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Efficient Sizing of Rigidly Fixed Portal Frames under Different Static Loads

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Abstract: *This work considered the height to span ratio (h/L) ratios that would give the most economical design for portal frames under different static loads. The equations for minimum depth for each section of the portal frame were first developed. These were used to formulate the equation for the volume of the frame. The volume of the frame was taken to be proportional to the cost of the portal frame. The usefulness or benefit of the frame was computed as the ratio of the frame's cross-sectional area to its perimeter. The economical height to span ratio (h/L) for any kind of load were the ratios that gave the least cost/benefit value. These values were found to depend on the ratio of the load to the grade of portal frame material w/σ and the thickness b of the portal frame.*
Keywords: *portal frames, frame cost*

I. Introduction

A structure is a system of connected components used to support a load [1]. Portal frames consist of vertical members called columns and a top member which may be horizontal, curved or pitched with monolithic joints at the junction of columns [2]. It is estimated that around 50% of the hot-rolled constructional steel used in the UK is fabricated into single-storey buildings [3]. Portal frames are mostly used in single storey industrial structures. BS 5950 Part 1[4] allows for a linear elastic analysis or a plastic analysis of portal frames. The elastic analysis produces heavier structures which are however stable with little need for stability bracing [5]. Under a plastic analysis the aim is for a hinge to be formed at the point where the highest moment occurs. Failure is deemed to have taken place when the plastic hinges form a mechanism [6, 7]. In the analysis and design of portal frames the engineer normally uses his experience in determining the member sizes. Work on an efficient method for selecting member sizes and rise/span (h/L) ratio was done by John Righiniotis [5], but this was based on a plastic analysis for steel structures and was not load specific. This work is based on an elastic analysis and it was tailored to meet the requirements for each design load characteristics. The aim of this work is to provide rise/span ratios that would give the lowest cost/benefit ratios for frames under different static loads.

II. Methods

Four portal frames (frame 1 – frame 4) under different static loads were selected. The internal stresses generated by external static loads were calculated using formulae obtained from design books. The maximum stresses (bending moment and axial force) on each element of the portal frame were used in calculating the minimum depth for each element. With these depths and an assumed thickness b, the volume of each element was calculated and summed up to give the volume of the frame. The cost of the frame was assumed to be directly proportional to the volume of the frame. Hence a cost coefficient Cc which is a ratio of the actual cost of the frame to the product of the unit cost (cost per unit volume) and the thickness b was calculated. This became a measure of the cost of the frame for the purpose of this work. The usefulness or benefit B of the frame was calculated as the ratio of the cross-sectional area to the perimeter of the frame. This is a measure of spread of the section with a square giving the maximum value. The ratio of the cost coefficient to the benefit of the frame Cc/B was plotted against the different values of height to span (h/L) ratios and the values of h/L corresponding to minimum Cc/B obtained. These were done for different values of the ratio of load to grade of frame material w/σ. The value of frame thickness was kept constant at b = 0.3m.

III. Calculation

From strength of materials, stress σ at a section of a loaded structural member is given by

$$\sigma = \frac{M}{Z} \pm \frac{N}{A} \qquad (1)$$

[8, 9]

Where M is the bending moment at the section, N is the axial force in the member, A is the cross-sectional area of the member and Z is the section modulus of the cross-section. For rectangular sections

$$Z = \frac{bd^2}{6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where b and d are the breadth and depth of the sections respectively.
By substituting equation (2) into equation (1) we have

$$\sigma = \frac{M}{bd^2} \pm \frac{N}{bd} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) is an expression of the maximum and minimum stress at a section of a loaded rectangular section. By assuming that the stress σ is the maximum stress that can be resisted by the material of the structure (i.e. the grade of the material), the depth d of the section can be expressed in terms of the stress σ, bending moment M and axial force N using the almighty formula as

$$d = \frac{N + \sqrt{N^2 + 24Mb\sigma}}{2b\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$d = \frac{1}{2b}\left(\frac{N}{\sigma}\right) + \sqrt{\frac{1}{4b^2}\left(\frac{N^2}{\sigma^2}\right) + \frac{6}{b}\left(\frac{M}{\sigma}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

