

Free Vibration Analysis of Sandwich Beam Structure Using Finite Element Approach

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Abstract: Sandwich beams offer designers a number of advantages, as the high strength to weight ratio, flexibility, high bending and buckling resistances. Sandwich construction results higher natural frequencies than none sandwich constructions, also it developed an adaptive tuned vibration absorber. In the present work, the natural frequencies and mode shapes of the sandwich beam structure are calculated under different boundary conditions. Three models are created using MSC-PATRAN/NASTRAN software, 1D beam, 2D shell and 3D solid. The results for AL solid beam, CPVC solid beam, and AL-CPVC sandwich construction beam were obtained and compared with the analytical results. The results show a good agreement between the finite element models and analytical models for AL solid beam, CPVC solid beam with less than 2% error. For AL-CPVC sandwich construction beam the analytical solution is over predict the natural frequencies with 27% for the first mode and increases with increasing the number of modes to reach 40% at the fourth mode. The effect of material application and the boundary conditions are studied.

Keywords:- Sandwich beam, Natural frequency, Mode shape, Analytic solution, Finite element analysis.

I. Introduction

A Structural Sandwich is a special form of composite comprising of a combination of different materials that are bonded to each other so as to utilize the properties of each separate component to the structural advantage of the whole assembly [1]. Sandwich materials are frequently used wherever high strength and low weight are important criteria. The most important application are found in the transport industry – such as in the aerospace, aircraft, automobiles, railroad and marine industries – where a high stiffness/ weight and strength/ weight ratio provides increased pay load capacity, improved performance and lower energy consumption [2-4]. A sandwich structure consists of three elements as shown in **Figure 1**, face sheets, core and the adhesive interface layers. The faces carry in-plane and bending loads, while the core resist transverse shear forces and keeps the facings in place [5]. The many advantages of sandwich constructions, the development of new materials and the need for high performance and low-weight structures insure that sandwich construction will continue to be in demand [6-7]. The purpose of the core is to maintain the distance between the laminates and to sustain shear deformations [8], by varying the core, the thickness and the material of the face sheet of the sandwich structures; it is possible to obtain various properties and desired performances.

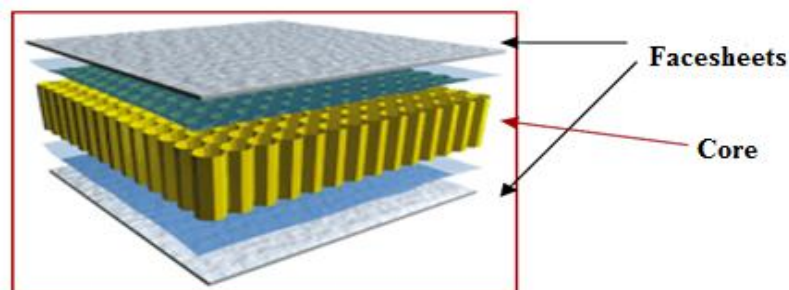


Figure 1: Main Components of Sandwich Structures

As a kind of new members in the family of lightweight structures, metallic sandwich beams, plates and shells with several kinds of metallic cores are developed, such as metallic foams, lattice materials, and woven materials. Studying the response of metallic sandwich beams under loading [1], obtain the solutions for the dynamic response of large deflections of solid monolithic beams as a degenerate case of the sandwich beams.

In a sandwich structure [9] the facesheets can be made of isotropic monocoque material, anisotropic monocoque material or a composite material. Aluminum, fiberglass, graphite and aramid are used as facesheet

materials, in order to minimize the weight of the structure composite facesheets are preferred. For all sandwich structures both in-plane and bending (primary loading) are carried by the facesheets, and the core carries transverse shear loads. The facesheets are identical in material and thickness, the variety of types of sandwich constructions depends upon the configuration of the core [10]. The core must be strong enough to withstand the compressive or crushing load placed on the panel, it must resist the shear forces involved. These are foam or solid core, honeycomb core, web core and corrugated or truss core. Solid cores can consist of balsa wood, vinyl sheet foam. The honeycomb materials are aluminum, glass fabric, asbestos fabric, plastic film. Foam cores are foamed plastic, foamed glass, foamed aluminum.

