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Free Convection and Mass Transfer Flow through a Porous Medium with Variable Viscosity and Thermal Conductivity

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Abstract : *The unsteady free convection and mass transfer flow past an exponentially accelerated infinite vertical plate with variable viscosity and thermal conductivity has been investigated in this work. The plate temperature is raised linearly with time and the concentration level near the plate is raised to* C'_∞ *.The governing boundary layer equations of momentum, temperature and concentration are the second order couple linear partial differential equation. Mathematical formulation of boundary layer equations have been non-dimensionalized by using dimensionless variables. These non-dimensional boundary layer equations are non-linear and partial differential equations and are solved by finite difference method. For the different physical parameters the results of velocity, temperature and concentrations are displayed in the form of level curves. Skin friction and Nusselt number are also described in graphically.*

Keywords: *Porous medium, Variable viscosity, Variable thermal conductivity, vertical plate, free convection*

I. Introduction

Natural convection induced by the simultaneous action of buoyancy forces from thermal and mass diffusion is of considerable interest in many industrial applications such as geophysics, oceanography, drying processes and solidification of binary alloy. The effect of the magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gases. The thermal physics of MHD problems with mass transfer is of interest in power engineering and metallurgy. When free convection flows occur at high temperature, radiation effects on the flow become significant. Many processes in engineering areas occur at high temperatures and knowledge of radiative heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles and space vehicles are examples of such engineering areas.

Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate and solve the governing equations analytically [1]. Muthucumaraswamy and Janakiraman (2006) studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion [2]. Suneetha and Bhaskar (2011) formulated in an (x, y, t) coordinate system with appropriate boundary conditions [3]. Effects on boundary layer flow and heat transfer of a fluid with variable viscosity along a symmetric wedge is presented here by Mukhopadhyay (2009) [4]. Effects of variable viscosity, Dufour, Soret, and thermal conductivity on free convective heat and mass transfer of non-Darcian flow past porous flat surface has been described by Animasuan and Oyem (2014) [5]. Rajput and Surendra (2011) studied the MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion [6].

Theoretical solution of unsteady flow past a uniformly accelerated infinite vertical plate has been presented by Muthucumaraswamy *et al.* (2009) in the presence of variable temperature and uniform mass diffusion [7]. Rajesh (2010) discussed the effect of a uniform transverse magnetic field on the free convection and mass- transform flow of an electrically- conducting fluid past an exponentially accelerated infinite vertical plate through a porous medium [8]. Sing *et al.*(2003) studied the two dimensional free convection and mass transfer flow of an incompressible viscous and a continuously moving infinite vertical porous plate in the presence of heat source, thermal diffusion, large suction and under the influence of uniform magnetic field applied normal to the flow is studied and perturbation technique is used to solve the governing equation [9]. Mehmood and Ali (2007) studied the effect of the wall slip on velocity field [10]. By Rajesh and Vijaya (2009) an analytical study is performed to study the effects of thermal radiation on unsteady free convection flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field [11]. Recently Effects of thermoporesis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non- Darcian MHD dissipative casson fluid flow with suction and n^{th} order chemical reaction has been discussed by Animasaun (2014) [12].In this research work we studied the unsteady free convection and mass transfer flow through a porous medium with variable viscosity and thermal conductivity. We used proper transformations to make the governing equations dimensionless. Then the dimensionless governing equations have solved by explicit finite difference method.

II. Mathematical Analysis

The unsteady free convection and mass transfer flow through a porous medium with variable viscosity and thermal conductivity has been considered. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The flow is assumed be in x'-direction which is taken along the vertical plate in the upward direction and y'-axis is taken to be normal to the plate. Initially the plate and the fluid are assumed at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all points. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 e^{a't'}$ is its own plane and the plate temperature and concentration are raised linearly with the time t and the fluid considered here is a absorbing or emitting radiation but a non-scattering medium. The viscous dissipation is assumed to be negligible. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following momentum, temperature and concentration equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) - \frac{\sigma B_0^2}{\rho}(u' - u_0 e^{a't'}) - \frac{\nu u'}{K'} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{1}{\rho C_p}\frac{\partial}{\partial y'}\left(\kappa\frac{\partial T'}{\partial y'}\right) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

The boundary conditions related with the problem are,

$$\left.\begin{aligned} &t' \le 0, u' = 0, T' = T'_\infty, C' = C'_\infty \text{ for all } y' \\ &t' > 0, u' = u_0 e^{a't'}, T' = T'_\infty + (T'_w - T'_\infty)At', C' = C'_w \text{ for all } y' = 0 \\ &u' = 0, T' \to T'_\infty, C' = C'_\infty \quad at \quad y' \to \infty \end{aligned}\right\} \tag{4}$$

