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Improvement of the Shell Element Implemented in FEAST^{SMT}

Sandra S Kumar[1], T J Raj Thilak[2] Ahsana Fathima[3]

[1,3](Department of Civil Engineering (Indira Gandhi Institute of Engineering & Technology for Women Mahatma Gandhi University, Kottayam, India [2](Scientist /Engineer VSSC/ISRO, Thiruvananthapuram, India)

Abstract: *The paper deals with shear locking problem in shell element. Shear locking does not mean complete rigidity, it refers to unwanted high-stiffness behavior that influences the solution but does not over whelm it, so that convergence with mesh refinement is slowed but not prevented. In the present study the Bilinear Degenerated Shell (BDS) element model is improved based on the bubble function for membrane strain energy and selective integration for the shear energy. After formulation of the shell element, implementation is carried out in FEAST^{SMT} (FINITE ELEMENT ANALYSIS OF STRUCTURES). Result of the shell element without any bubble function terms showed sensitivity to shear locking problem. Use of bubble functions and selective integration greatly improves the element performance. The results were compared with those available in literatures.*

Keywords–*Bilinear degenerated shell, Bubble function, Shear Locking, Selective integration, Shell Element*

I. INTRODUCTION

The Finite Element Method (FEM) is a numerical technique to find approximate solutions of partial differential equations. It was originated from the need of solving complex elasticity and structural analysis problems in Civil, Mechanical and Aerospace engineering. FEM allows for detailed visualization and indicates the distribution of stresses and strains inside the body of a structure. Many of the Finite Elements (FE) software are powerful yet complex tool meant for professional engineers with the training and education necessary to properly interpret the results. Several modern FEM packages are specifically developed to fluid, thermal, electromagnetic and structural working environments. FEM allows entire component/system to be constructed, refined and optimized before the component/system is manufactured, thus reducing the lead time for design and cost. This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications. One must take the advantage of the advent of faster generation of personal computers for the analysis and design of engineering product with precision level of accuracy. The shell element formulation can be broadly classified in to classical shell element formulation and degenerated shell element formulation. The classical shell element formulation requires C^1 continuity and hence not suitable for general purpose finite element packages. Whereas, the BDS formulation requires only C^0 continuity and is widely implemented in general purpose finite element packages. The main disadvantage of degenerated shell element is locking problem [1]. Linear elements in FEM do not accurately model the curvature present in the actual material under bending, and a shear stress is introduced. This spurious shear stress causes the element to lock and is called shear locking of the element [2]. Shear locking is an error that occurs in finite element analysis due to the inability of the linear element to represent bending behaviour of an element. The different remedial measures for shear locking problems are reduced integration method or selective method and bubble function etc.

II. SHELL ELEMENT

In many areas of structural and especially aerospace designs, we require analysis of shell subjected to different type of loads. Classical solution involves tedious calculation and is extremely difficult especially for shell of arbitrary shape. The finite element method is suited for the analysis of shell because of its flexibility in accounting for arbitrary geometry, loading and different material properties [2].

The Bilinear Degenerated Shell Element evolves from eight node three dimensional brick element by introducing the assumption that normal to the mid-plane remains straight before and after deformation, the normal stress remains zero and there is no deformation in thickness direction. By this approach, the displacements and rotations of the shell reference surface are taken as degrees of freedom. This formulation makes it possible to develop element for the analysis of moderately thick shells, but in the case of thin shell shear locking problems can occur.

2.1Formulation of Shell Element

Since there is no change in geometry in the thickness direction, the strain energy can be integrated analytically over the entire shell thickness. This not only simplifies its derivation but also reduce the effort in formulating element stiffness. The present element is an isoparametric element and hence the same shape function can be

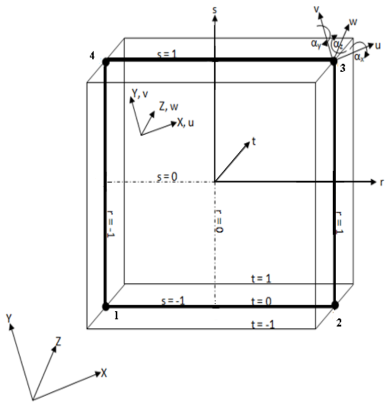

Fig. 1 Bilinear Degenerated Shell Element

used to interpolate the geometry and the field variables and is applicable for both thick and thin shells.

