IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684,p-ISSN: 2320-334X, Volume 13, Issue 2 Ver. I (Mar. - Apr. 2016), PP 33-39
www.iosrjournals.org

Design of Nonlinear Piping Water System Network by Using Newton Raphson Method

Abdulhamid El Idris Saad
Mechanical Engineering Department, Faculty of engineering, University Of Tobruk, Libya.

Abstract: *Cost of electricity and material of the network that consists of a reservoir that gives water to five tanks in different places has reasonable effects by using Newton Raphson methods. In the first design, the cost of electricity was 270936.44$ compare to 293913$ of the second design. Newton Raphson method was used in the first design to get the diameters and the head (the input power). In the second design, the fact of the average velocity of pipe network between (0.899m/s-2.097m/s) was used to get the diameters and the head. Small diameters give less cost of materials but they increase the head which increase the cost of electricity. However, large diameters give high cost of materials but they decrease the head which decreases the cost of electricity.*
Keywords: *NewtonRphson, network, head, cost, electricity*

I. Introduction

A water distribution network is a system containing pipes, reservoirs, pumps, valves of different types, which are connected to each other to provide water to consumers. The problem of optimal design of water distribution networks has various aspects to be considered such as hydraulics, reliability, material availability, water quality.

A large amount of money is invested around the world to provide or upgrade piped water supply facilities. Nearly 80% to 85% of the cost of a total water supply system is contributed toward water transmission and the water distribution network. Water distribution system design has attracted many researchers due to the enormous cost. The high cost of water supply systems has motivated a great and intense effort to obtain more economically viable systems.

The problem of design optimization for new water supply systems is defined by the best possible combination of reducing costs for its components. In practice, this optimization can take numerous forms, depending on the various kinds of components which comprise a water supply system, the diverse criteria for correct functioning and the design constraints of such a network.

A general strategy to solve such optimization problems of water supply systems can be defined in terms of a balanced combination of: least cost for the layout and size using new components. Pipe sizing, the head loss, and the pump selection will be the main important points in this project. Analysis will take place by setting up a system of a nonlinear equations as a results of internal flow in pipe such as, the continuity equation, Bernoulli equation, major , and minor losses. This system cannot be solved analytically. Therefore, numerical method by using MATLAB software is used to solve the nonlinear systems of the network.

II. The Case Study

To save an electricity cost , a town's water supply system uses gravity - driven from five large storage tanks during the day and then refills there tanks from (10 pm to 6am) at a cheaper night rate of (7 cents per kw.h) . The total resupply needed each night varies from (20 e^{+5}) (7575m^3) gallons to any one tank . The tanks can be filled to an elevation of (30.48 m). A single constant - speed pump drawing water from a large river at ground elevation (zero ft) and valves into five different cost iron supply lines will be used to do the job .
Distance from the pump to the five tanks varies more or less evenly from 1 to 3 Km .Each line averages one elbow every (30.48m) and has butterfly valves that can be controlled at any desirable angle. The dimension of the system and the properties of water is shown in table 1. The goal is to determine pipe lines sizes that achieve a minimum total cost over a 5 years period.
Use the suggested cost data below:
- Valves : $100 plus $100 per cm of pipe size diameter.
- Pipe lines : 50 cent per meter of diameter per meter of length.

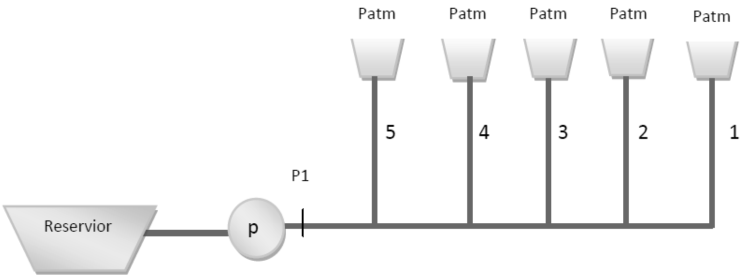

Fig 1. The network diagram of the system.