Here, $A = \frac{U_0^2}{\nu}$

It is assumed that consider that the viscosity as well as the thermal conductivity varies as a linear function of temperature. (Animasaun, I.L, 2014)

$\mu(T') = \mu^*\left(1 + \lambda(T' - T'_\infty)\right)$ and $\kappa(T') = \kappa^*\left(1 + \delta(T' - T'_\infty)\right)$

Let γ and ε denote the non-dimensional viscosity and thermal conductivity parameter and be given by $\gamma = \lambda(T'_w - T'_\infty)$ and $\varepsilon = \delta(T'_w - T'_\infty)$

Then the fluid viscosity and thermal conductivity takes the following form

$\mu = \mu^*(1 + \gamma\theta)$ and $\kappa = \kappa^*(1 + \varepsilon\theta)$

On introducing the following non-dimensional quantities,

$$\begin{aligned} &U = \frac{u'}{U_0},\ Y = \frac{y'U_0}{\vartheta},\ S_c = \frac{\vartheta}{D},\ M = \frac{\sigma B_0^2 \vartheta}{\rho U_0^2},\ P_r = \frac{\mu C_p}{\kappa},\ a = \frac{a'\vartheta}{U_0^2},\ G_r = \frac{g\beta\vartheta(T'_w - T'_\infty)}{U_0^3}, \\ &y' = \frac{Y\vartheta}{U_0},\ \tau = \frac{t'U_0^2}{\vartheta},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ k = \frac{U_0^2 K'}{\vartheta^2},\ G_c = \frac{g\beta^*\vartheta(C'_w - C'_\infty)}{U_0^3}, \end{aligned} \tag{5}$$

Using the equation (5) the non dimensional form of the equation (1) to (4) is as follows,

$$\frac{\partial U}{\partial \tau} = G_r\theta + G_c C + \frac{\partial^2 u}{\partial Y^2} + \frac{\gamma}{1 + \gamma\theta}\left(\frac{\partial \theta}{\partial Y}\right)\left(\frac{\partial U}{\partial Y}\right) - MU - \frac{U}{k} \tag{6}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{P_r}\left(\frac{\partial^2 \theta}{\partial Y^2} + \frac{\varepsilon}{1 + \varepsilon\theta}\left(\frac{\partial \theta}{\partial Y}\right)^2\right) \tag{7}$$

$$\frac{\partial C}{\partial \tau} = \frac{1}{S_c}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

Subjected to the boundary conditions

$\tau \le 0,\ U = 0,\ \theta = 0,\ C = 0$ for all Y

$\tau > 0,\ U = e^{at},\ \theta = t,\ C = 1$ for all $Y = 0$

Skin friction and Nusselt number:

The dimensionless skin-friction coefficient is generally known as the shear stress at the plate and defined as follows, $\tau_x = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and $\tau_y = \mu\left(\frac{\partial w}{\partial y}\right)_{y=0}$. The dimensionless rate of heat transfer is known as the Nusselt number and is defined as follows, $N_u = -\mu\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$.

III. Numerical Solution

Using the explicit finite difference approximation, the following appropriate set of finite difference of the equation (6) to (8) are obtained as,

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} = G_r\theta_{i,j} + G_m C_{i,j} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + \left(\frac{\varepsilon}{1+\varepsilon\theta_{i,j}}\right)\left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y}\right)\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right) - MU_{i,j} - \frac{U_{i,j}}{K} \tag{10}$$

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta\tau} = \frac{1}{P_r}\left(\left(\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2}\right) + \left(\frac{\gamma}{1+\gamma\theta_{i,j}}\right)\left(\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y}\right)^2\right) \tag{11}$$

$$\frac{C'_{i,j} - C_{i,j}}{\Delta\tau} = \frac{1}{S_c}\left(\frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{(\Delta Y)^2}\right) \tag{12}$$

Then the initial and boundary condition takes the following form as,

$$\tau \le 0: U^0_{i,0} = 0,\ \theta^0_{i,0} = 0,\ C^0_{i,0} = 0, \text{ for all } i$$

$$\tau > 0: U^0_{0,j} = \exp(a.j.\Delta t),\ \theta^0_{0,j} = j.\Delta t,\ C^0_{0,j} = 1 \tag{13}$$

$$U^n_{L,j} = 0,\ \theta^n_{L,j} = 0,\ C^n_{L,j} = 0$$

Where L corresponds to ∞

IV. Results And Discussions

In order to get the clear insight of the physical problem, numerical results are displayed with the help of graphs. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters. In the present study we have chosen the default value of each different physical parameter like as thermal Grashof number (G_r), mass Grashof number (G_m), magnetic parameter (M), accelerated parameter (a), permeability parameter (k), Schmidt number for water vapor, Ammonia and Benzine (S_c), viscosity variation parameter (ε), thermal conductivity parameter (γ), Prandtl number for air, carbon dioxide and water (P_r) and time (τ).