Since the stress σ is the grade of the material, M the bending moment at the section, N the axial force at the section, d is therefore the minimum depth of section that can overcome these internal stresses. When d is expressed in the form of equation (4a) it would be seen that d depends on the ratio of the internal stress M and N to the grade of the material. But under an elastic analysis of structures, the internal stresses are proportional to the load w. hence d is dependent on the ratio of the load w to the grade of material (w/σ).

For portal frames consisting of two vertical columns and a horizontal beam, equation (4) can be expressed as

$$d_i = \frac{N_i + \sqrt{N_i^2 + 24M_i b\sigma}}{2b\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$i = 1,2,3$ i is the element number

The cost of a portal frame is proportional to its volume. For a portal frame made up of prismatic members the cost can be expressed as

$$cost = K \sum_{i=1}^{3} L_i A_i \quad \dots \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where L_i is the length of the element i, A_i is the cross-sectional area of the element i and K is a constant of proportionality equivalent to the cost of a unit volume of the material of the portal frame.
For portal frames made up of rectangular elements of constant thickness b equation (6) reduces to

$$C_c = \sum_{i=1}^{3} L_i d_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where Cc is a cost coefficient equal to cost/Kb.
Equation (7) was used to calculate the cost coefficient of a portal frame as the sum of the cost coefficient of the individual elements of the portal frame.

A portal frame is useful when internally it is roomy i.e. it has space to permit its use for different purposes. To satisfy this requirement the portal frame has to be less compact.
The degree of compactness of a solid can be expressed as the ratio of its surface area to its volume.

$$Compactness = \frac{Surface\ Area}{Volume} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

But since we are treating the portal frame as a 2D structure, it has to be rewritten as

$$Compactness = \frac{Perimeter}{Area} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The less compact the frame is the more beneficial it would be for range of uses, hence

$$Benefit \propto \frac{Area}{Perimeter} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

By keeping the constant of proportional as unity

$$Benefit = \frac{hL}{2(h+L)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Where h is the height of the portal frame, L is the width or span of the portal frame.

IV. Results and Discussion

The equations for the determination of the internal moments M and N of a loaded portal frame is dependent on the ratio of the second moment of area of the beam section I_2 to the second moment of area of the column section I_1.[5, 9]. If we designate this as m then

$m = \frac{I_2}{I_1} = \frac{d_2^3}{d_1^3}$ . . . . (12)

where d_1 is the depth of the column member while d_2 is the depth of the horizontal beam member.
(The frame thickness b is the same for the beam and the column)

While m can be calculated from equation (12) using equation (5) to obtain the required d, m must first be known before equation (5) can be evaluated. There is therefore need to estimate a suitable value of m that will produce a set of internal stress M and N which on substitution into equation (5) will yield the same value of m. For Frame 1 (a portal frame with a uniformly distributed vertical load w) a graph of the estimated m, m_e against m_e plotted on the same graph with a graph of the calculated m, m_c against m_e is shown in Figure 1. The consensus value of m which is the value at the point where line m_e intersects line m_c is the value of m that can be used to evaluate m. These were evaluated for different values of the ratio of height to length (h/L) of the portal frame and presented in Table 1. For values of $w/\sigma > 0.75$ the two lines (me and mc) do not meet but can be close at some values of m as shown in Figure 2. For the cases where m_e and m_c do not intersect there is no sizing of the portal frame that would result in each member of the frame being stress optimally at the same time.

By using the consensus values of m and plotting a graph of the cost coefficient per unit benefit (Cc/B) against h/L for different values of w/σ, we obtain cost-benefit curves with minima at certain values of h/L, these minima were obtained at the highest values of h/L that has got a consensus m. These curves for certain values of w/σ are shown in Figure 3. A detailed results of the values of h/L, m, d_1 and d_2 corresponding to minimum Cc/B is given in Table 2.