A sandwich beam with different facesheet and core materials were modeled to analyze the impact response of sandwich panels using the numerical software LS-DYNA [11]. Two facesheet materials were 2024-T3 (AL -Aluminum) alloy sheet metal and (GFRP- Glass Fiber Reinforced Plastic). The core materials were Balsa wood with density 80kg/m^3 , Balsa Wood with density 150 kg/m^3 , cork and Polystyrene foam. The modelled impact response was used to determine the energy absorption capabilities of different sandwich panel materials at low velocities. They found that the sandwich panel which was most effective at absorbing energy and had the highest resistance when impacted at low velocity consisted of (GFRP) face sheets with Balsa80 core material; conversely the AL alloy face sheet and Balsa80 core absorbed the least energy when impacted.

Critical energy release rate of the carbon fiber/aluminum foam sandwich beams with as received and treated interfacial aramid fibers were measured to study the influence of the surface treatment on aramid fibers [12]. It was found that reinforcements in critical energy release rate were achieved for all samples with treated aramid fiber as measured under double cantilever beam condition. The interfacial characteristics of the short aramid fibers with different surface condition were investigated and discussed based on scanning electron microscopy observations. It is suggested that advanced bonding between aramid fibers and epoxy resin was conducted after surface treatment, and more energy was therefore absorbed through fiber bridging during crack opening and extension process.

Static and dynamic three-point bending tests were carried out in order to investigate the structural response of two different typologies of aluminum foam sandwich panels, consisting of a closed-cell aluminum foam core with either two integral or two glued faces. Impact measurements were performed by a bi-pendulum testing machine designed by the authors [13]. It was found that different collapse modes can be obtained for samples with identical nominal dimensions, depending on the support span distance and on the own aluminum foam sandwich panels properties. As far as energy dissipation is concerned, no strain rate sensitivity was found for initial impact velocity up to about 1.2 m/s.

The initiation of failure in composite sandwich beams is heavily dependent on properties of the core material. Several core materials, including PVC (Poly Vinyl Chloride) foams and balsa wood with aluminum facesheet were characterized [14]. The failure modes were investigated experimentally in axially loaded composite sandwich columns and sandwich beams under bending. They found that, plastic yielding or cracking of the core occurs when the critical yield stress or strength of the core is reached. Also in the case of cantilever beams or beams under three-point bending, entailing shear interaction between the facesheets and core, the Hoff and Mautner formula predicts a value for the critical wrinkling stress which is proportional to the cubic root of the product of the core Young's and shear modules in the thickness direction. The ideal core should be highly anisotropic with high stiffness and strength in the thickness direction.

In the present work, the natural frequencies and mode shapes of the sandwich beam structure are calculated under different boundary conditions. Three models are created using MSC-PATRAN/NASTRAN software, 1D beam, 2D shell and 3D solid. The results for AL solid beam, CPVC (Chlorinated Poly Vinyl Chloride) solid beam, and AL-CPVC sandwich construction beam were obtained and compared with the analytical results. The results show a good agreement between the finite element models and analytical models for AL solid beam, CPVC solid beam with less than 2% error. For AL-CPVC sandwich construction beam the analytical solution is over predict the natural frequencies with 27% for the first mode and increases with increasing the number of modes to reach 40% at the fourth mode. The effect of material application and the boundary conditions are studied.

II. Finite Element Models

Different Models of beam and sandwich beam under different boundary conditions as presented in Table 1 are created using MSC-PATRAN/NASTRAN software. The models are 1D beam, 2D shell and 3D solid. The 1D beam model consists of 100 elements with 2-nodes. The 2D shell model consists of 60 QUAD elements with 4-nodes. The 3D solid model consists of 2400 HEXA elements with 8-nodes. The face sheet material and core materials which used are (AL) and (CPVC).