2.1.1Derivation for Stiffness Matrix

The degenerated shell element formulated from the brick element is shown in figure 1. The thickness of the shell element in the direction normal to the mid-surface at each node is specified as an input. Using the shape functions, the coordinates at any point in the element can be uniquely given in terms of nodal coordinate and thickness as,

$$X(r,s,t) = \sum_{i=1}^{i=4} N^I(r,s)\{X^I + \frac{1}{2} t h^I e_{z_3}^I \} \quad (1)$$

where X^I are the co ordinates of the reference surface, h^I the thickness and $e_{z_3}^I$ is the normal at node I. The interpolation function to describe the reference surface in terms of two dimensional isoparametric elements is given by,

$$N^I(r,s) = \frac{1}{4}(1+r^I r)(1+s^I s) \quad (2)$$

The displacement vector at any point (r,s,t) in the element can be given in the form,

$$u(r,s,t) = \sum_{i=1}^{i=4} N^I(r,s)\ \{u^I + u^I_\alpha(t)\} \quad (3)$$

where u^I is the nodal displacement vector on reference surface along the global x,y,z directions, and u^I_α is the relative nodal displacement x,y,z directions produced by a normal rotation at the node.

The vector u^I_α is to be expressed in terms of rotation vector α^I each of the global axes at the node. Using the shell assumption that straight normal to the reference mid-surface remain straight after deformation, the displacement vector based on the local z co-ordinates, produced by the normal rotation α' about the normal axes, is w_α an expressed by,

$$w_\alpha(t) = \frac{1}{2} th \begin{Bmatrix} \alpha'_2 \\ -\alpha'_1 \\ 0 \end{Bmatrix} \quad (4)$$

For infinitesimal rotation, the usual transformation from w_α^I to u^I_α and α'^I to α^I,in view of (4) lead to

$$u^I_\alpha(t) = \frac{1}{2} th\Phi\alpha^I \quad (5)$$

where,

$$\Phi = \begin{bmatrix} 0 & \theta_{33} & -\theta_{23} \\ -\theta_{33} & 0 & \theta_{13} \\ \theta_{23} & -\theta_{13} & 0 \end{bmatrix} \tag{6}$$

Substituting (5) in to (3) yields the expression of the displacement vector at any in the shell element in term of nodal variables:

$$u(r,s,t)=\sum_I N^I(r,s)\{u^I+\tfrac{1}{2}th\Phi\alpha^I\} \tag{7}$$

From the stress- strains relation, assuming $\varepsilon_{33} = 0$,the strain components along the local axes of the shell element are given by,

$$\varepsilon\,(r,s,t) = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_1}{\partial z_1} \\ \frac{\partial w_2}{\partial z_2} \\ \frac{\partial w_1}{\partial z_2} + \frac{\partial w_2}{\partial z_1} \\ \frac{\partial w_1}{\partial z_3} + \frac{\partial w_3}{\partial z_1} \\ \frac{\partial w_2}{\partial z_3} + \frac{\partial w_3}{\partial z_2} \end{Bmatrix} \tag{8}$$

The stiffness matrix can be split into two parts: transverse shear stiffness matrix and the membrane stiffness matrix. This will allow us to use appropriate of integration order for each part. Accordingly,

$$\{\varepsilon_m\}= \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} = \sum_{i=1}^{4} B_m^I \quad \delta^I \tag{9}$$

$$\{\varepsilon_s\} = \begin{bmatrix} \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix} == \sum_{i=1}^{4} B_s^I \quad \delta^I \tag{10}$$

The standard form of element stiffness as derived by the general finite element procedure, is

$K^{IJ} = \int [B^I]^T D B^J dv$

in which B^I is the strain displacement matrix relating the local strain vector to the nodal variable

$\delta^I = \begin{Bmatrix} u^I \\ \alpha^I \end{Bmatrix}$, such that

$\varepsilon\,(r,s,t) = \sum_I B^I\ \delta^I$

To formulate the matrix B_m , B_s

$$\begin{Bmatrix} \varepsilon_m \\ \varepsilon_s \end{Bmatrix} = \sum_I \begin{bmatrix} B^I{}_{1m} & B^I{}_{2m} + t\, B^I{}_{3m} \\ B^I{}_{1s} & B^I{}_{2s} + t\, B^I{}_{3s} \end{bmatrix} \begin{Bmatrix} u^I \\ \alpha^I \end{Bmatrix} \tag{11}$$