The analysis was carried out on Frame 2 (a portal frame with a uniformly distributed horizontal load w), but unlike in Frame 1 the estimated m, me and the calculated m, mc only intersect at m = 0 for all values of w/σ and h/L. This is shown in Figure 4. The difference between the values of m_e and m_c increased at higher values of m_e hence lower values of m are preferable. A value of m = 0.25 was adopted. A graph of the cost coefficient per unit benefit (Cc/B) against h/L for different values of w/σ is presented in Figure 5. The values of h/L, d_1, d_2 and d_3 are presented in Table 2. Since the loading and internal stress distribution is not symmetrical the values of d_1 and d_3 obtained from equation (5) are different.

For frame 3 (a portal frame with a vertical concentrated load P at the centre) just like in frame 1 there is a consensus value of m only for limited values of h/L for each value of P/σ. For P/σ = 0.001, m exist only for h/L= 0.1 – 0.35. For values of h/L > 0.35 graphs similar to that presented in Figure 2 are obtained. This implies that for values of h/L > 0.35 there is no proportioning of the members of frame 3 that would result in each member being stressed optimally. At a value of m = 5.3, the difference between me and mc is a minimum. By adopting m = 5.3 the graph of Cc/B against h/L is plotted and presented in Figure 6. From the graph it would be seen that the cost coefficient per benefit decreased progressively with an increase in h/L. Hence the least cost is obtained by the smallest possible value of h/L.

For frame 4 (a portal frame with a concentrated horizontal load) there is no consensus value of m for all the possible values of P/σ and h/L. The graph of the estimated m, m_e against m_e plotted on the same graph with a graph of the calculated m, m_c against m_e is similar to that obtained in Figure 4. The lines m_e and m_c only intersect at m = 0 but there difference increased at higher values of m_e.

Just like in Frame 2 a value of m = 0.25 was adopted and the graph of cost coefficient per unit benefit against h/L produced is presented in Figure 7. The values of h/L, d1 and d2 corresponding to a minimum Cc/B at different values of P/σ are presented in Table 2.

V. Conclusion

As seen in the discussion above, the ratio of load to grade of frame material (material of the portal frame) and the height to width ratio (h/L) of a portal frame affect the cost of the frame. For portal frames supporting mostly a uniformly distributed vertical load (frame 1) the economical h/L ratio for various w/σ ratio can be obtained from Table 2. For frames supporting mostly a horizontal uniformly distributed load (frame 2) the ratio h/L = 0.35 proved to be the most economical for values of the ratio w/σ ranging from 0.001 to 0.03. For values of w/σ above 0.03, values of h/L = 0.4 should be adopted.

Frames designed primarily to support a vertical concentrated load (frame 3) should be assigned the maximum possible value of h/L as the higher the value of h/L the lower the frame's cost. Finally frames that support most a horizontal concentrated force should be designed with a h/L ratio of 0.45 as this would give the most economic design.