Table 1: Different Models of Sandwich Beam with Different Materials and boundary conditions

Model Type	Face sheet Material	Core Material	End Conditions			
			Fixed-Free	Hinged-Hinged	Fixed-Hinged	Fixed-Fixed
1D beam	AL	AL	Fixed-Free	Hinged-Hinged	Fixed-Hinged	Fixed-Fixed
2D shell	CPVC	CPVC	Fixed-Free	Hinged-Hinged	Fixed-Hinged	Fixed-Fixed
3D solid	AL	CPVC	Fixed-Free	Hinged-Hinged	Fixed-Hinged	Fixed-Fixed

The most important mechanical properties of sandwich beam materials are the compressive strength, shear modulus and shear strength at failure as presented in **Table 2**. Stiffness and strength increases with the density of core material.

Table 2: Mechanical Properties of Different Materials of the Used Sandwich Beam [15]

Material	Density (gm/cm ³)	Modulus of Elasticity (GPa)	Poisson's ratio	Strength (MPa)	Shear Modulus (GPa)
Aluminum	2.70	69	0.33	227.4	26
CPVC	1.56	3.15	0.40	52	1.13

The dimensions of sandwich beam were (L), (B), (t), (t_c), and (t_f). As shown in **Figure 2**,

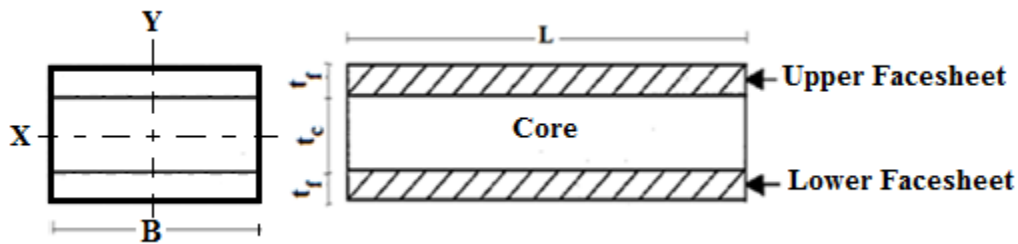


Figure 2: Sandwich Beam Structure

All models of sandwich beams have the same dimensions for face and core as presented in **Table 3**.

Table 3: Dimensions of Sandwich Beam

Length of sandwich Beam, L	0.180m
Width of Sandwich Beam, B	0.025m
Facesheet Thickness, t _f	0.003m
Core Thickness, t _c	0.010m
Beam Thickness, t	0.016m

III. Analytical Solution Method

III.I. Section Characterization

The section characteristics; modulus weighted centroid, axial stiffness and bending rigidity of composite area shown in Figure 3 are determined using the concept of idealization in [16]

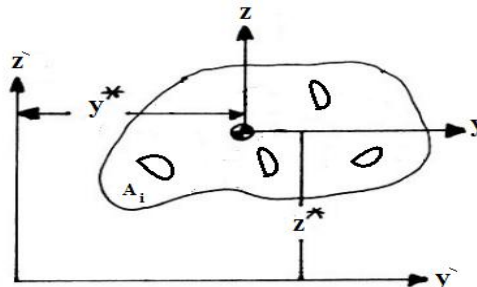


Figure 3: Representation of General Sandwich Beam Cross-Section.

