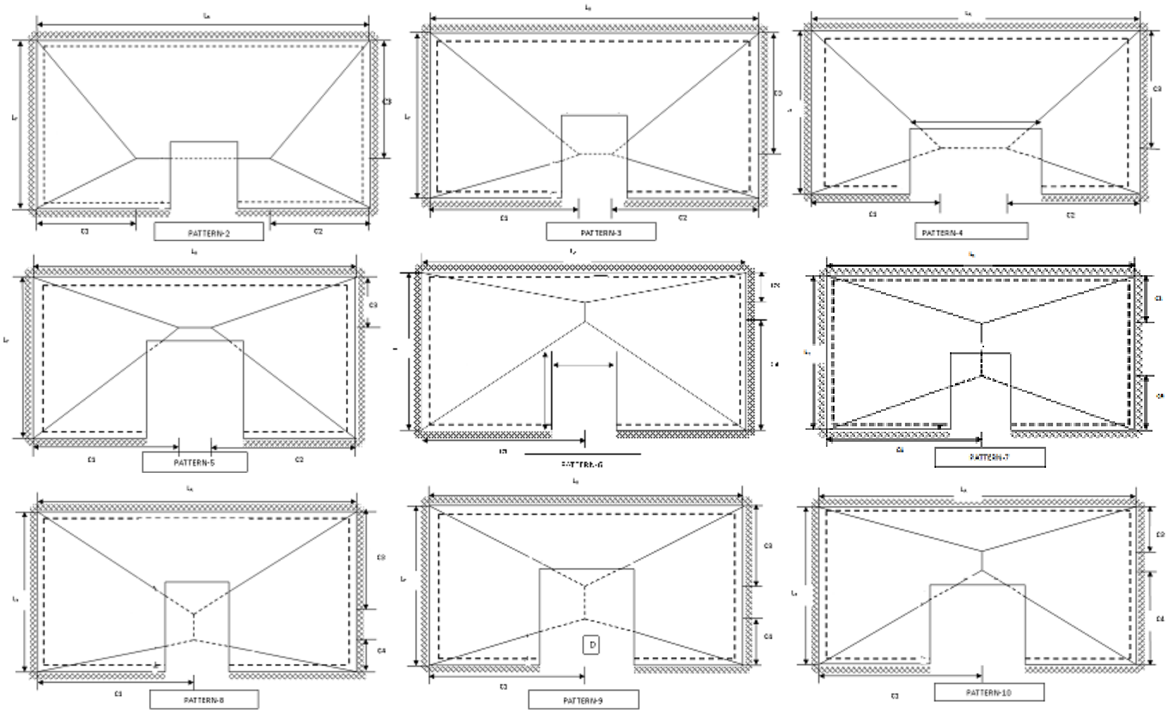


Fig 2 Patterns from 2 to 10 of a continuous slab

Virtual Work Equations For Continuous Slab (CS)

External Work Done by Segment 'A': $\frac{1}{2}L_Y C_1 \frac{W_{ult}}{3} = \frac{W_{ult} L_X L_Y}{6r_1}$

External Work Done by Segment 'B': $\frac{1}{2}C_1 C_3 \frac{W_{ult}}{3} + (L_X - C_1 - C_2)C_3 \frac{W_{ult}}{2} + \frac{1}{2}C_2 C_3 \frac{W_{ult}}{3}$

$$= W_{ult} L_X L_Y \left[\frac{1}{6r_1 r_3} + \frac{1}{2r_3} - \frac{1}{2r_1 r_3} - \frac{1}{3r_2 r_3} + \frac{1}{6r_2 r_3} \right]$$

$$= W_{ult} L_X L_Y \left[\frac{1}{2r_3} - \frac{1}{3r_1 r_3} - \frac{1}{3r_2 r_3} \right]$$

External Work Done by Segment 'C': $\frac{1}{2}L_Y C_2 \frac{W_{ult}}{3} = \frac{W_{ult} L_X L_Y}{6r_2}$

External Work Done by Segment

'D': $\frac{1}{2}C_1(L_Y - C_3)\frac{W_{ult}}{3} + (L_X - C_1 - C_2)(L_Y - C_3)\frac{W_{ult}}{2} + \frac{1}{2}C_2(L_Y - C_3)\frac{W_{ult}}{3} - \alpha L_X \beta L_Y \frac{\left(\beta L_Y / 2\right)}{(L_Y - C_3)} W_{ult}$