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Appendix

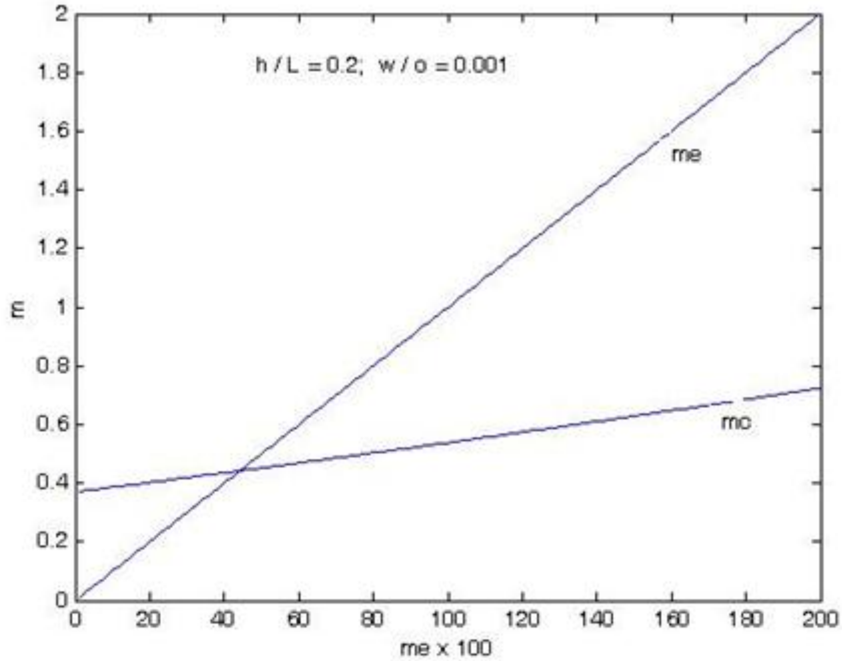


Figure 1: Graph of m_e and m_c against me for Frame 1

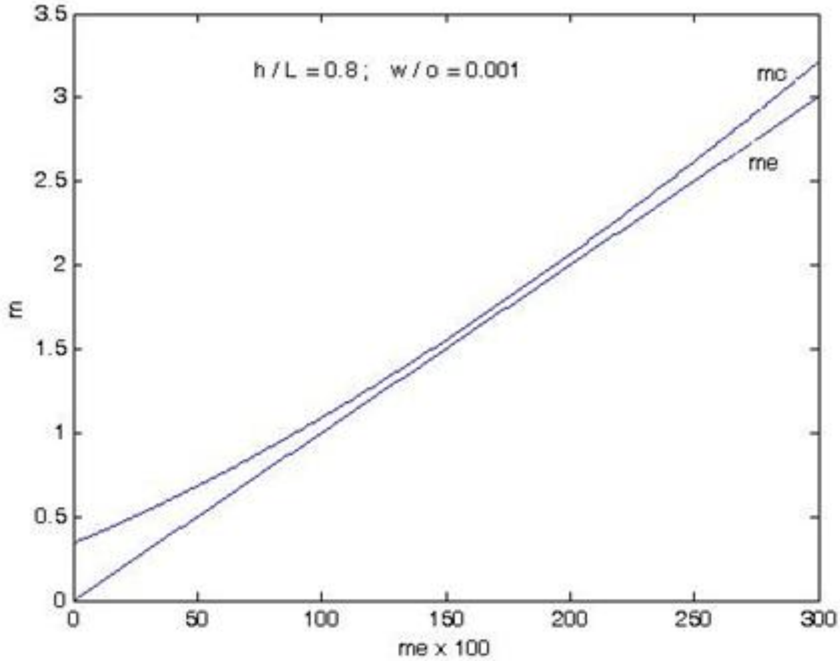


Figure 1: Graph of m_e and m_c against me for h/L > 0.75 in Frame 1

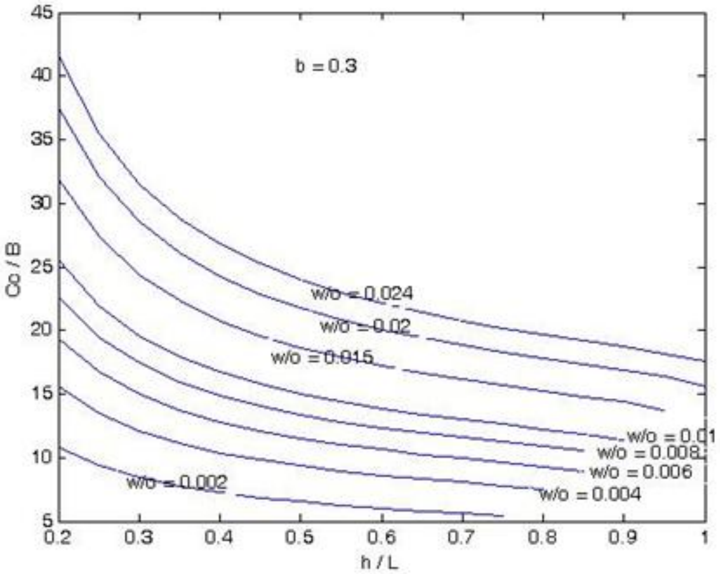


Figure 3: Graph of C_c/B against h/L for Frame 1

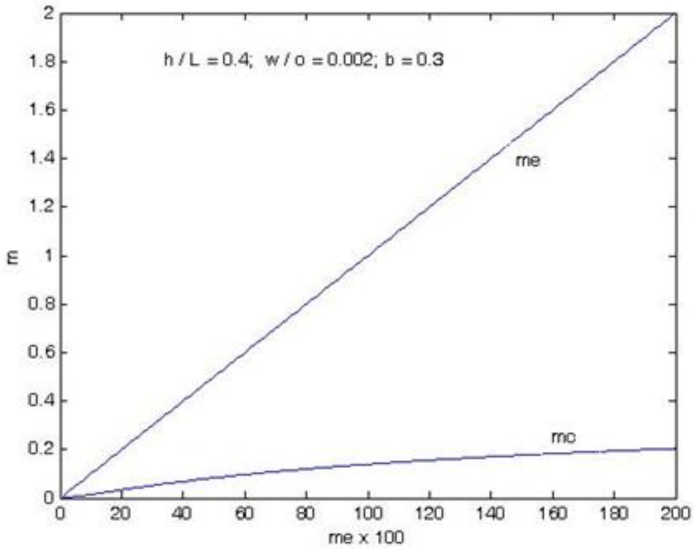


Figure 4: Graph of m_e and m_c against me for Frame 2