The effects of different physical parameters on velocity profiles have shown in the Fig (1) to (10). Fig (1) depicts the effect of accelerated parameter (a) on velocity profiles. It is clear from the figure that velocity increases near the plate for the increase of accelerated parameter. From the Fig (2) it is observed that velocity increases with the increasing value of viscosity variation parameter (ε). The effects of thermal Grashof number (G_r) and mass Grashof number (G_m) on velocity profile described in the Fig (3) and (4). It shows that the velocity increases because of increasing value of thermal Grashof number and mass Grashof number. The Fig (5) represents the velocity distribution for different values of permeability parameter (k). It is noticed that from the figure velocity decreases for the decreasing value of permeability parameter. From Fig (6) to (8) reveals the velocity distribution for the several values of magnetic parameter (M), Prandtl number (P_r) and Schmidt number (S_c). The following graphs illustrate velocity profiles decreases for the increasing value of magnetic parameter, Prandtl number and Schmidt number. Lastly, in the Fig (9) to (10) the velocity distribution changes with the changing value of time (τ) and thermal conductivity parameter (γ).

It displays that velocity increases with the increasing value of time and thermal conductivity parameter. The temperature distribution for the different values of above parameters is described in the Fig (11) to (12). We see from the graph (11) to (12) is that temperature decreases for the increasing value of Prandtl number (P_r). On the other hand temperature increases for the increasing value of viscosity variation parameter (ε). In Fig (13), we see the effects of Schmidt number (S_c) on concentration profiles. It is noticed that concentration increases for the decreasing value of Schmidt number. At last Fig (14) to (15) focused the skin friction and Nusselt number profiles. In Fig (14) we observed that skin friction increases for the increasing value of viscosity variation parameter (ε). But in case of Nusselt number the effect is reversed comparing to the effects of skin friction. The effect of thermal conductivity parameter (γ) on skin friction is displayed in the Fig (15). It shows that if we increase the value of thermal.

Nomenclature: C'_∞ Concentration in the fluid far away from the plate, A is Constant, y' Coordinate axis normal to the plate, C Dimensionless Concentration, Y Dimensionless Concentration axis normal to the plate, U Dimensionless velocity, B_0 External magnetic field, G_m Mass Grashof number, P_r Prandtl number, N_u Nusselt number, S_c Schmidt number, C' Species concentration in the fluid, C_p Specific heat at constant pressure , T'_∞ Temperature of the fluid far away from the plate, T' Temperature of the fluid near the plate, T'_w Temperature of the plate, κ Thermal conductivity of the fluid, G_r Thermal Grashof number, t' Time, u' Velocity of the fluid in the x'-direction, U_0 Velocity of the plate, a Accelerating parameter, D Mass diffusivity coefficient, g Acceleration due to gravity, k Permeability parameter, M Magnetic field parameter , τ Dimensionless time , ρ Density of the fluid, τ_x, τ_y Dimensionless skin friction, θ Dimensionless temperature, ϑ Kinematic viscosity, β^* Volumetric coefficient of expansion Concentration, β Volumetric coefficient of thermal expansion, w Conditions on the wall, ∞ Free stream conditions.

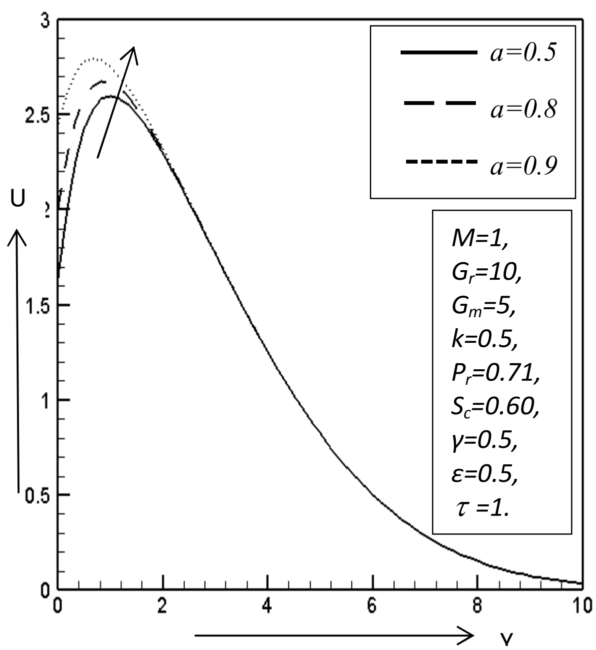

Fig 1: Velocity profile for different values of *a* against *Y*.