The elastic area (A^*) of the crosssection is given by;

$$A^* = \sum_{i=1}^n E_i A_i \tag{1}$$

The modulus weighted centroid is determined as follows:

$$y^* = \frac{1}{A^*} \sum E_i A_i y_i \tag{2}$$

$$z^* = \frac{1}{A^*} \sum E_i A_i z_i \quad (3)$$

The bending stiffness w.r.t. modulus weighted centroid is determined in two steps. In the first step, the bending stiffness ($EI_{y\setminus}$) and ($EI_{z\setminus}$) with respect to $y\setminus - z\setminus$ coordinates;

$$EI_{y\setminus} = \sum E_i (I_{y_0} + z_i^2 A_i) = I_{y\setminus} \quad (4)$$

$$EI_{z\setminus} = \sum E_i (I_{z_0} + y_i^2 A_i) = I_{z\setminus} \quad (5)$$

In the second step, the bending stiffness (EI_y) and (EI_z) with respect to y & z coordinates (modulus weighted centroid);

$$EI_y = I_{y\setminus} - (z^*)^2 A^* \quad (6)$$

$$EI_z = I_{z\setminus} - (y^*)^2 A^* \quad (7)$$

Where;

A^* , is the axial stiffness of the cross-section

$y_i\setminus$ & $z_i\setminus$, is the coordinates of the i^{th} Element with respect to $(y\setminus)$ & $(z\setminus)$ coordinates

E_i , is the membran elastic constant

I_{y_0} & I_{z_0} , is the second moment of Inertia of elemental area (A_i) about its centroid

EI_y, EI_z , is the bending rigidity about y and (z) coordinates respectively

III.2. Transverse Vibration of Sandwich Beam

Equation of motion of beam vibrating in transverse direction (simply supported) as shown in **Figure 4**, can write as [17];

$$\frac{\partial^4 y}{\partial x^4} + \left(\frac{\rho A}{EI}\right) \frac{\partial^2 y}{\partial t^2} = 0 \quad (8)$$

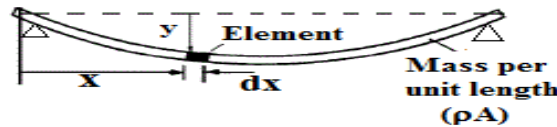


Figure 4: Transverse Vibration of Simply Supported Beam

The solution of equation (8) can be written as;

$$y = X (B_1 \sin \omega t + B_2 \cos \omega t) \quad (9)$$

Where, X is a function of (x) which defines the beam shape of the normal mode of vibration.

Hence;

$$\frac{\partial^4 y}{\partial x^4} = \left(\frac{\rho A}{EI}\right) \omega^2 y = \lambda^4 y \quad (10)$$

Where;

$$\lambda = \rho A \omega^2 / EI \quad (11)$$

Where;

ρ , is the material density

λ , is the eigen value

ω , is the angular natural frequency

E , is the Young's modulus

This is the beam equation. The general solution to the beam equation is

$$X = C_1 \cos \lambda x + C_2 \sin \lambda x + C_3 \cosh \lambda x + C_4 \sinh \lambda x \quad (12)$$

Where;

$C_{1,2,3,4}$ are constants to be determined from the boundary conditions.

IV. Boundary Conditions

1- Fixed-Free

$$\begin{matrix} \text{at } x = 0 & \text{at } x = L \\ X = 0 & X'' = 0 \\ X' = 0 & X''' = 0 \end{matrix}$$

2- Hinged-Hinged

$$\begin{matrix} \text{at } x = 0 & \text{at } x = L \\ X = 0 & X = 0 \\ X'' = 0 & X'' = 0 \end{matrix}$$

3- Fixed-Hinged

$$\begin{matrix} \text{at } x = 0 & \text{at } x = L \\ X = 0 & X = 0 \\ X'' = 0 & X'' = 0 \end{matrix}$$

4- Fixed-fixed

$$\begin{matrix} \text{at } x = 0 & \text{at } x = L \\ X = 0 & X = 0 \\ X' = 0 & X' = 0 \end{matrix}$$

Apply the boundary conditions for the previous cases, the constants $C_1, C_2, C_3,$ and C_4 are determined. The frequency equation for different end conditions is given below. The natural frequency for isotropic beam is given by;

$$\omega = \frac{\alpha}{L^2} \sqrt{\frac{EI}{\rho A}} \quad \text{rad/s} \tag{13}$$

The values of α for different end conditions are given in **Table 4**.