$= W_{ult} L_X L_Y \left[\frac{(r_3-1)}{2r_3} - \frac{(r_3-1)}{3r_1 r_3} - \frac{(r_3-1)}{3r_2 r_3} - \frac{\alpha\beta^2 r_3}{2(r_3-1)}\right]$

Total Work Done: work done by the segments (A+B+C+D)

$= W_{ult} L_X L_Y \left[\frac{1}{6r_1} + \frac{1}{2r_3} - \frac{1}{3r_1 r_3} - \frac{1}{3r_2 r_3} + \frac{1}{6r_2} + \frac{(r_3-1)}{2r_3} - \frac{(r_3-1)}{3r_1 r_3} - \frac{(r_3-1)}{3r_2 r_3} - \frac{\alpha\beta^2 r_3}{2(r_3-1)}\right] = W_{ult} L_Y^{\,2} r\left[\frac{1}{2} - \frac{1}{6r_1} - \frac{1}{6r_2} - \frac{\alpha\beta^2 r_3}{2(r_3-1)}\right]$

Energy Absorbed By Yield Lines

$K'_X m \frac{L_y}{C_1} + I_1 m \frac{L_y}{C_1} + K'_Y m \frac{L_X}{C_3} + I_2 m \frac{L_X}{C_3} + K'_X m \frac{L_y}{C_2} + I_3 m \frac{L_y}{C_2} + K'_Y m \frac{L_X}{(L_Y - C_3)} + I_4 m \frac{(L_X - \alpha L_X)}{(L_Y - C_3)}$

$= m\left[(K'_X + I_1)\frac{r_1}{r} + (K'_Y + I_2) r r_3 + (K'_X + I_3)\frac{r_2}{r} + K'_Y \frac{r r_3}{(r_3-1)} + I_4 \frac{r r_3(1-\alpha)}{(r_3-1)}\right]$

Equating total work done by the segments to the energy absorbed by yield line we get Equation-1 for failure pattern-1

Equation-1 for Failure Pattern-1:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{(K'_X + I_1)\frac{r_1}{r} + (K'_Y + I_2) r r_3 + (K'_X + I_3)\frac{r_2}{r} + K'_Y \frac{r r_3}{(r_3-1)} + I_4 \frac{r r_3(1-\alpha)}{(r_3-1)}}{r\left[\frac{1}{2} - \frac{1}{6r_1} - \frac{1}{6r_2} - \frac{\alpha\beta^2 r_3}{2(r_3-1)}\right]}$$

Equation-2 for Failure Pattern-2:

$$\frac{W_{ult} L_y^{\,2}}{m_{ult}} = \frac{(K'_X + I_1)\frac{r_1}{r} + K'_Y r r_3(1-\alpha) + I_2 r r_3 + (K'_X + I_3)\frac{r_2}{r} + (K'_Y + I_4)\frac{r r_3(1-\alpha)}{(r_3-1)}}{r\left[\frac{(1-\alpha)}{2} - \frac{1}{6r_1} - \frac{1}{6r_2} - \frac{\alpha(1-\beta)^2 r_3}{2}\right]}$$

Equation-3 for Failure Pattern-3:

$$\frac{W_{ult} L_y^{\,2}}{m_{ult}} = \frac{\left[K'_X \frac{r_1^{\,2}(1-\alpha)}{2r} + I_1 \frac{r_1}{r} + (K'_Y + I_2) r r_3(1-\alpha) + K'_X \frac{r_2^{\,2}(1-\alpha)}{2r} + I_3 \frac{r_2}{r} + (K'_Y + I_4)\frac{r r_3(1-\alpha)}{(r_3-1)}\right]}{r\left[\frac{r_1(1-\alpha)^2}{8} + \frac{r_2(1-\alpha)^2}{8} - \frac{r_1^{\,2}(1-\alpha)^3}{48} - \frac{r_2^{\,2}(1-\alpha)^3}{48} + \frac{\alpha r_3(1-\beta)^2}{2}\right]}$$