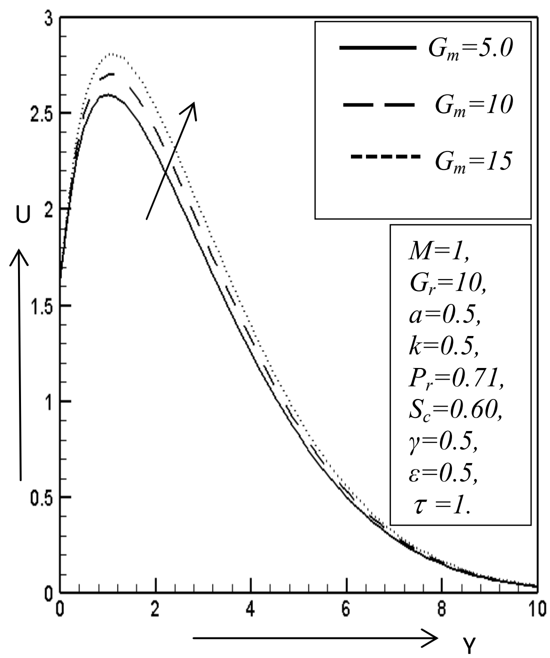

Fig 3: Velocity profile for different values of G_m against *Y*.

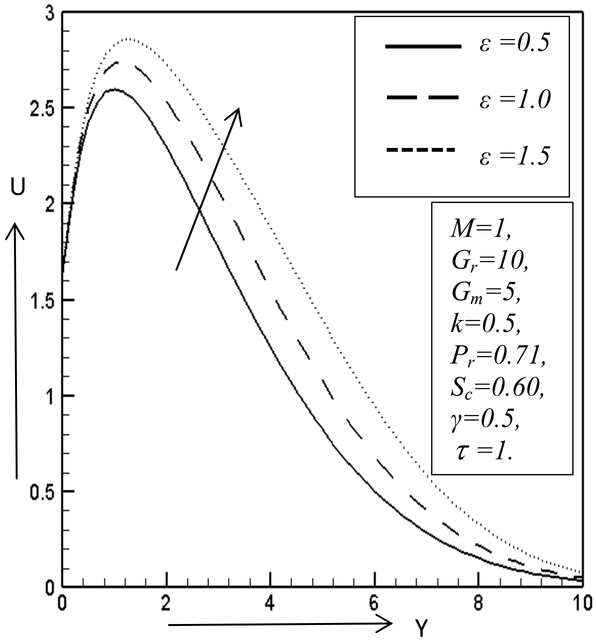

Fig 2: Velocity profile for different values of ε against Y.

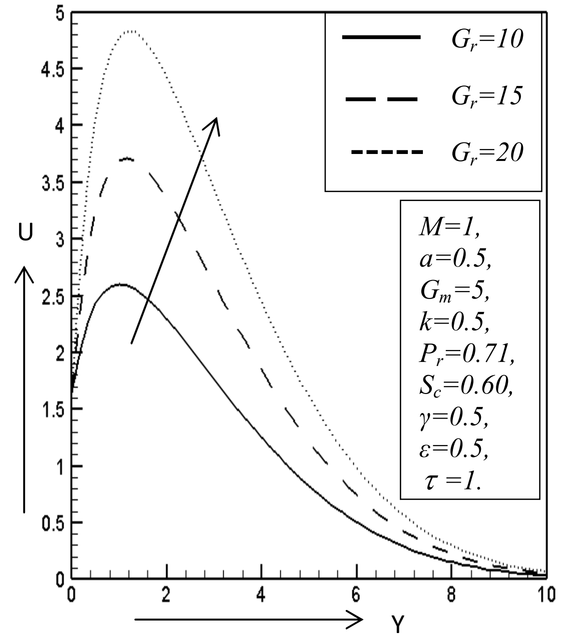

Fig 4: Velocity profile for different values of G_r against Y.

