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# Soil Structure Interaction Calculus, For Rigid Hydraulic Structures, Using FEM

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***Abstract:*** *The interaction between the foundation and the deformable soil calculated by finite element method is based on various models representing terrain behavior. Of these models, most commercial calculation programs implemented in their content models Winkler and Pasternak. Article shows the influence of these computing models on conventional rigid hydraulic construction. It was calculated the stiffness matrix structure and deformations developed, by considering these two models.*
***Keywords****: FEM, Pasternak model, rigid structure, stiffness matrix*

## I. Introduction

The traditional method for simulation the mathematical load-deformation response of a beam in uniaxial bending is a differential equation (Horvath 2002) [1]. The basic form of the matrix formulation for beam flexure is

$$[S]\{d\} = \{q\} \tag{1}$$

where:
[*S*] = stiffness matrix; {*d*} = displacement vector; {*q*} = load (force) vector.
The relevance of equation (1) is that all of the variations in beam behavior can be explained as variations solely in the formulation of the stiffness matrix, [*S*].
In Winkler model (Fig.1) the flexural behavior of this beam is given by equation (2)

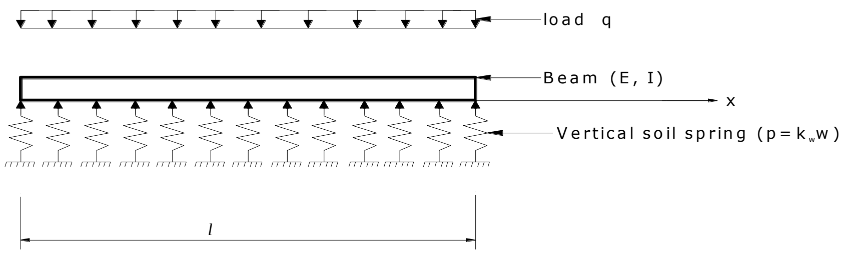


Fig. 1 The Winkler model

$$EI\frac{d^4 w(x)}{dx^4} + p(x) = q(x) \tag{2}$$

where subgrade reaction in one (*x*-axis) direction only is

$$p(x) = k_w w(x)$$

$k_w$ = Winkler coefficient of subgrade reaction
E = elasticity modulus of beam
I = beam moment of inertia
Solving ecuation (2) by FEM is expressed by relation (3)

$$([S_e] + [S_w])\{d\} = \{q\} \tag{3}$$

wherein elastic stiffness matrix expression [$S_e$] and subgrade reaction matrix [$S_w$] are determined with the following shape function(4) according to Cook [2] Chang [3] Teodoru [4]

$$N_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad ; \quad N_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2};$$
$$N_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad ; \quad N_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2}. \tag{4}$$

Stiffness matrix are:

$$[S_e] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \tag{5}$$

$$[S_w] = \frac{k_w l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{6}$$

In Pasternak model (fig.2) The flexural behavior of this beam is given by equation (7)

$$EI\frac{d^4w(x)}{dx^4} + p(x) - g\frac{d^2w(x)}{x^2} = q(x) \tag{7}$$

where $g =$ the shear stiffness of the shear layer. Solving ecuation (7) by FEM is expressed by relation (8)

$$\left([S_e] + [S_w] + [S_g]\right)\{d\} = \{q\} \tag{8}$$

wherein elastic stiffness matrix expression $[S_e]$ is subgrade reaction matrix $[S_w]$ are the same like those from relations (5) and (6) and matrix $[S_g]$ is given by equation (9)

$$[S_g] = \frac{g}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \tag{9}$$

The introduction of second parameter for soil (shear stiffness) have the same effect like siffness grovth of the beam (the terms of stiffness matrix is increase)

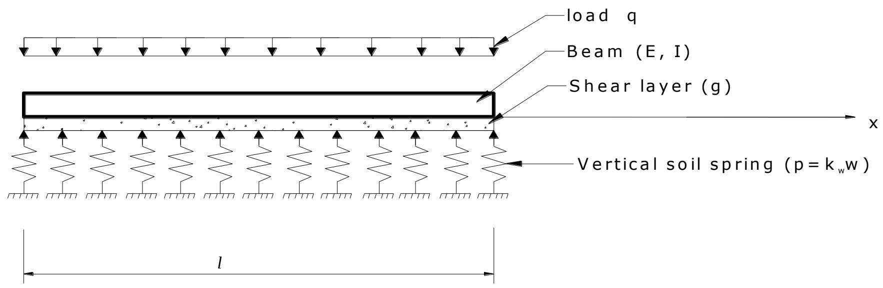


Fig. 2 The Pasternak model

Stiffness matrix is obtained considering continuum bearing on soil like in fig. 3