Equation-4 for Failure Pattern-4:

$$\frac{W_{ult} L_y^{\,2}}{m_{ult}} = \frac{\left[\begin{array}{l} K'_X \frac{r_1}{r}\left((1-\beta) + \frac{r_1^{\,2}(r_3-1)(1-\alpha)}{2r_3}\right) + I_1 \frac{r_1}{r} + K'_Y r r_3\left[\frac{r_3(1-\beta)}{r_1} + \frac{r_3(1-\beta)}{r_2}\right] + I_2 r r_3 + \\ K'_X \frac{r_2}{r}\left[(1-\beta) + \frac{r_2(r_3-1)(1-\alpha)}{2r_3}\right] + I_3 \frac{r_2}{r} + (K'_Y + I_4)\frac{r r_3(1-\alpha)}{(r_3-1)} \end{array}\right]}{r\left[\beta(r_1 + r_2)\frac{(1-\alpha)^2}{8} - \frac{r_3^{\,2}(1-\beta)^3}{6}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) + \frac{r_3(1-\beta)^2}{2} - \frac{(r_3-1)(1-\alpha)^3}{48r_3}\left(r_1^{\,2} + r_2^{\,2}\right)\right]}$$

Equation-5 for Failure Pattern-5:

$$\frac{W_{ult} L_y^{\,2}}{m_{ult}} = \frac{\left[\begin{array}{l} K'_X \frac{r_1}{r}\left((1-\beta) + \frac{r_1(r_3-1)(1-\alpha)}{2r_3}\right) + I_1 \frac{r_1}{r} + (K'_Y + I_2) r r_3 + K'_X \frac{r_2}{r}\left[(1-\beta) + \frac{r_2(r_3-1)(1-\alpha)}{2r_3}\right] + \\ I_3 \frac{r_2}{r} + K'_Y\left(\frac{r r_3}{(r_3-1)}\right)\left(1 + (1-\alpha) - \frac{\beta r_3}{r_1(r_3-1)} - \frac{\beta r_3}{r_2(r_3-1)}\right) + I_4 \frac{r r_3(1-\alpha)}{(r_3-1)} \end{array}\right]}{r\left[\begin{array}{l} \frac{1}{2} - \frac{1}{6r_1} - \frac{1}{6r_2} + \frac{\beta^3 r_3^{\,2}}{6r_1(r_3-1)^2} + \frac{\beta^3 r_3^{\,2}}{6r_2(r_3-1)^2} + \frac{\beta r_1(1-\alpha)^2}{8} + \\ \frac{\beta r_2(1-\alpha)^2}{8} - \frac{r_3\beta^2}{2(r_3-1)} - \frac{r_1^{\,2}(r_3-1)(1-\alpha)^3}{48r_3} - \frac{r_2^{\,2}(r_3-1)(1-\alpha)^3}{48r_3} \end{array}\right]}$$

Equation-6 for Failure Pattern-6:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{\left(K'_X + I_1\right)\frac{r_1}{r} + \left(K'_Y + I_2\right) r r_3 + \left(K'_X + I_3\right)\frac{r_1}{r(r_1 - 1)} + K'_Y r r_4 + I_4 r r_4 (1-\alpha)}{r\left[\frac{1}{2} - \frac{1}{6r_3} - \frac{1}{6r_4} - \frac{\alpha\beta^2 r_4}{2}\right]}$$

Equation-7 for Failure Pattern-7:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} K'_x \frac{r_1}{r}\left((1-\beta) + \frac{r_1(1-\alpha)}{2r_4}\right) + I_1 \frac{r_1}{r} + \left(K'_y + I_2\right) r r_3 + \\ K'_x \frac{r_1}{r(r_1-1)}\left((1-\beta) + \frac{r_1(1-\alpha)}{2r_4(r_1-1)}\right) + I_3 \frac{r_1}{r(r_1-1)} + \left(K'_y + I_4\right) r r_4 (1-\alpha) \end{array}\right]}{r\left[\frac{(1-\beta)}{2} - \frac{1}{6r_3} + \frac{\beta r_1 (1-\alpha)^2}{8} - \frac{r_1^2 (1-\alpha)^3}{48 r_4} + \frac{\beta r_1 (1-\alpha)^2}{8(r_1-1)} - \frac{r_1^2(1-\alpha)^3}{48 r_4 (r_1-1)^2}\right]}$$