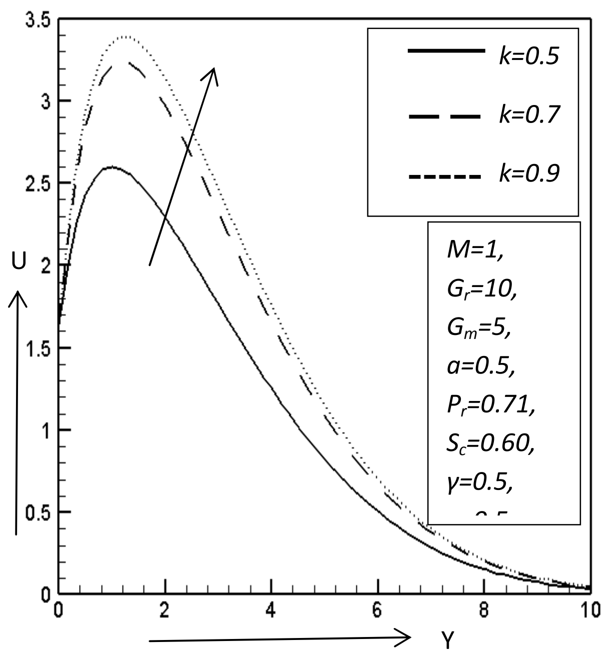

Fig 5: Velocity profile for different values of k against Y.

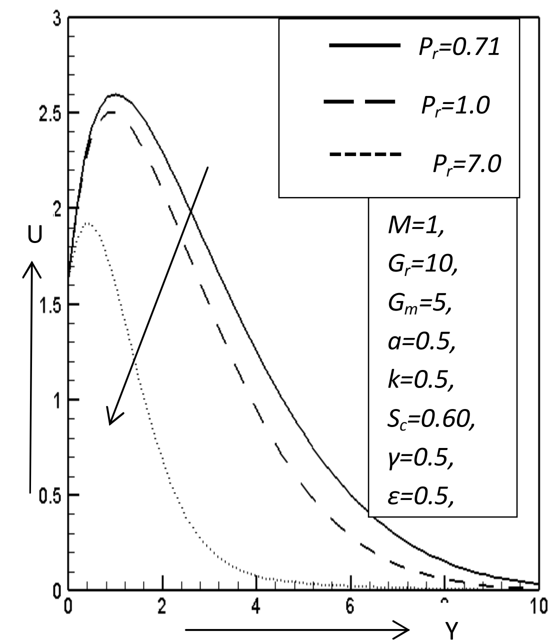

Fig 7: Velocity profile for different values of P_r against Y.

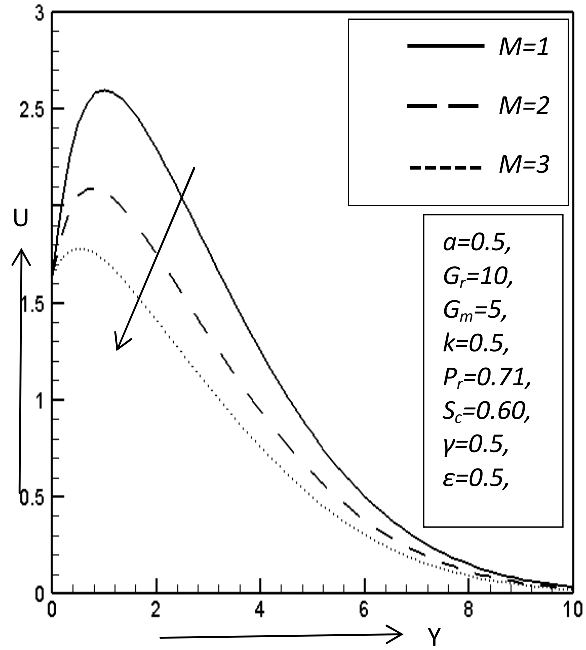

Fig 6: Velocity profile for different values of M against Y.