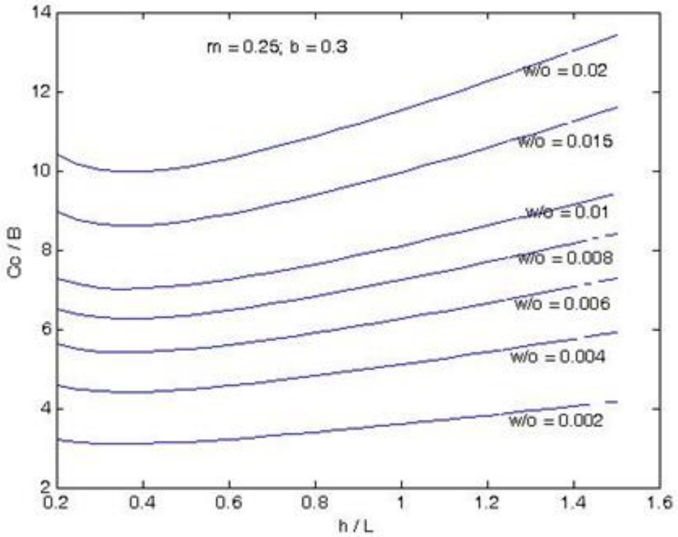


Figure 5: Graph of C_c/B against h/L for Frame 2

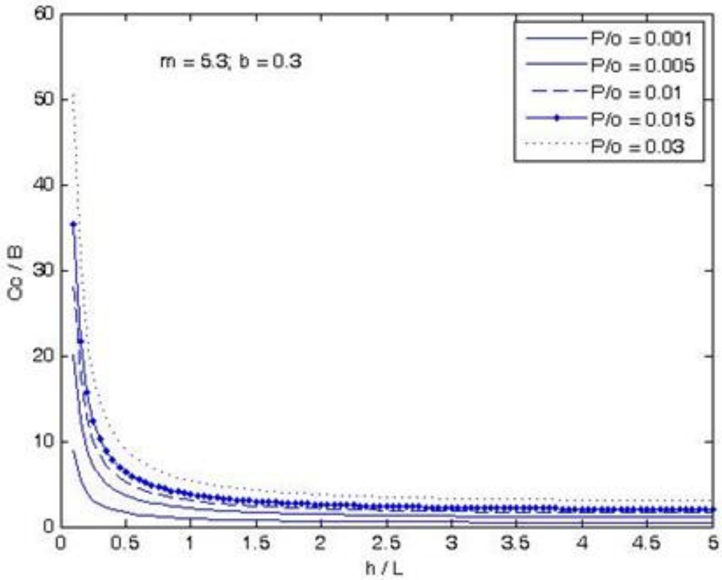


Figure 6: Graph of C_c/B against h/L for Frame 3

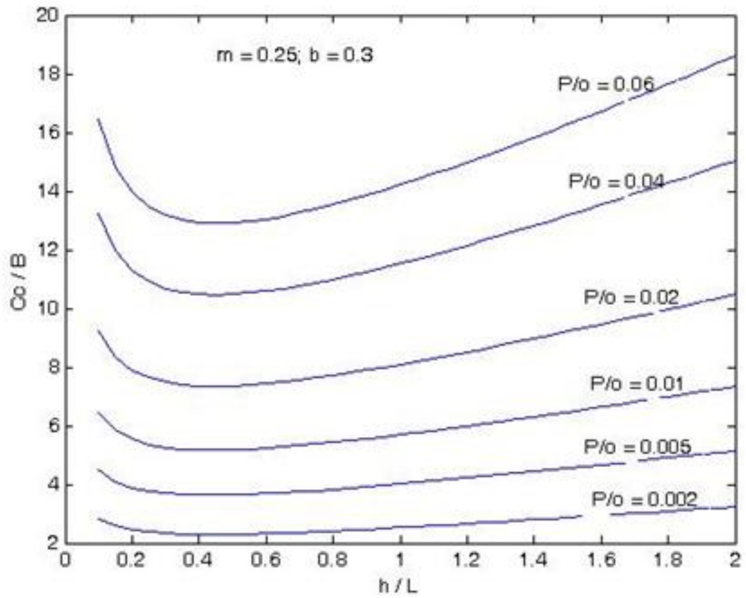


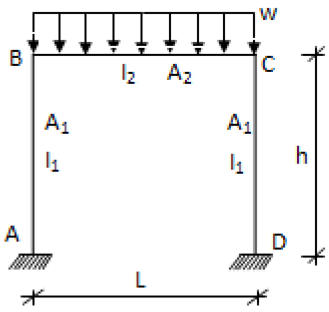
Figure 7: Graph of C_c/B against h/L for Frame 4

Table 1: Values of Consensus m for different values of h/L

h / L	m	d_1	d_2
0.20	0.4430	0.8158	0.6216
0.25	0.4550	0.6489	0.4989
0.30	0.4720	0.5373	0.4183
0.35	0.4950	0.4572	0.3616
0.40	0.5220	0.3969	0.3195
0.45	0.5560	0.3496	0.2874
0.50	0.5960	0.3114	0.2621
0.55	0.6480	0.2797	0.2420
0.60	0.7140	0.2526	0.2258
0.65	0.8050	0.2289	0.2129
0.70	0.9420	0.2071	0.2030
0.75	1.2120	0.1850	0.1973

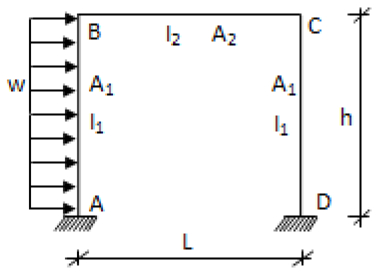
Table 2: Values of h/L, m, d_1 and d_2 corresponding to minimum C_c/B for different values of w/σ