Equation-8 for Failure Pattern-8:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} K'_x \frac{r_1}{r}\left(\frac{r_1(1-\alpha)}{2r_4} + \frac{r_1(1-\alpha)}{2r_3}\right) + I_1 \frac{r_1}{r} + K'_y r r_3 (1-\alpha) + I_2 r r_3 + \\ K'_x \frac{r_1}{r(r_1-1)}\left(\frac{r_1(1-\alpha)}{2r_3(r_1-1)} + \frac{r_1(1-\alpha)}{2r_4(r_1-1)}\right) + I_3 \frac{r_1}{r(r_1-1)} + \left(K'_y + I_4\right) r r_4 (1-\alpha) \end{array}\right]}{r\left[\frac{r_1^2(1-\alpha)^2}{8(r_1-1)} - \frac{r_1^2(1-\alpha)^3}{48r_3} - \frac{r_1^2(1-\alpha)^3}{48r_4} - \frac{r_1^2(1-\alpha)^3}{48r_3(r_1-1)^2} - \frac{r_1^2(1-\alpha)^3}{48r_4(r_1-1)^2} + \frac{\alpha r_3 (1-\beta)^2}{2}\right]}$$

Equation-9 for Failure Pattern-9:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} K'_X\left((1-\beta) + \frac{(1-\alpha) r_1}{2r_4}\right)\frac{r_1}{r} + I_1 \frac{r_1}{r} + K'_Y r r_3 \left(\frac{r_3(1-\beta)}{r_1} + \frac{r_3(1-\beta)(r_1-1)}{r_1}\right) \\ + I_2 r_3 r + K'_X \frac{r_1}{r(r_1-1)}\left((1-\beta) + \frac{r_1(1-\alpha)}{2r_4(r_1-1)}\right) + I_3 \frac{r_1}{r(r_1-1)} + \left(K'_Y + I_4\right) r r_4 (1-\alpha) \end{array}\right]}{r\left[\frac{r_3(1-\beta)^2}{2} + \frac{r_1^2 \beta (1-\alpha)^2}{8(r_1-1)} - \frac{r_3^2(1-\beta)^3}{6} - \frac{r_1^2(1-\alpha)^3}{48r_4} - \frac{r_1^2(1-\alpha)^3}{48r_4(r_1-1)^2}\right]}$$

Equation-10 for Failure Pattern-10:

$$\frac{W_{ult} L_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} K'_x \frac{r_1}{r}\left((1-\beta) + \frac{r_1(1-\alpha)}{2r_4}\right) + I_1 \frac{r_1}{r} + \left(K'_y + I_2\right) r r_3 + K'_x \frac{r_1}{r(r_1-1)}\left((1-\beta) + \frac{r_1(1-\alpha)}{2r_4(r_1-1)}\right) + \\ I_3 \frac{r_1}{r(r_1-1)} + K'_y r r_4 \left(1 - \beta r_4 + (1-\alpha)\right) + I_4 r r_4 (1-\alpha) \end{array}\right]}{r\left[\frac{1}{2} - \frac{1}{6r_3} - \frac{1}{6r_4} + \frac{\beta^3 r_4^2}{6} + \frac{\beta r_1^2 (1-\alpha)^2}{8(r_1-1)} - \frac{r_1^2(1-\alpha)^3}{48r_4} - \frac{r_1^2(1-\alpha)^3}{48r_4(r_1-1)^2} - \frac{\beta^2 r_4}{2}\right]}$$

The respective equations for corresponding failure patterns can be obtained for other edge conditions (CS, SS, TSC, TLC) by making respective negative yield lines zero.

Minimization of the Virtual Work Equations

The value $\frac{W_{ult} L_y^2}{m_{ult}}$ of these equations consist of the unknown non dimensional parameters r_1, r_2, r_3 and r_4 which define the positions of the yield lines. A computer program has been developed for various values of the non dimensional parameters r_1, r_2, r_3 and r_4 within their allowable ranges in order to find the minimum value of $\frac{W_{ult} L_y^2}{m_{ult}}$ for the yield line failure patterns considered. In this computer program, the values of r_1, r_2, r_3 and r_4 were varied at increments of 0.1. Using the above equations, one can develop useful charts basing on orthogonality which may be used either for design or analysis in general. The governing failure patterns for different data and edge conditions of the slab are presented in Table 1.

EXAMPLE: Continuous Slab (CS) (Negative moment to positive moment ratio in both directions is same and unity) Transform an orthotropic slab to an equivalent Isotropic Slab in which the ratio of Negative moment to positive moment in both directions is same and unity using affine theorem. Since $I_1/K'_x = I_2/K'_y = 1.0$, the transformation of the given orthotropic slab (Fig.11a)) in X – direction is transformed to an equivalent isotropic slab (Fig.11 (b)) by dividing with $\sqrt{\mu}$. This principle is illustrated in Fig.11 using the above methodology. Few numerical examples are presented in Table 2.