Table 4: Values of (α) given in equation (13) for Different End Condition

End Condition	Frequency Equation	1 st Mode	2 nd Mode	3 rd Mode	4 th Mode
Hinged-Hinged	$\sin \lambda L = 0$	π^2	$4 \pi^2$	$9 \pi^2$	$16 \pi^2$
Clamped-Free	$\cos \lambda L \cosh \lambda L = -1$	13.52	22.4	61.7	121.0
Fixed-Hinged	$\tan \lambda L = \tanh \lambda L$	15.4	50.0	104.0	178.3
Fixed-Fixed or Free-Free	$\cos \lambda L \cosh \lambda L = 1$	22.4	61.7	121.0	199.9

V. Results And Discussions

The results of calculating natural frequencies of sandwich beam under different boundary conditions and based on analytical and finite elements techniques are presented in following tables. Also, the relative error of finite element results to the analytical results is presented. In all finite element models, the material was applied as a laminated composite.

The natural frequency of sandwich beam is calculated using equation (13) for different boundary conditions. The bending rigidity (EI) is calculated based on the characterization technique equations (1) to (12).

The natural frequencies of sandwich beam (Al-PVC) are presented in **Table 5 (a, b, c, and d)**. It was noted that the analytical model upper estimated the natural frequencies for different boundary conditions by (28%) to (38%), for 1D and 3D models while this percent changes from (27%) to (33%) for 2D model. The big different between the analytical is attributed to the way of modeling the bending rigidity. In the analytical model, the effect of location of material is taken into consideration while in the (FE) models, the modulus of elasticity E is calculated based on the rule of mixture irrespective the location of the material relative to the centroidal axis. So, the analytical bending rigidity equals (1.9) of the finite element models which lead to the analytical natural frequencies higher than that of the finite element model by (37%).

To verify this result, the natural frequencies of sandwich beam made of CPVC and Al are calculated and presented in **Tables (6, 7)** respectively. It was noted from the results that a good agreement between the analytical and finite element models by (1%) to about (3%) relative error for the first and second modes. This error increase as the mode shape increase to reach about (13%) for the fourth mode.

The effect of material application is presented in **Table 8**. In this case the natural frequencies and mode shape of the 3D solid model of AL-CPVC sandwich beam are determined. In the first model the material was applied as isotropic material. In the second model the material was applied as a laminated composite. It was found from the results that the model with isotropic material is very bad to predict the natural frequencies. Investigating the mode shapes, it is noted that the structure does not behave as a beam as shown in **Figure 6b** and **Figure 8b**. The hypothesis of the beam theory is violated and there is an interaction between the three beams (upper face, core, and lower face).

Table 5: Natural Frequencies of Sandwich AL-CPVC Structure

(a) Fixed –Free

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	412.1	297.44	300.0	299.6	27.83	27.20	27.30
2 nd	2622.5	1811.8	1855.1	1810.9	30.91	29.26	30.95
3 rd	7223.7	4862.0	5101.1	4817.3	32.69	29.38	33.31
4 th	14166.4	9007.	9752.9	8848.5	36.42	31.15	37.54

(b) Hinged –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	1155.5	830.4	835.7	828.9	28.13	27.67	28.26
2 nd	4622.1	3224.4	3311.8	3199.4	30.24	28.35	30.78
3 rd	10399.6	6928.8	7333.7	6830.7	33.37	29.48	34.32
4 th	18488.2	11623.0	12744.0	11397.0	37.13	31.07	38.36

(c) Fixed –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	1803.0	1274.6	1306.0	1282.8	29.31	27.57	28.85
2 nd	5853.9	3960.1	4167.0	3963.0	32.35	28.82	32.30
3 rd	12176.1	7818.0	8507.0	7774.1	35.79	30.13	36.15
4 th	20875.0	12540.0	14148.0	12402.0	39.93	32.23	40.59