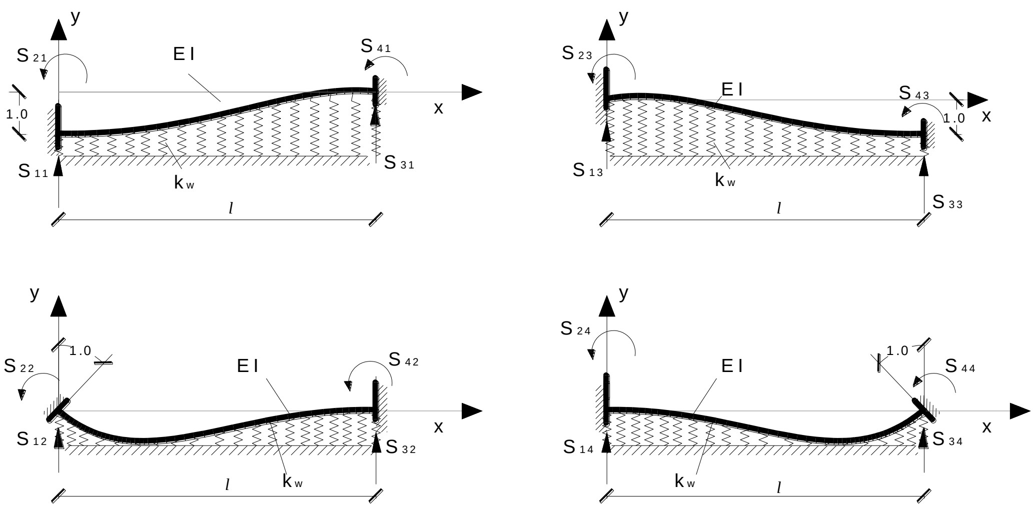


Fig. 3 Stiffness matrix calculation by continuum bearing

## II. Stiffness Matrix Calculation By Punctual Bearing Of The Beam

In beam on elastic foundation calculus by FEM, subgrade reaction matrix of Winkler spring was given by Bowles [5] in configuration (10)

$$[S_w] = \frac{k_w l}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

This expression is direct suggestion by calculus scheme from fig. 4, where can be see that only elements $S_{11}$ and $S_{33}$ of stiffness matrix have values different of zero values. (There's an element stiffness matrix $S_{ij}$ is generalized force that develops on *i* direction when in the direction of *j* is imposed on movement or rotation unit)

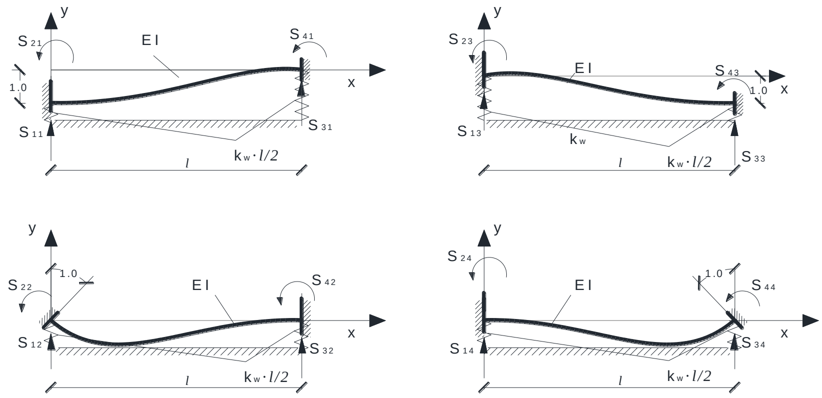


Fig. 4 Stiffness matrix calculation by nodal bearing

In ecuation (7) apart from term which include Winkler springs and for which stiffness matrix member $S_{11}$ and $S_{33}$ are easy to find (intuit) , apear and terms which include shearing efect for which stiffness matrix intuition is not simple. The term of the equation that considers the earth shear, contain second derivative of beam deformation($d^2w/dx^2$). To calculate the stiffness matrix expressing shear earth [$S_g$] in case of nodal bearing, on use similar functions to those for calculating matrix form [$S_w$]