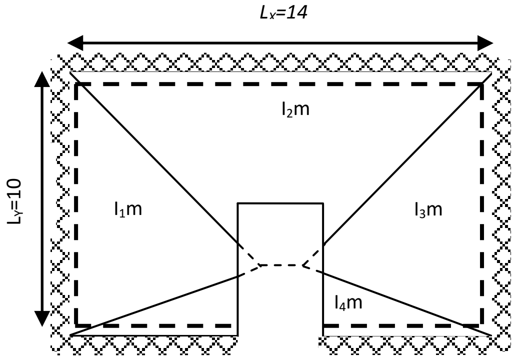


Figure: 3(A)

Orthotropic slab for **CS** condition

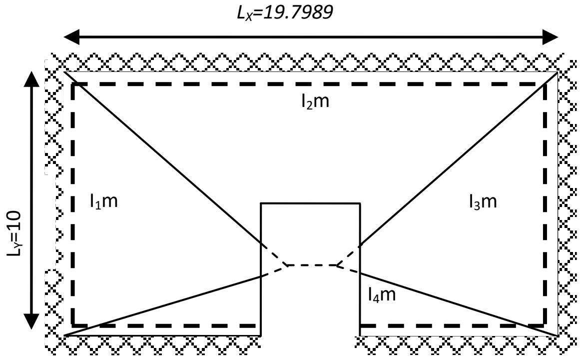


Figure: 3(B)

Equivalent isotropic slab for CS

$K'_X = I_1 = I_3 = 0.5$, $K'_Y = I_2 = I_4 = 1.0$, $\alpha = 0.4$, $\beta = 0.6$, $I_1/K'_X = I_2/K'_Y = 1.0$, $\mu = 0.5$, $r = L_X/L_Y = 14/10 = 1.4$, $\sum K = 3$ In order to check on affine theorem a computer program is used to evaluate the value of $W_{ult}L_Y^2/m_{ult}$ and the value is 29.81401.

As per affine theorem, the transformed $K'_X = I_1 = I_3 = K'_Y = I_2 = I_4 = 1.0$, $\alpha = 0.4$, $\beta = 0.6$, $L_X = L_X/\sqrt{\mu} = 14/\sqrt{0.5} = 19.7989$ m, $r = 19.7989/10 = 1.9798$, $\mu = 1.0$, $\sum K = 4$, the value of $W_{ult}L_y^2/m_{ult}$ is obtained from Graph 4.1 for r = 1.9798.Taking this value one can design the given orthotropic slab without using computer program.

Table 2: Numerical examples based on Theorem's VI & VII of Johansen[3] for CS condition

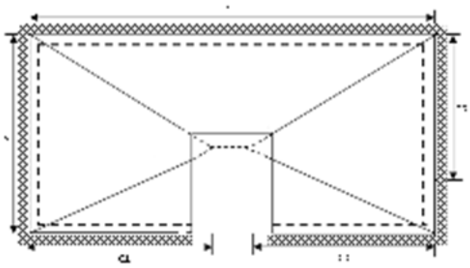
Example (CS)		Orthogonal Moment Coefficients	Aspect Ratio (*r*)	Strength $W_{ult}L_y^2/m_{ult}$	Aspect Ratio (*r**)
Sl. No.	Openings				
1	α=0.5,β=0.6	K'_x =0.25, K'_y =1.0 I_1= I_3= 0.25, I_2= I_4= 1.0, μ=0.25, ∑K =2.25	1.0	30.20383	2.0
2	α=0.4,β=0.6	K'_x =0.5, K'_y =1.0 I_1= I_3= 0.5, I_2= I_4= 1.0, μ=0.5, ∑K =2.5	1.4	29.81401	1.98
3	α=0.3,β=0.5	K'_x =0.667, K'_y =1.0, I_1= I_3= 0.667, I_2= I_4= 1.0, μ=0.667, ∑K =3.33	1.6	28.03296	1.959
4	α=0.2,β=0.5	K'_x =1.0, K'_y =1.0, I_1= I_3= 1.0, I_2= I_4= 1.0, μ=1.0, ∑K =4.0	1.3	38.76392	1.3
5	α=0.3,β=0.6	K'_x =1.5, K'_y =1.0, I_1= I_3= 1.5, I_2= I_4= 1.0, μ=1.5, ∑K =5.0	1.9	35.4455	1.551
6	α=0.2,β=0.6	K'_x =2.0, K'_y =1.0, I_1= I_3= 2.0, I_2= I_4= 1.0, μ=2.0, ∑K =6.0	1.7	42.76975	1.202

NOTE: 1. r*: equivalent isotropic slab aspect ratio,
2. $I_1/K'_x = I_2/K'_y = 1.0$

Note: In the case of SS, TLC and TSC edge conditions of the slab; the affine theorem cannot be applied because the negative moment is not present along one of the edges of the given slab. Therefore one can design the slab as orthotropic using any available computer program.