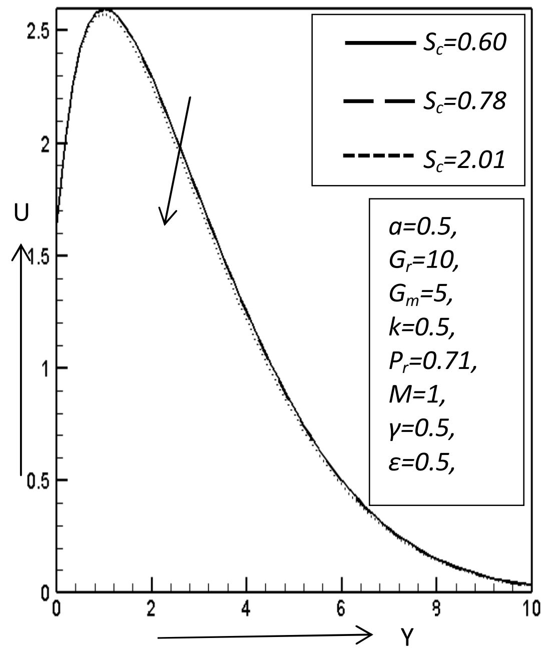

Fig 8: Velocity profile for different values of S_c against Y

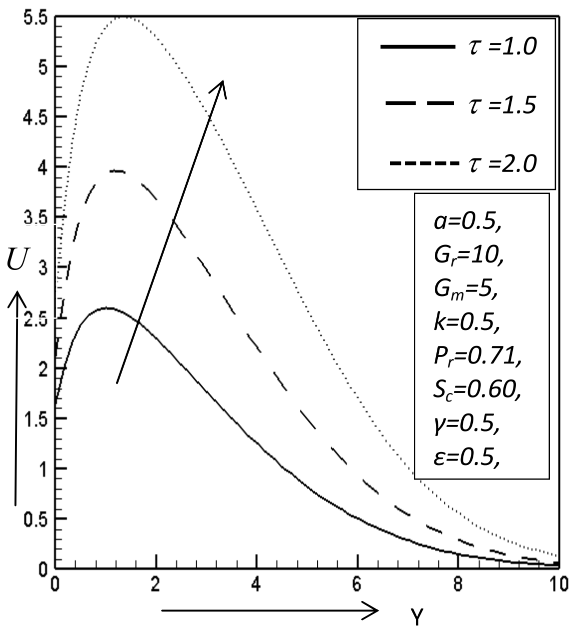

Fig 9: Velocity profile for different values of t against Y.

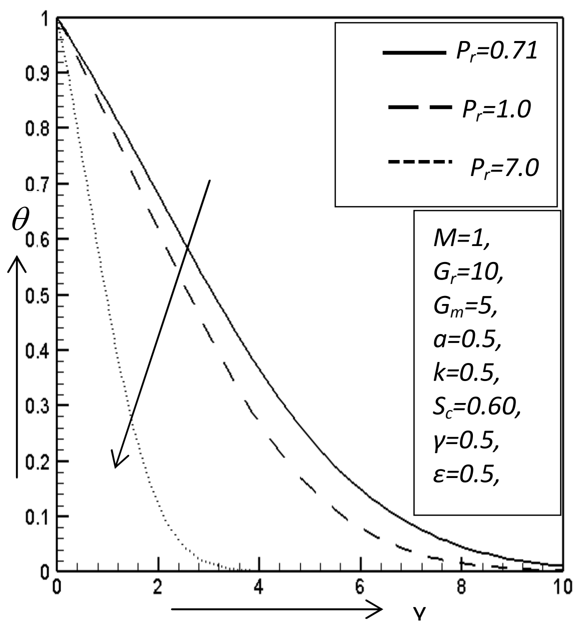

Fig 11: Temperature profile for different values of P_r against Y.

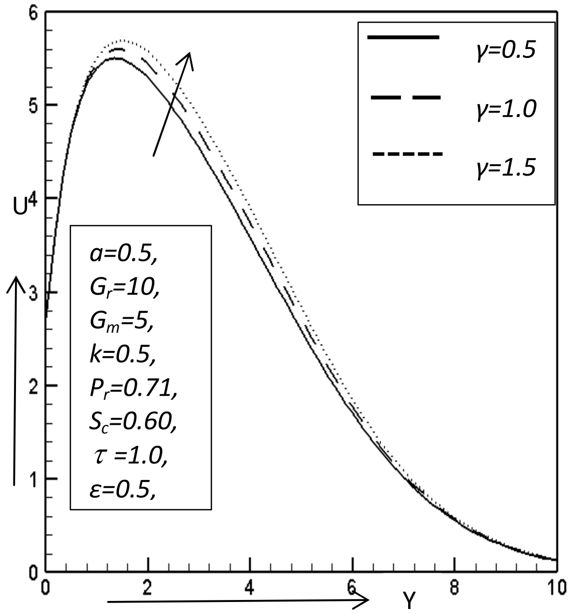

Fig 10: Velocity profile for different values of γ against Y.