Relation (10) for [$S_w$] rezult by solving with Galerkin method of differential ecuation (7)

Seeing that expression $w_e(x) = N_1(x)w_1+N_2(x)\theta_1+N_3(x)w_2+N_4(x)\theta_2$ (11)

is an approximal solution of differential ecuation (7) it rezult an residuum

$$\Re(x) = EI\frac{d^4 w_e(x)}{dx^4} - g\frac{d^2 w_e(x)}{x^2} + kw_e(x) - q(x) \neq 0 \tag{12}$$

in which k=$k_w$·1 considering an unitar width beam or k=$k_w$·B for a beam of B width; after Chung [6]

With this reziduum on form balanced reziduum functionals with shape functions

$$\pi_i = \int_0^l N_i(x)\cdot\Re(x,t)\,dx = EI\int_0^l N_i(x)\frac{d^4 w_e(x)}{dx^4}dx - g\int_0^l N_i(x)\frac{d^2 w_e(x)}{dx^2}dx + \\ + k\int_0^l N_i(x)w_e(x)dx - \int_0^l N_i(x)q(x)dx = 0 \tag{13}$$

From first integral of expresion (13) on obtain nodal force vector and elastic stiffness matrix of the beam(5). From the third integral obtain subgrade reaction matrix of Winkler spring, considring relation (11) write in form: $w(x)=[N(x)]\{d_e\}$, cu $\{d_e\}=\{w_1\ \ \theta_1\ \ w_2\ \ \theta_2\}$

$$[S_w] = k\int_0^l N_i(x)w_e(x)dx = k\int_0^l N_i(x)N_j dx = k\int_0^l \begin{bmatrix} N_1(x) \\ N_2(x) \\ N_3(x) \\ N_4(x) \end{bmatrix}[N_1(x)\quad N_2(x)\quad N_3(x)\quad N_4(x)]dx \tag{14}$$

Following stiffness matrix became

$$\left[S_w\right] = k\int_0^l \begin{bmatrix} (N_1)^2 & N_1N_2 & N_1N_3 & N_1N_4 \\ N_2N_1 & (N_2)^2 & N_2N_3 & N_2N_4 \\ N_3N_1 & N_3N_2 & (N_3)^2 & N_3N_4 \\ N_4N_1 & N_4N_2 & N_4N_3 & (N_4)^2 \end{bmatrix} dx \tag{15}$$

In relation (15) if accepted for shape function the relations(16)

$$N_1(x) = \begin{cases} 1, & x \le \dfrac{l}{2} \\ 0, & \dfrac{l}{2} \le x \le l \end{cases} ; \qquad N_2(x) = 0,\ x \in [0, l] \tag{16}$$

$$N_3(x) = \begin{cases} 0, & x < \dfrac{l}{2} \\ 1, & \dfrac{l}{2} \le x \le l \end{cases} ; \qquad N_4(x) = 0,\ x \in [0, l]$$

subgrade reaction matrix of Winkler spring become

$$\left[S_w\right] = k \begin{bmatrix} \dfrac{l}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{l}{2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

In this way was find the same subgrade reaction matrix of Winkler spring, like that given by Bowles(1996)

Folowing on use shape function for matrix $[S_g]$ calculation

If from ecuation (13) using the first two integral and consider shear stress attached to $g$ parameter , after Zhaohua apud Teodoru [4]

$$EI\frac{d^3w_e}{dx^3} = Q + g\frac{dw_e}{dx}$$

obtain integration by parts

$$N_i(x)EI\frac{d^3w_e}{dx^3}\Big|_0^l - N_i'(x)EI\frac{d^2w_e}{dx^2}\Big|_0^l + EI\int_0^l N_i''(x)\frac{d^2w}{dx^2}dx - gN_i(x)\frac{dw_e}{dx}\Big|_0^l + g\int_0^l N_i'(x)\frac{dw_e}{dx}dx =$$

$$= N_i(x)Q(x)\Big|_0^l + gN_i(x)\frac{dw_e}{dx}\Big|_0^l + N_i(x)M(x)\Big|_0^l + EI\int_0^l N_i''(x)\frac{d^2w_e}{dx^2}dx - gN_i(x)\frac{dw_e}{dx}\Big|_0^l + g\int_0^l N_i'(x)\frac{dw_e}{dx}dx \tag{18}$$