Table 3 shows strength and failure pattern for continuous slab (CS) based on the principle $\mu = r^2$ for different values of coefficient of orthotropy(μ) and their corresponding orthogonal affine moment coefficients.

Table 3: Continuous slab (CS), based on the principle $\mu = r^2$

Coefficient of orthotropy (μ)	Aspect Ratio of Slab $r = \sqrt{\mu}$	Orthogonal moment coefficient (affine)	$\frac{W_{ult} L_y^2}{m_{ult}}$	Failure pattern for all aspect ratios
0.25	0.5	K'_x=0.25, K'_y=1.0 $I_1 = I_3 = 0.25$, $I_2 = I_4 = 1.0$	59.25	
0.5	0.7071	K'_x=0.5, K'_y=1.0 $I_1 = I_3 = 0.5$, $I_2 = I_4 = 1.0$	59.25	Failure pattern-3 $C_1 = L_x/r_1$. r_1=2.4333 $C_2 = L_y/r_2$. r_2=2.4146 $C_3 = L_y/r_3$. r_3=1.8111 Coefficients of opening α = 0.4, β = 0.6
0.667	0.8165	K'_x=0.667, K'_y=1.0 $I_1 = I_3 = 0.667$, $I_2 = I_4 = 1.0$	59.25	
1.0	1.0	K'_x=1.0, K'_y=1.0 $I_1 = I_3 = 1.0$, $I_2 = I_4 = 1.0$	59.25	
1.44	1.2	K'_x=1.44, K'_y=1.0 $I_1 = I_3 = 1.44$, $I_2 = I_4 = 1.0$	59.25	
1.5	1.2247	K'_x=1.5, K'_y=1.0 $I_1 = I_3 = 1.5$, $I_2 = I_4 = 1.0$	59.25	
2.0	1.4142	K'_x=2.0, K'_y=1.0 $I_1 = I_3 = 2.0$, $I_2 = I_4 = 1.0$	59.25	
2.25	1.5	K'_x=2.25, K'_y=1.0 $I_1 = I_3 = 2.25$, $I_2 = I_4 = 1.0$	59.25	
4.0	2.0	K'_x=4.0, K'_y=1.0 $I_1 = I_3 = 4.0$, $I_2 = I_4 = 1.0$	59.25	

Analysis the safe uniformly distributed load on a rectangular two - way slab with longer side opening supported two long edges continuous as shown in Fig. 4., for the following data.

A slab 9 m X 6 m with an opening size of 2.7 m X 1.8 m reinforced with 12 mm φ bars @ 110 mm c/c perpendicular to long span and 10 mm φ bars @ 150 mm c/c perpendicular to long span is considered. Two meshes are used one at top and the other at bottom. Thickness of the slab is 120 mm. The characteristic strength of concrete is 20 MPa and steel is Fe 415. Calculate the intensity of live load on the slab. According to IS 456-2000,

$$m_{ult} = 0.87 f_y A_{st} z \text{ , where } z = d\left(1 - \left(f_y A_{st} / f_{ck} bd\right)\right)$$

Assuming Effective depth of slab in long span direction = 100 mm

Effective depth of slab in long span direction = 100 mm

Area of the steel perpendicular to long span = 1028.1575 mm^2

Area of the steel perpendicular to long span = 523.5987 mm^2

The ultimate moments in long and long span directions can be found using above expression.

Therefore m_{ult} parallel to long span = 29.201 kNm/m

m_{ult} parallel to long span = 16.850 kNm/m

For aspect ratio of slab r = 9/6 =1.5 and taking m_{ult} = 29.201 kNm/m, the orthogonal coefficients will be K'_X = 0.577, K'_Y = 1.0, $I_1 = I_3 = 0$, $I_2 = I_4 = 1.0$. With these orthogonal coefficients and for α = 0.3, β = 0.3, r = 9/6 =1.5; ten predicted failure patterns are evaluated by using computer program to find the governing failure pattern and the final results are as follows.