Explicitly integrating through the thickness direction. we get

$$K_m^{IJ} = \int_{-1}^{1}\int_{-1}^{1} \begin{bmatrix} 2(B_{1m}^I)^T D_m B_{1m}^J & 0 \\ 0 & \frac{2}{3}(B_{3m}^I)^T D_m B_{3m}^J \end{bmatrix} |J(r,s,0)|\ dr\ ds$$

Similarly shear part can be obtained as

$$K_s^{IJ} = \int_{-1}^{1}\int_{-1}^{1} \begin{bmatrix} 2(B_{1s}^I)^T D_s B_{1s}^J & 2(B_{1s}^I)^T D_s B_{2s}^J \\ 2(B_{2s}^I)^T D_s B_{1s}^J & 2(B_{2s}^I)^T D_s B_{2s}^J + \frac{2}{3}(B_{3s}^I)^T D_s B_{3s}^J \end{bmatrix} |J(r,s,0)|\ dr\ ds$$

B_1^I is the strain contribution due to in plane displacement at node I, $B_2^I + tB_3^I$ is the strain contribution due to the rotation at node I, which also includes curvature effect. Due to the orthogonality conditions of the coordinate system in the mid plane $B_{2m}^I = 0$.

2.2Formulation of Bubble Function

To overcome this shear locking problem bubble function is implemented [4].Bubble function means adding nodeless degrees of freedom to the equations to simulate bending behaviour for linear elements. In this paper the Enhanced Assumed Strain method (EAS) is applied to degenerating shell element. The formulation is based up on the degenerated solid approach which taken in to account transverse shear effects. In this EAS method compatible strain enhanced by an extra strain field, ie,

$$\varepsilon = \mathrm{B}\delta + \acute{\varepsilon} \tag{12}$$

The additional strain, $\acute{\varepsilon} = M_{\xi}$

$$\mathrm{M}_{\xi}=\begin{bmatrix} \xi & 0 & 0 & 0 & \xi\eta & 0 & 0 \\ 0 & \eta & 0 & 0 & 0 & \xi\eta & 0 \\ 0 & 0 & \xi & \eta & 0 & 0 & \xi\eta \end{bmatrix} \tag{13}$$

The membrane stiffness matrix is give as,

$$\mathrm{K}= \int B^T\, D\, B \,\mathrm{dv} \tag{14}$$

$$[B]=[B_d \quad B_a]$$

$$\mathrm{K}= \begin{bmatrix} {B_d}^T \\ {B_a}^T \end{bmatrix} D\ [B_d \quad B_a\ ] \tag{15}$$

$$\mathrm{K}=\begin{bmatrix} {B_d}^T D B_d & {B_d}^T D B_a \\ {B_a}^T D B_d & {B_a}^T D B_a \end{bmatrix} \tag{16}$$

$$\mathrm{K}=\begin{bmatrix} K_{dd} & K_{da} \\ K_{ad} & K_{aa} \end{bmatrix} \tag{17}$$

General global equation is,

$$[\mathrm{K}][\mathrm{D}] = [\mathrm{F}] \tag{18}$$

$$\begin{bmatrix} K_{dd} & K_{da} \\ K_{ad} & K_{aa} \end{bmatrix} \begin{Bmatrix} u_d \\ u_a \end{Bmatrix} = \begin{Bmatrix} f_d \\ f_a \end{Bmatrix} \tag{19}$$

$$K_{dd}\, u_d + K_{da}\, u_a = f_d \tag{20}$$

$$K_{ad}\, u_d + K_{aa}\, u_a = f_a \tag{21}$$

where, u_d are the nodal dof to be retained u_a are nodeless d.o.f due to the bubble function and to be condensed.