Table 1. Pipe lines dimensions and water properties of our network at 20 C

$No\ of\ pipe$	1	2	3	4	5
$Length, m$	4828.032	4023.36	3218.688	2414.016	1609.344
$No\ of\ elbow$	159	132	106	80	53
$No\ of\ valve$	4 each pipe				
$Dz, elevation(m)$	30.48				
$Ke, elbow, 90\deg rees$	0.75				
$Kv, valve, butteifiy, 60\deg rees$	118				
$Water\ dencity\ (kg/m^3)$	999				
$Vis\cos ity(N/m.s)$	$1.012*10^{-6}$				
$Gravity(m/s^2)$	9.81				
$Patm(Pa)$	101325				
Roughness	e=0.00085m				

The modified Bernoulli's equations of the five pipes (Bruce and Donald, 2009).

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+\frac{fl}{D}\frac{V_1^2}{2g}+ke\frac{V_1^2}{2g}+kv\frac{V_1^2}{2g} \tag{1}$$

The last three terms of R.H.S of equation (1) are the losses due to the length of the pipe (viscosity effect), the minor losses due to elbows, and the minor losses due to valves. Rearrangement equation(1):
$V_o=0$ (the velocity at the top of the tank)

$$p_1+\frac{\rho V_1^2}{2}=p_{atm}+\rho g z_2+\frac{fl}{d_1}\frac{\rho V_1^2}{2}+ke\frac{\rho V_1^2}{2}+kv\frac{\rho V_1^2}{2} \tag{2}$$

$$p_1-p_{atm}=\rho g z_2+\frac{fl}{d_1}\frac{\rho V_1^2}{2}+ke\frac{\rho V_1^2}{2}+kv\frac{\rho V_1^2}{2}-\frac{\rho V_1^2}{2} \tag{3}$$

$$p_1-p_{atm}=\rho g z_2+\frac{\rho V_1^2}{2}\left(\frac{fl}{d_1}+ke+kv-1\right) \tag{4}$$

$$p_1-p_{atm}=\rho g z_2+\frac{\rho V_1^2}{2}\left(\frac{fl}{d_1}+ke+kv-1\right) \tag{5}$$

The Bernoulli equation is different cordoning to the distance between the tanks and the reservoir and the number of valves as following :
For the first pipe :

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_1^2}{2}\left(\frac{4828.032 f}{d_1} + 159 ke + 4kv - 1\right) \tag{6}$$

For the second pipe :

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_2^2}{2}\left(\frac{4023.36 f}{d_2} + 132 ke + 4kv - 1\right) \tag{7}$$

For the third pipe :

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_3^2}{2}\left(\frac{3218.688 f}{d_3} + 106 ke + 4kv - 1\right) \tag{8}$$

For the forth pipe :

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_4^2}{2}\left(\frac{2414.016 f}{d_4} + 80 ke + 4kv - 1\right) \tag{9}$$

For the fifth pipe :

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_5^2}{2}\left(\frac{1609.344 f}{d5} + 53 ke + 4kv - 1\right) \tag{10}$$

There is an empirical equation that fit to the Moody chart which is called the Colebrook formula to calculate the friction factor (Philip, 2011):

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{e/d}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right] \tag{11}$$

The continuity equation of the network

For a pipe that branches out into five parallel connected pipes,

- The total flow rate is the sum of the flow rates in the individual pipes.
- The pressure drop (or head loss) in each individual pipe connected in parallel must be the same (Yunus andJohn, 2006)

$$Q_{tot} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 \tag{12}$$

where:

$$Q = A * V \tag{13}$$

$$A = \frac{\pi * D^2}{4} \tag{14}$$

.

The unknowns are the head and diameters of the pipes

The method of the design

In this design, we assumed that the flow rate in each pipe are equal which is the total flow rate divided by five.

- The total resupply needed each night is $7570 m^3$
- The resupply of each tank needed each night is $7570/5 = 1514 m^3$
- The total flow rate each night is $\frac{7570}{8*60*60} = 0.2628 m^3/s$
- The flow rate of each tank $\frac{0.2628}{5} = 0.0525\ m^3/s$
- Number of hours needed to fill each tank at the same time $\frac{7570}{0.2628} = 480 \min .(8\ hours)$

The system of equations got from the network is nonlinear. Therefore, a numerical method is needed. Newton Raphson Method can be used to solve a nonlinear system of equations. In order to see how a system of equations can be solved numerically (Otto and Denier, 2005) :