(d) Fixed –Fixed

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	2622.5	1809.6	1891.2	1837.1	31.00	27.89	29.95
2 nd	7223.7	4725.2	5100.4	4767.6	34.59	29.39	34.00
3 rd	14166.4	8700.0	9731.3	8721.7	38.59	31.31	38.43
4 th	23403.8	13428.0	15569.0	13386.0	42.62	33.48	42.80

(a) Fixed –Free

Table 6: Natural Frequencies of CPVC-Beam Structure

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	113.49	112.77	113.66	114.04	0.63	-0.15	-0.49
2 nd	722.18	686.07	689.51	688.42	5.00	4.52	4.67
3 rd	1989.22	1837.90	1843.40	1828.40	7.61	7.33	8.08
4 th	3901.06	3397.60	3399.20	3352.40	12.91	12.86	14.06

(b) Hinged –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	318.20	314.78	315.01	314.15	1.07	1.00	1.27
2 nd	1272.79	1220.60	1223.60	1212.10	4.10	3.86	4.77
3 rd	2863.78	2618.00	2627.90	2585.10	8.58	8.24	9.73
4 th	5091.17	4382.30	4397.60	4308.70	13.92	13.62	15.37

(c) Fixed –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	496.50	482.74	487.05	487.81	2.77	1.90	1.75
2 nd	1612.01	1497.20	1510.70	1504.60	7.12	6.28	6.66
3 rd	3352.98	2949.70	2974.20	2946.00	12.03	11.30	12.14
4 th	5748.42	4721.30	4749.80	4691.40	17.87	17.37	18.39

(d) Fixed –Fixed

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	722.18	684.74	695.89	700.40	5.18	3.64	3.02
2 nd	1989.22	1784.10	1810.00	1812.90	10.31	9.01	8.86
3 rd	3901.06	3277.70	3316.70	3308.10	15.98	14.98	15.20
4 th	6444.81	5048.80	5088.60	5065.30	21.66	21.04	21.40

Table 7: Natural Frequencies of AL-Beam Structure

(a) Fixed –Free

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	403.73	401.30	403.34	404.03	0.60	0.10	-0.07
2 nd	2569.18	2444.80	2450.40	2442.50	4.84	4.62	4.93
3 rd	7076.72	6562.20	6563.30	6498.90	7.27	7.26	8.17
4 th	13878.18	12160.00	12130.00	11940.00	12.38	12.60	13.97

(b) Hinged –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	1132.00	1120.40	1121.00	1117.90	1.02	0.97	1.25
2 nd	4528.00	4351.10	4358.10	4316.90	3.91	3.75	4.66
3 rd	10188.01	9352.10	9373.30	9217.90	8.20	8.00	9.52
4 th	18112.02	15692.00	15714.00	15383.00	13.36	13.24	15.07

(c) Fixed –Hinged

Mode No.	Analytical	1D	2D	3D	1D %error	2D %error	3D %error
1 st	1766.31	1719.80	1730.40	1730.20	2.63	2.03	2.04
2 nd	5734.79	5344.60	5377.00	5346.20	6.80	6.24	6.78
3 rd	11928.35	10554.00	10610.00	10490.00	11.52	11.05	12.06
4 th	20450.24	16934.00	16983.00	16738.00	17.19	16.95	18.15

(d) Fixed –Fixed

Mode No.	Analytical	1D (100 element)	2D	3D	1D 100 %error	2D %error	3D %error
1 st	2569.18	2442.00	2469.30	2477.20	4.95	3.89	3.58
2 nd	7076.72	6378.30	6440.80	6430.70	9.87	8.99	9.13
3 rd	13878.18	11747.00	11836.00	11768.00	15.36	14.72	15.21
4 th	22927.67	18136.00	18208.00	18066.00	20.90	20.59	21.20

Table 8: Natural Frequencies of AL-CPVC Structure for Isotropic and Laminated Composite