By static condensation [2], the lower partition is solved for u_a,which is then substituted into the upper partition. Thus,

$$u_a = K_{aa}^{-1}[f_d - K_{ad}\, u_d] \tag{22}$$

$$[K_{dd} - K_{da} K_{aa}^{-1} K_{ad}]u_d = f_d - K_{da} {K_{aa}}^{-1} f_a \tag{23}$$

Condensed [K] Condensed [f]

2.3 Selective Integration Method

Selective integration method is one of the methods used to alleviate shear locking. In numerical analysis, quadrature rule is an approximation of the definite integral of a function, usually stated as a weighted sum of function values at specified points within the domain of integration. An n-point Gaussian quadrature rule is a quadrature rule constructed to yield an exact result for polynomials of degree $2n - 1$ or less by a suitable choice of the sample points x_i and weights w_i for i = 1... n. The domain of integration for such a rule is conventionally taken as [a, b] ans can be calculated by the following expression.

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i) \qquad (24)$$

Selective reduced order integration requires one order lesser quadrature rule for selected components.

Table 1 Location of gauss points for calculating various strains

Strain Component	r	s	t
$\varepsilon_x, \varepsilon_y$	$\pm\frac{1}{\sqrt{3}}$	$\pm\frac{1}{\sqrt{3}}$	0
γ_{xy}	0	0	$\pm\frac{1}{\sqrt{3}}$
γ_{xz}	0	$\pm\frac{1}{\sqrt{3}}$	0
γ_{zy}	$\pm\frac{1}{\sqrt{3}}$	0	0

2.4 RESULTS AND DISCUSSIONS

The present element is implemented in FEASTSMT an indigenous software developed by VSSC/ISRO. The element is validated with two standard benchmark problems available in the literature.

- **Cantilever beam subjected to in plane shear loading**

Geometric properties: length =6 unit, width=0.2unit, depth=0.1 unit.

Material properties: E =1×10^7 unit, ν = 0.3

Boundary Condition: Fixed at one end and unit in plane shear force at other end.

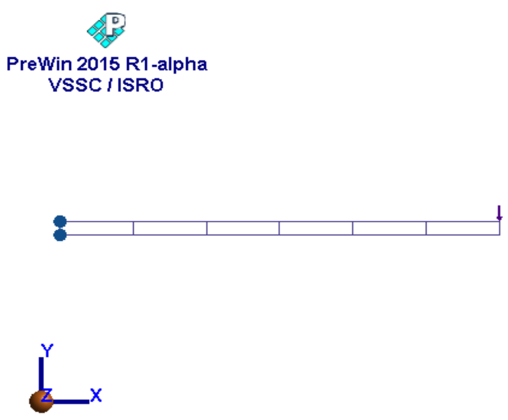

Table 2 Comparison of tip displacement

FEAST^{SMT} (Full integration)	0.009747
FEAST^{SMT} (bubble function)	0.104756
FEAST^{SMT} (selective integration)	0.0983
Ref. [6]	0.1081

- **Hemispherical shell subjected to radial load**

Geometric properties: thickness=0.04 unit

Boundary Condition: Symmetric boundary conditions at the side edges and free at top and bottom edges and concentrated radial loads of 2 units along X and Y directions.

Material Property: E = 6.825E+07, ν =0.3

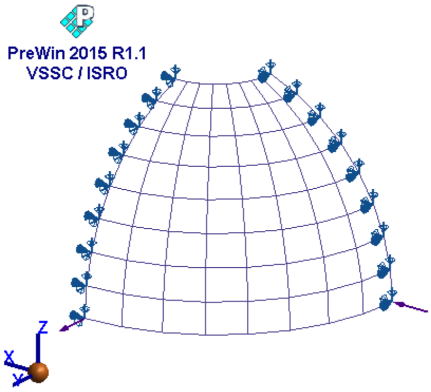

Table 3 Comparison of radial displacement at the point of application of load

FEAST^{SMT} (Full integration)	0.0059
FEAST^{SMT}(bubble function)	0.185
FEAST^{SMT}(selective integration)	0.185
Ref. [6]	0.187

III. CONCLUSION

The paper gives details about formulation of shear locking free four node shell element. Approaches to reduce shear locking by means of selective order integration and bubble function have been introduced to construct shell element. After implementation in FEAST^{SMT}, method is validated by cantilever subjected to in plane shear loading and hemispherical shell subjected to radial load. The result obtained from FEAST^{SMT} is compared with the literatures.

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