Step#1: Setting up the whole system:

$$f_1(P_1, d_1, d_2, d_3, d_4, d_5) = p_1 - p_{atm} - \rho g z_2 - \frac{\rho V_1^2}{2}\left(\frac{4828.032 f}{d_1} + 159ke + 4kv - 1\right) \quad (15)$$

$$f_2(P_1, d_1, d_2, d_3, d_4, d_5) = p_1 - p_{atm} - \rho g z_2 - \frac{\rho V_2^2}{2}\left(\frac{4023.36 f}{d_2} + 132ke + 4kv - 1\right) \quad (16)$$

$$f_3(P_1, d_1, d_2, d_3, d_4, d_5) = p_1 - p_{atm} - \rho g z_2 - \frac{\rho V_3^2}{2}\left(\frac{4023.36 f}{d_3} + 132ke + 4kv - 1\right)$$

(17)

$$f_4(P_1, d_1, d_2, d_3, d_4, d_5) = p_1 - p_{atm} - \rho g z_2 - \frac{\rho V_4^2}{2}\left(\frac{4023.36 f}{d_4} + 132ke + 4kv - 1\right)$$

(18)

$$f_5(P_1, d_1, d_2, d_3, d_4, d_5) = p_1 - p_{atm} - \rho g z_2 - \frac{\rho V_5^2}{2}\left(\frac{4023.36 f}{d_5} + 132ke + 4kv - 1\right)$$

(19) $$f_6(P_1, d_1, d_2, d_3, d_4, d_5) = Q_{tot} - V_1 \frac{\pi}{4} d_1^2 - V_2 \frac{\pi}{4} d_2^2 - V_3 \frac{\pi}{4} d_3^2 - V_4 \frac{\pi}{4} d_4^2 - V_5 \frac{\pi}{4} d_5^2$$

(20)

We have six equations and six unknowns. Newton Raphson method is used to solve this nonlinear system. The nonlinearity can be seen in the square diameters of the equation of continuity.

Step#2 : the initial guess of :(give a values to the unknowns: $P_1, d_1, d_2, d_3, d_4, d_5$)

Step#3: compute the velocities from the continuity equation: (for given discharge, and assumed diameter)

$$V_1 = \frac{Q_1}{\pi/4\, d_1^2}, V_2 = \frac{Q_2}{\pi/4\, d_2^2}, V_3 = \frac{Q_3}{\pi/4\, d_3^2}, V_4 = \frac{Q4}{\pi/4\, d_4^2}, V_5 = \frac{Q_5}{\pi/4\, d_5^2} \quad (21)$$

Step#4: compute Reynolds's numbers

$$\mathrm{Re}1 = \frac{V_1\, d_1}{\upsilon}, \mathrm{Re}2 = \frac{V_2\, d_2}{\upsilon}, \mathrm{Re}3 = \frac{V_3\, d_3}{\upsilon}, \mathrm{Re}4 = \frac{V_4\, d_4}{\upsilon}, \mathrm{Re}5 = \frac{V_5\, d_5}{\upsilon} \quad (22)$$

Step#5: compute the friction factor of each pipe:

$$f = \left[-1.8 Log\left((\frac{e/d}{3.7})^{1.11} + \frac{6.9}{\mathrm{Re}} \right)\right]^{-2} \quad (23)$$

Step #6 applying the Newton Raphson matrix to our network:

In general : The Newton Raphson equation for one nonlinear equation is (Jaan , 2005):

$$x_{k+1} = x_k - \left(\frac{\partial f}{\partial x}\right)^{-1} f(x) \quad (24)$$

In our case, we have a system of nonlinear equation,so we have a matrix as shown below:

$$\begin{bmatrix} P_1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}_{K+1} = \begin{bmatrix} P_1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}_{K} - \begin{bmatrix} \frac{\partial f_1}{\partial P_1} & \frac{\partial f_1}{\partial d_1} & \frac{\partial f_1}{\partial d_2} & \frac{\partial f_1}{\partial d_3} & \frac{\partial f_1}{\partial d_4} & \frac{\partial f_1}{\partial d_5} \\ \frac{\partial f_2}{\partial P_1} & \frac{\partial f_2}{\partial d_1} & \frac{\partial f_2}{\partial d_2} & \frac{\partial f_2}{\partial d_3} & \frac{\partial f_2}{\partial d_4} & \frac{\partial f_2}{\partial d_5} \\ \frac{\partial f_3}{\partial P_1} & \frac{\partial f_3}{\partial d_1} & \frac{\partial f_3}{\partial d_2} & \frac{\partial f_3}{\partial d_3} & \frac{\partial f_3}{\partial d_4} & \frac{\partial f_3}{\partial d_5} \\ \frac{\partial f_4}{\partial P_1} & \frac{\partial f_4}{\partial d_1} & \frac{\partial f_4}{\partial d_2} & \frac{\partial f_4}{\partial d_3} & \frac{\partial f_4}{\partial d_4} & \frac{\partial f_4}{\partial d_5} \\ \frac{\partial f_5}{\partial P_1} & \frac{\partial f_5}{\partial d_1} & \frac{\partial f_5}{\partial d_2} & \frac{\partial f_5}{\partial d_3} & \frac{\partial f_5}{\partial d_4} & \frac{\partial f_5}{\partial d_5} \\ \frac{\partial f_6}{\partial P_1} & \frac{\partial f_6}{\partial d_1} & \frac{\partial f_6}{\partial d_2} & \frac{\partial f_6}{\partial d_3} & \frac{\partial f_6}{\partial d_4} & \frac{\partial f_6}{\partial d_5} \end{bmatrix}^{-1} \begin{bmatrix} f_1(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_2(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_3(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_4(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_5(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_6(P_1, d_1, d_2, d_3, d_{4,} d_5) \end{bmatrix} \quad (25)$$

$\frac{\partial f_1}{\partial P_1}$: is the first derivative of the first function f_1 respect to P_1

$$\begin{bmatrix} P_1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}_{K+1} = \begin{bmatrix} P_1 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}_{K} - \begin{bmatrix} 1 & \frac{-2*4828032\rho V_1^2 f}{4d_1^2} & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{-2*4828032\rho V_2^2 f}{4d_2^2} & 0 & 0 & 0 \\ 1 & 0 & 0 & \frac{-2*4828032\rho V_3^2 f}{4d_3^2} & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{-2*4828032\rho V_4^2 f}{4d_4^2} & 0 \\ 1 & 0 & 0 & 0 & 0 & \frac{-2*4828032\rho V_5^2 f}{4d_5^2} \\ 0 & -2\frac{\pi}{4}V_1 d_1 & -2\frac{\pi}{4}V_2 d_2 & -2\frac{\pi}{4}V_3 d_3 & -2\frac{\pi}{4}V_4 d_4 & -2\frac{\pi}{4}V_5 d_5 \end{bmatrix}^{-1} \begin{bmatrix} f_1(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_2(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_3(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_4(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_5(P_1, d_1, d_2, d_3, d_{4,} d_5) \\ f_6(P_1, d_1, d_2, d_3, d_{4,} d_5) \end{bmatrix}$$

(26)

Step # 5: These steps can be repeated several time starting with the initial guess, so Matlab code is used to do the job to get the next results:

P_1=642200 Pa d_1=0.3231m d2=0.3171m d_3=0.3106m d_4=03033m d_5=0.2954m

Step # 6: compute the head (the input power) by :

$$H = \frac{P_1 - P_{atm}}{\rho\, g} \quad (27)$$

H=55.1923m

The Cost of the system

The cost of the valves can be calculated by:

\$100 plus \$100 per 0.0254m of pipe size diameter

$= 100 * 4 * 5 + 100 * 4 * (0.3231 + 0.3171 + 0.3106 + 0.3033 + 0.2954) / 0.0254$

$= \$26400$

The cost of Pipe lines can be calculated by:

50 cent per meter of diameter per meter of length

$= 0.5 * (0.3231 * 4828.032 + 0.3171 * 4023.36 + 0.3106 * 3218.688 + 0.3033 * 2414.016 + 0.2954 * 1609.344)$

$= \$99272.44$

$$\text{Material cost} = 26400 + 99272.44 = \$125672.4$$

The cost of electricity for the 5 year period:

$H = \frac{P_1 - P_{atm}}{\rho g} = \frac{6.4222*10^5 - 101325}{999*9.81} = 55.19\, m$

$Pinp = \rho * g * Q * H$

$= 999 * 9.81 * 0.2628 * 55.19$

$= 142.14\,KW$

$=$

$Power \cos t = 0.07 * 142.14 * 5 * 8 * 365 = \$145267$

Finally, the total cost of the whole system:

$Total \cos t = material \cos t + power \cos t$

$= 12567244 + 145267 = \$27093644$

The second design (compared to Newton Raphson)

The second design was done under the fact that the speed in pipe network is restricted between 2.95ft/s (0.899m/s) and 6.88ft/s (2.097m/s). The speed was assumed to the first pipe line as 0.899m/s. and then adding an equaled rate until it reaches 2.097m/s in the fifth line.The following steps show the method of the second design.