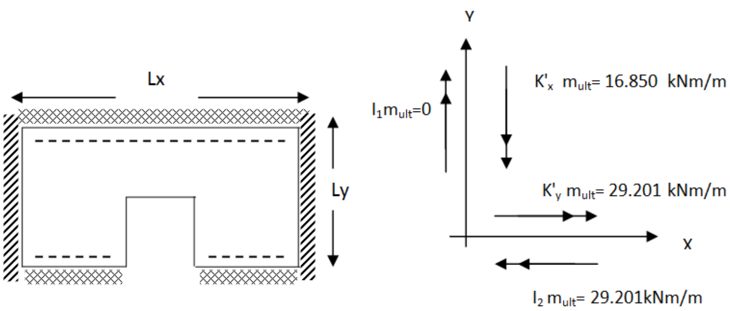


Figure 4 Analysis problem for TLC condition

$W_{ult}\, L_y^2 / m_{ult} = 22.234$, $r_1 = 3.757$, $r_2 = 3.757$, $r_3 = 1.411$ and the failure pattern 2.
$W_{ult} = 22.234 \times 29.201/6^2 = 18.0355$ kN/m^2
W_{dl} = (dead load including finishing) = 0.12 x 25 + 0.5 = 3.5
$W_{ult} = 1.5 \times (w_{ll} + w_{dl}) = 18.0355$ kN/m^2
$w_{ll} = (18.0355 /1.5) - 3.5 = 8.5237$ kN/m^2
The intensity of live load on the slab is 8.523 kN/m^2

Design a simply supported slab 6 m X 3 m with long side opening of 1.8 m X 0.6 m to carry a uniformly distributed load of 3 kN/m^2 .Use M20 mix and Fe 415 grade steel.
Aspect ratio of slab = $L_x/L_y = 6/3 = 2$
αL_x= 1.8 m, βL_y= 0.6 m
$\alpha = 0.3$, $\beta = 0.2$
Ten predicted failure patterns are evaluated by using computer program to find the governing failure pattern by taking $K'_X = 1.8$, $K'_Y = 1.2$, $I_1 = I_2 = I_3 = I_4 = 0$,
$W_{ult}\, Ly^2 / m_{ult} = 22.76537$ and failure **pattern is 2**
Unknown parameters:
$r_1 = (L_x/C_1) = 3.55714$, $r_2 = (L_x/C_2) = 3.55714$, $r_3 = (L_y/C_3) = 1.211$
Overall thickness of slab = 120 mm
Dead loads including finishing = 3.5 kN/m^2
Total load = 6.5 kN/m^2
Ultimate total load =1.5 x 6.5 = 9.75 kN/m^2
$m_{ult} = 9.75 \times 3^2/ 22.76537 = 3.85453$ kNm/m
The orthogonal moments are $K'_X\ m_{ult}$ =1.2 X 3.85453 = 4.62544 kNm/m.
$K'_y\ m_{ult}$ = 1.8 X 3.85453 = 6.93815 kNm/m, $I_2 m_{ult} = I_1 m_{ult} = 0$
Effective depth: $d = \sqrt{(6.93815 \times 10^6/0.138 \times 20 \times 1000)} = 50.13$ mm
Adopt effective depth as 100 mm and overall depth as 120 mm
Area of steel along long span =0.36 X 20 X 1000 X 0.48 X 100) / (0.87 X 415) = 957.2 mm^2
Use 10 mm bars @ 80 mm c/c
Area of steel along long span = 909.34 mm^2
Use 10 mm bars @ 85 mm c/c