(a) Fixed –Free

Mode No.	Analytical	3Disot	3D_Comp	3D_isot %error	3D_Comp %error
1 st	412.1	389.1	299.6	5.6	27.3
2 nd	2622.5	1883.7	1810.9	28.2	30.9
3 rd	7223.7	4209.1	4817.3	41.7	33.3
4 th	14166.4	6677.5	8848.5	52.9	37.5

(b) Hinged –Hinged

Mode No.	Analytical	3Disot	3D_Comp	3D_isot %error	3D_Comp %error
1 st	1155.5	1018.3	829.0	11.9	28.3
2 nd	4622.0	3170.5	3199.4	31.4	30.8
3 rd	10399.6	5588.0	6830.7	46.3	34.3
4 th	18488.2	8076.8	11397.0	56.3	38.4

(c) Fixed –Hinged

Mode No.	Analytical	3Disot	3D_Comp	3D_isot %error	3D_Comp %error
1 st	1803.0	1370.2	1282.8	24.0	28.9
2 nd	5853.9	3466.6	3963.0	40.8	32.3
3 rd	12176.1	5839.4	7774.1	52.0	36.2
4 th	20875.0	8332.1	12402.0	60.1	40.6

(d) Fixed –Fixed

Mode No.	Analytical	3Disot	3D_Comp	3D_isot %error	3D_Comp %error
1 st	2622.5	1729.7	1837.1	34.0	29.9
2 nd	7223.7	3735.7	4767.6	48.3	34.0
3 rd	14166.4	6090.3	8721.7	57.0	38.4
4 th	23403.9	8589.9	13386.0	63.3	42.8