Step#1: compute the diameters from the range of velocities:

- The ranges are (V1=0.899, V2=1.197, V3=1.499, V4=1.801, V5=2.097) m/s
- The assumed discharge are Q1=Q2=Q3=Q4=Q5=0.0525 m^3/s
- The diameters can be calculated by:

$$d = \sqrt{\frac{4}{\pi} * A} \qquad (28)$$

d1=0.272m d2=0.236m d3=0.211m d4=0.192m d5=0.178m

step#2: compute the pressure from Bernoull's equation:

computing Re, friction factor, and then P1

$$p_1 - p_{atm} = \rho g z_2 + \frac{\rho V_1^2}{2}\left(\frac{4828.032\, f}{d_1} + 159ke + 4kv - 1\right) \qquad (29)$$

P1=853783.5 Pa

Step# 3: The head

$$H = \frac{P_1 - P_{atm}}{\rho\, g} \qquad (30)$$

H=76.78 m

Cost of system

The cost of Valves:

\$100 plus \$100 per 0.0254m of pipe size diameter

$= 100 * 4 * 5 + 100 * 4 * (0.272 + 0.236 + 0.211 + 0.192 + 0.178) / 0.0254$

$= \$19149.6$

The cost of Pipe lines:

50 cent per 0.0254m of diameter per m of length

$= 0.5 * (0.272 * 4828.032 + 0.236 * 4023.36 + 0.211 * 3218.688 + 0.192 * 2414.016 + 0.178 * 1609.344) / 0.0254$

$= \$72674$

$material \cos t = 19149.6 + 72674 = \$91823.6$

The cost of electricity for the 5 year period:

$Pinp = \rho * g * Q * H$

$= 999 * 9.81 * 0.2628 * 76.78$

$= 197.74\,KW$

$=$

$Power \cos t = 0.07 * 197.74 * 5 * 8 * 365 = \$20209028$

Finally, the total cost of the whole system:

$Total \cos t = material \cos t + power \cos t$

$= 918236 + 20209028 = \$29391388$

III. Conclusion

In the first design of using Newton Raphson method, the cost of the pipes and electricity had reasonable effects on the total cost. The largest three diameters d1,d2,d3 with the longest three pipes L1,L2,L3, and smallest three diameters d4,d5 with the shortest pipes L4,L5.

So, the ratio L/d resulted a reasonable head, and then less cost of electricity compare to the second design, and the size of the diameters resulted a reasonable cost of pipe lines because of the size of the diameters that were got by Nowton Raphson . In the second design, the assumption of the recommended velocity in pipes gave much more cost compare to the first design. That may because of the not accurate range of velocities in pipe.

Nomenclature

Q discharge
P Pressure
H head
L Length of pipe
D Diameter of pipe
V Velocity
F friction factor
Re reynold's number
Z The elevation
A Cross section area
Patm Atmospheric pressure
Ke loss coefficient of elbow
Kv loss coefficient of valve
g gravity
e: roughness
$P_{inp:}$ the input power

References

[1]. Bruce R. Munson ,Donald Young .(2009), Theodore H. Okishi , Wade W. Huebsch , fundamentals of fluid mechanics , Sixth Edition , John Wiley &Sons.INC, United State of America .

[2]. Jaan Kiusalaas .(2005), Numerical methods in engineering with matlab , first edition, Cambridge University, United State of America.

[3]. Philip j. Pritchard , John C. Leylegian (2011) , Introduction to fluid mechanics , Eighth Edition , John Wiley &Sons.INC . United State of America.

[4]. S. R. Otto and J. P. Denier. (2005), An Introduction to programming and numerical method in matlab , first edition, Springer verlag London limited, United State of America.

[5]. Yunus A. Cengel , John M. Cimbala.(2006), fluid mechanics fundamentals and applications , first edition, McGraw-Hill, United State of America.