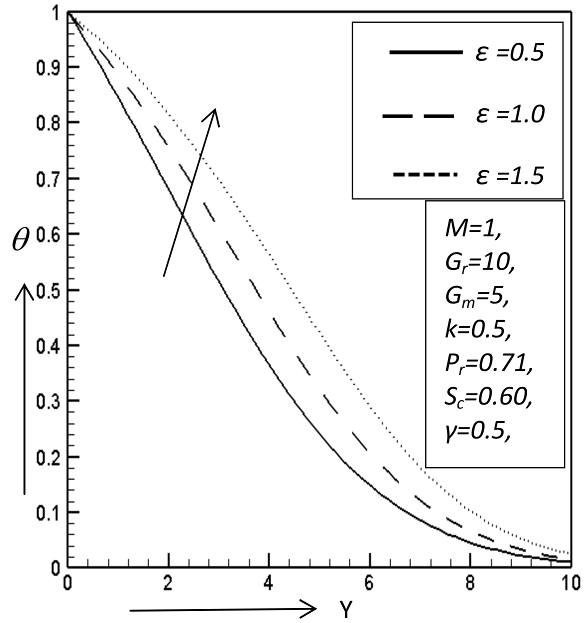

Fig 12: Temperature profile for different values of ε against Y.

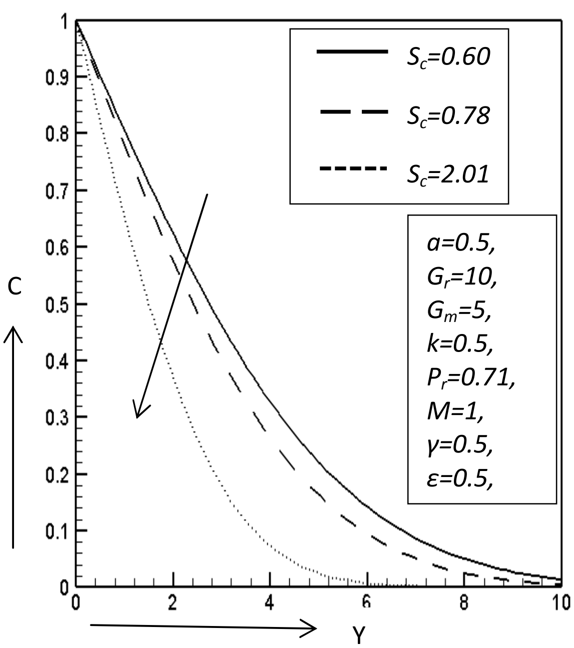

Fig 13: Concentration profile for different values of S_c against Y.

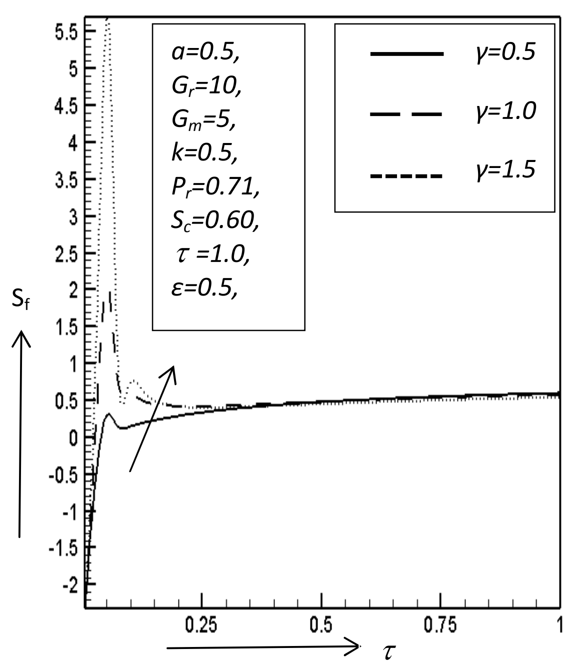

Fig 15: Skin friction profile for different values of γ against τ .