FRAME 1



w/σ	h / L	m	d_1	d_2
0.002	0.75	1.063	0.2694	0.2749
0.003	0.80	1.311	0.2991	0.3274
0.004	0.80	1.1670	0.3542	0.3729
0.005	0.80	1.079	0.4029	0.4132
0.006	0.85	1.408	0.3965	0.4444
0.007	0.85	1.253	0.4390	0.4732
0.008	0.85	1.158	0.4773	0.5012
0.009	0.85	1.089	0.5131	0.5279
0.010	0.90	1.471	0.4837	0.5500
0.011	0.90	1.310	0.5198	0.5687
0.012	0.90	1.212	0.5520	0.5885
0.013	0.90	1.140	0.5822	0.6082
0.014	0.90	1.084	0.6108	0.6275
0.015	0.95	1.461	0.5668	0.6432
0.016	0.95	1.322	0.5979	0.6562
0.017	0.95	1.23	0.6259	0.6706
0.018	0.95	1.161	0.6521	0.6853
0.019	0.95	1.107	0.6769	0.7002
0.020	1.00	1.554	0.6184	0.7165
0.022	1.00	1.284	0.6756	0.7342
0.024	1.00	1.150	0.7225	0.7569
0.026	1.05	1.515	0.6801	0.7811
0.028	1.05	1.284	0.7311	0.7946

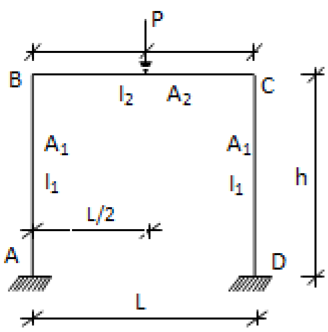
FRAME 2



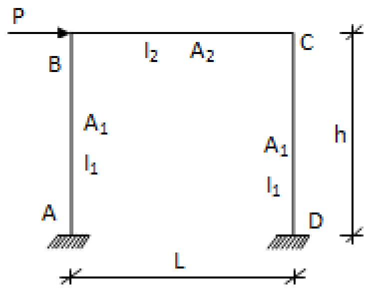
w/σ	h / L	d_1	d_2	d_3
0.001	0.35	0.3010	0.100	0.2262
0.002	0.35	0.4258	0.1421	0.3200
0.004	0.35	0.6023	0.2025	0.4526
0.006	0.35	0.7379	0.2495	0.5545
0.008	0.35	0.8521	0.2895	0.6404
0.010	0.35	0.9529	0.3250	0.7162
0.015	0.35	1.1674	0.4017	0.8775
0.020	0.35	1.3483	0.4674	1.0136
0.025	0.35	1.5078	0.5261	1.1336
0.030	0.35	1.6521	0.5799	1.2422
0.040	0.40	1.9007	0.7050	1.4254
0.050	0.40	2.1259	0.7955	1.5943
0.060	0.40	2.3297	0.8786	1.7476
0.080	0.40	2.6919	1.0295	2.0197
0.10	0.40	3.0114	1.1660	2.2599

FRAME 3

w/σ	h / L	d_1	d_2	d_3



Adopt the highest value of h/L possible for all values of w/σ

FRAME 4	w/σ	h / L	d_1	d_2	d_3
	0.01	0.45	0.5667	0.2924	0.5667
	0.02	0.45	0.8023	0.4185	0.8023
	0.03	0.45	0.9834	0.5174	0.9834
	0.04	0.45	1.1364	0.6021	1.1364
	0.05	0.45	1.2713	0.6778	1.2713
	0.06	0.45	1.3934	0.7474	1.3934
	0.07	0.45	1.5059	0.8117	1.5059
	0.08	0.45	1.6106	0.8724	1.6106
	0.09	0.45	1.7091	0.9300	1.7091
	0.10	0.45	1.8024	0.9850	1.8024
	0.15	0.45	2.2116	1.2316	2.2116