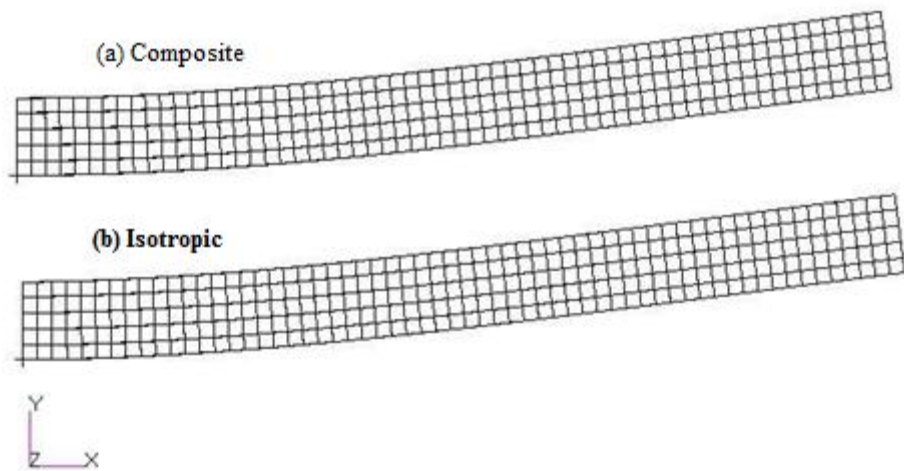


Figure 5: 1st Mode Shape of Fixed-Free Composite and Isotropic Sandwich Beam

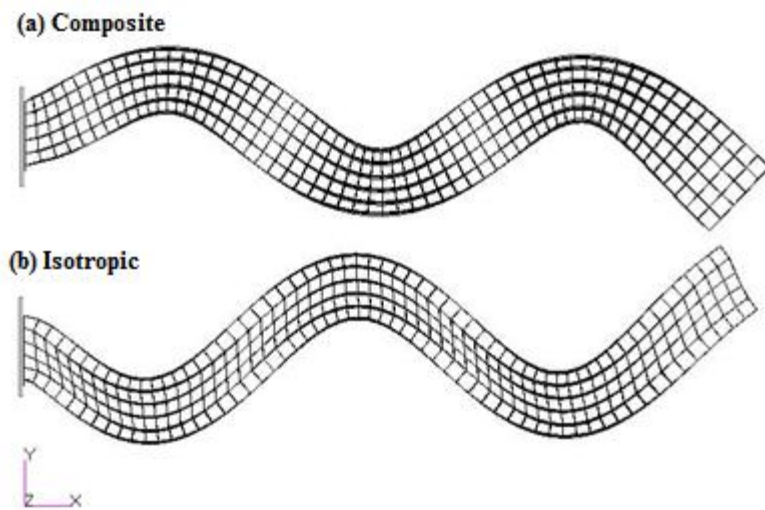


Figure 6: 4th Mode Shape of Fixed-Free Composite and Isotropic Sandwich Beam

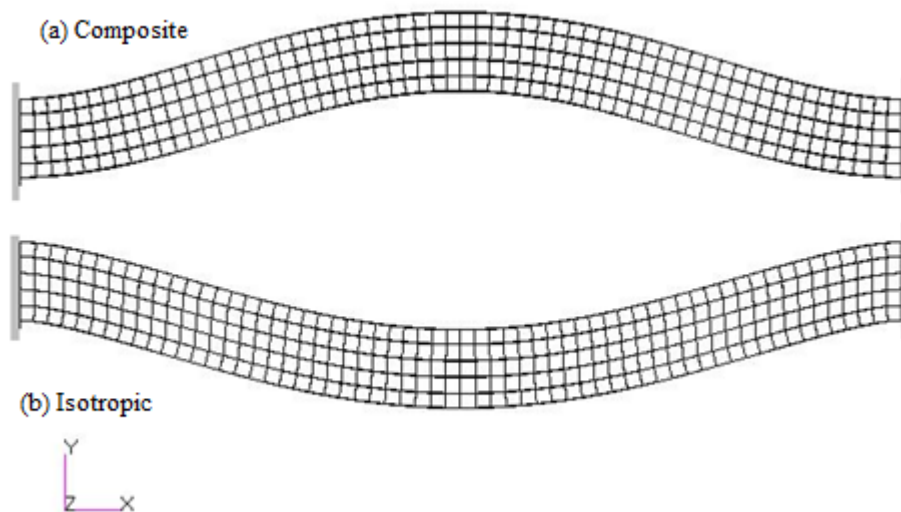


Figure 7: 1st Mode Shape of Fixed-Fixed Composite and Isotropic Sandwich Beam

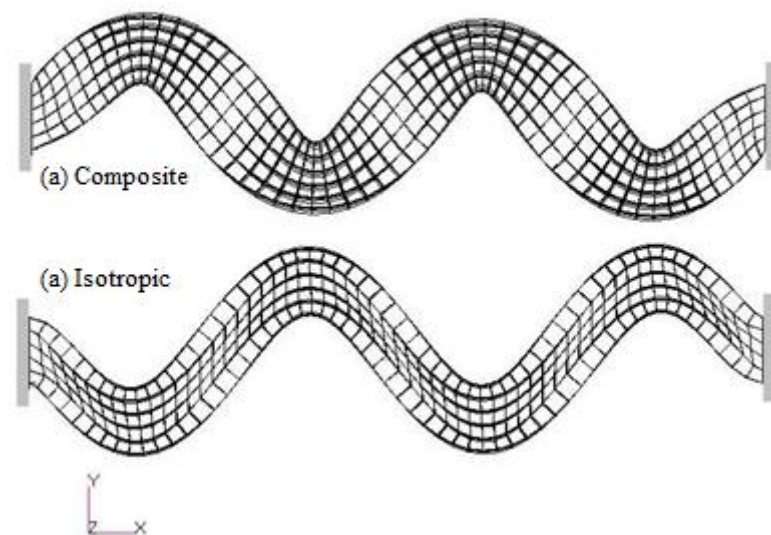


Figure 8: 4th Mode Shape of Fixed-Fixed Composite and Isotropic Sandwich Beam

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