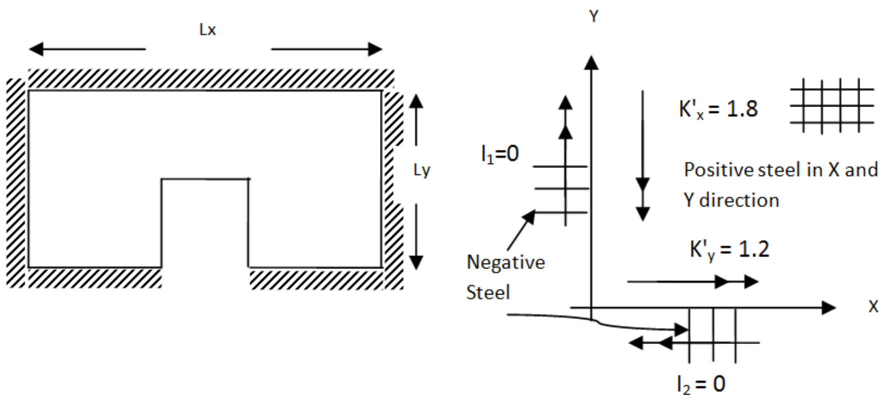


Figure 5Design problem for SS condition

V. Conclusions

1. The equations for orthotropic slabs with unequal central long side opening whose aspect ratio is different from the aspect ratio of slab subjected to udl supported on 4 edge conditions are presented.
2. Design charts for two edge conditions two long sides continuous and two long sides continuous are presented for different aspect ratios.
3. Few numerical examples are presented based on theorem of VI and VII of affine theorem for orthotropic slabs with unequal openings.
4. One chart for Affine Transformation for different sizes of openings is presented.

References

[1] Johansen, K.W., “Yield-Line Theory “, Cement and Concrete Association, London, 1962, pp. 181.
[2] Goli, H B. and Gesund, H., “Linearity in Limit State Design of Orthotropic Slabs” J. of Structural Division, ASCE,, Oct.1979, Vol.105, No. ST10, pp.1901-1915.
[3] Rambabu, K. and Goli, H B. “Simplified approach to design orthogonal slabs using affine theorem”, Journal of Structural Engineering, Chennai, India, Dec 2006-Jan 2007,Vol.33,No.5, pp.435-442.
[4] Islam, S. and Park, R., “Yield line analysis of two-way reinforced concrete slabs with openings”, The Structural Engineer, June 1971, Vol. 49, No. 6, pp. 269-275.
[5] Zaslavsky. Aron, Yield line analysis of rectangular slabs with central opening”, Proc. Of ACI, December 1967, Vol. No.64, pp. 838-844.
[6] Siva Rama Prasad, Ch., and Goli, H. B., “Limit State Coefficients for Orthogonal Slabs.” International Journal of Structural Engineering, India, Jan-June 1987, Vol.7, No.1, pp. 93-111.
[7] Sudhakar, K.J., and Goli, H B., “Limit State Coefficients for Trapezoidal- Shaped Slabs Supported on Three Sides”, Journal of Structural Engineering, Chennai, India, June-July 2005, Vol.32, No.2, pp.101-108.
[8] Veerendra Kumar and Milan Bandyopadhyay “ Yield line analysis of two way reinforced concrete slabs having two adjacent edges discontinuous with openings” ,Journal of Structural Engineering, Chennai, India, June-July 2009,Vol.36,No.2, pp.82-99.
[9] “Indian Standard Plain and Reinforced Concrete-Code of Practice”, IS 456:2000, BIS New Delhi

Notations:

	Continuous edge
	Simply supported edge
	Free edge
	Negative yield line
CS	A slab supported on all sides continuously (restrained)
I_1 *and* I_2	Negative moment coefficients in their corresponding directions
$I_1 m_{ult}$	Negative ultimate yield moment per unit length provided by top tension Reinforcement bars placed parallel to x-axis.
$I_2 m_{ult}$	Negative ultimate yield moment per unit length provided by top tension Reinforcement bars placed parallel to y-axis.
$K^1_x m_{ult}$	Positive ultimate yield moment per unit length provided by bottom tension Bars placed parallel to X-axis
$K^1_y m_{ult}$	Positive ultimate yield moment per unit length provided by bottom tension Bars placed parallel to Y-axis
K1	$\frac{K'_y}{K'_x}$
K2	$\frac{I_2}{K'_y}$
L_x, L_y	Slab dimensions in X and Y directions respectively
m_{ult}	Ultimate Yield moment per unit length of the slab
r	Aspect ratio of slab defined by L_x/L_y.
r_1, r_2, r_3, r_4	Non dimensional parameters of yield line propagation
SS	A slab simply supported on all sides
TLC	A slab restrained on two long edges and other two sides simply supported
TSC	A slab restrained on two long edges and other two sides simply supported
udl	Uniformly Distributed Load
W_{ult}	Ultimate uniformly distributed load per unit area of slab.
α, β	coefficients of opening in the slab
μ	Coefficient of orthotropy $= \frac{[K'_x + I_1]}{[K'_y + I_2]}$