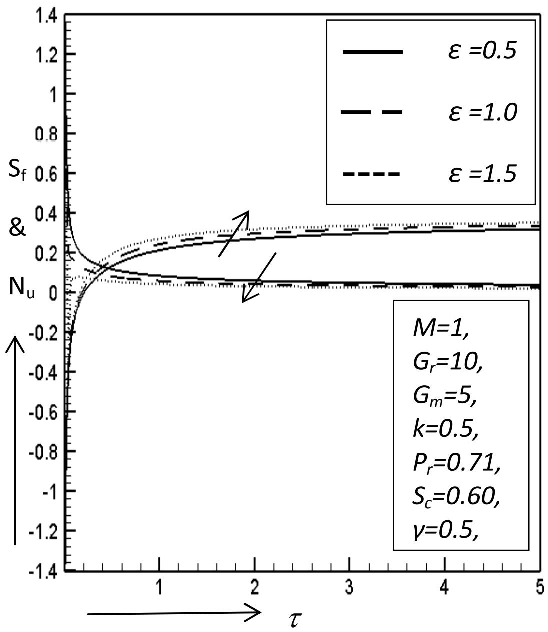

Fig 14: Skin friction and Nusselt number profile for different values of ε against τ .

V. Conclusion

It is clear from the above discussion that velocity profiles increases for the increasing value of accelerated parameter (a), viscosity variation parameter (ε), thermal Grashof number (G_r), mass Grashof number (G_m), permeability parameter (k), time (τ) and thermal conductivity parameter (γ). On the other hand velocity profiles decreases for the increasing value of magnetic parameter (M), Prandtl number (P_r) and Schmidt number (S_c). Temperature profiles increases for the increasing value of viscosity variation parameter (ε) and decreases for the increasing value of Prandtl number (P_r). Concentration decreases for the increasing value of Schmidt number (S_c). Skin friction increased with the increasing value of viscosity variation parameter (ε) and thermal conductivity parameter (γ). Nusselt number decreases for the increasing value of viscosity variation parameter (ε).

References

[1] Raptis, A. Perdikis, C. (1999) Radiation and free convection flow past a moving plate. Int. J. of App. Mech. and Engg. 4: 817-821.
[2] Muthucumaraswamy, R. Janakiraman (2006) MHD and Radiation effects on moving isothermal vertical plate with variable mass diffusion. Theoret. Appl. Mech. 33(1): 17-29.
[3] Suneetha, S. Bhaskar, N. Reddy (2011) Radiation and Darcy Effects on Unsteady MHD Heat and Mass Transfer Flow of a Chemically Reacting Fluid Past an Impulsively Started Vertical Plate with Heat Generation. Int. J. of Appl. Math and Mech. 7(7): 1-19.
[4] Mukhopadhyay, S. (2009) Effects of Radiation and Variable Fluid Viscosity on Flow and Heat Transfer along a Symmetric Wedge. Journal of Applied Fluid Mechanics. 2 (2): 29-34.
[5] Animasaun, I. L. and Oyem, A.O., (2014), Effects of variable viscosity, Dufour, Soret, and thermal conductivity on free convective heat and mass transfer of non- Darcian flow past porous flat surface, American J. of Computational mathematics, 4, p357-365.
[6] Rajput, U. S. Surendra Kumar (2011) MHD Flow Past an Impulsively Started Vertical Plate with Variable Temperature and Mass Diffusion. Applied Mathematical Sciences. 5 (3): 149 – 157.
[7] Muthucumaraswamy, R. Sundar Raj, M. Subramanian, V. S. A. (2009) Unsteady Flow Past an Accelerated Infinite Vertical Plate With Variable Temperature and Uniform Mass Diffusion. Int. J. of Appl. Math and Mech. 5(6): 51-56.
[8] Rajesh, V. (2010) MHD Effects on Free Convection and Mass Transform Flow Through a Porous Medium with Variable Temperature. Int. J. of Appl. Math and Mech. 6(14): 1-16.
[9] Shing, N. P. Ajay Kumar Shing, Atul Kumar Singh (2003) MHD Free Convection and Mass Transfer Flow Past a Flat Plate. The Arabian Journal for Science and Engineering. 32 (1A): 93-114.

[10] Mehmood, A. Ali, A. (2007) The Effect of Slip Condition on Unsteady MHD Oscillatory Flow of a Viscous Fluid in a Planer Channel. Rom. Journ. Phys. 52 (1-2): 85-91, Bucharest.

[11] Rajesh, V. Vijaya Kumar Varma, S. (2009) Radiation and Mass Transfer Effects on MHD Free Convection Flow Past an Exponentially Accelerated Vertical Plate with Variable Temperature. ARPN Journal of Engineering and Applied Sciences. 4 (6): 20-26.

[12] Animasaun, I. L. (2014), Effects of thermoporesis, variable viscosity and thermal conductivity on free convective heat and mass transfer of non-Darcian MHD dissipative casson fluid flow with suction and n^{th} order chemical reaction, J. of Nigerian mathematical society, 34, p11-31.