From ecuation (18) the last member give stiffness matrix wich simulate shear stres in soil

$$\left[S_g\right] = g\int_0^l N_i'(x)\frac{dw_e(x)}{dx}dx = g\int_0^l N_i'(x)N_j'dx = g\int_0^l \begin{bmatrix} N_1'(x) \\ N_2'(x) \\ N_3'(x) \\ N_4'(x) \end{bmatrix} \left[N_1'(x) \quad N_2'(x) \quad N_3'(x) \quad N_4'(x)\right] dx \tag{19}$$

Forward stiffness matrix become

$$\left[S_g\right] = g\int_0^l \begin{bmatrix} (N_1')^2 & N_1'N_2' & N_1'N_3' & N_1'N_4' \\ N_2'N_1' & (N_2')^2 & N_2'N_3' & N_2'N_4' \\ N_3'N_1' & N_3'N_2' & (N_3')^2 & N_3'N_4' \\ N_4'N_1' & N_4'N_2' & N_4'N_3' & (N_4')^2 \end{bmatrix} dx \tag{20}$$

where (‘) denotes differentiation with respect to x

If using shape functions (16) like those used for subgrade reaction matrix of Winkler spring calculation, $[S_w]$ (17) , shear matrix is $[S_g] = 0$
If using for matrix $[S_w]$ calculation linear shape function (21)

$$N_1(x,t) = 1 - \frac{x}{l}, \quad N_2(x) = 0, \quad N_3(x) = \frac{x}{l}, \quad N_4(x) = 0 \tag{21}$$

it obtain the folowing stiffness matrix

$$[S_w] = \frac{kl}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$[S_g] = \frac{g}{l}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

Stiffness matrix obtained with relation (22) and (23) as well those given by (17) and $Sg=0$ are very approximal because of rough shape function expresion used (16) and (21).
In folowing example on use the interaction model with continuum bearing. The goal of calculus example is to find stiffness matrix and displacements for a special structure with large rigidity

## III. Calculus Example

### 3.1. Design structure and calculus schedule

The structure is bottom discharge at an earth dam(Ibaneasa dam from Botosani county – Romania).
The conduit is made by steel concrete with polygonal cross section (fig. 5) - internal quadratic and external trapezoid.

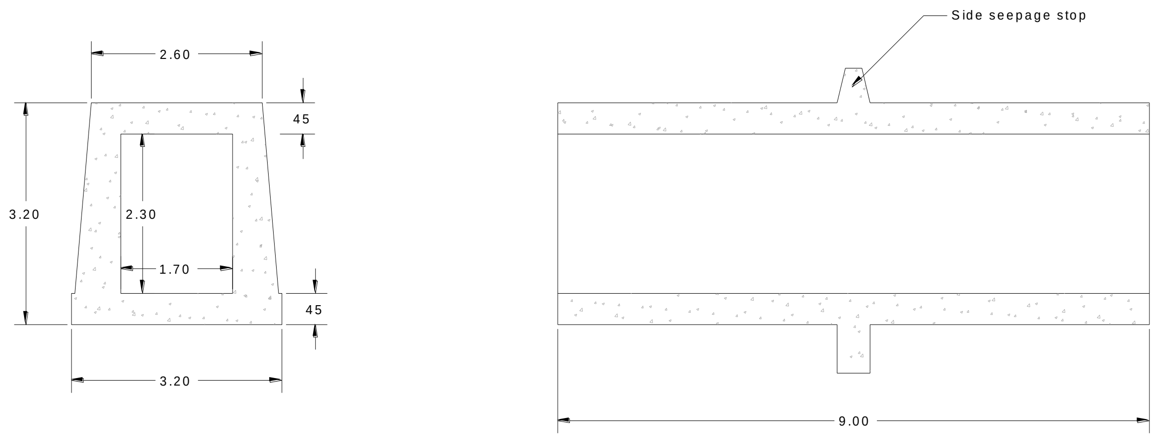


Fig. 5 Cross and longitudinal section by bottom discharge (concrete steel)

The conduit is separated in 9m length transom. It shall be calculate a central transom of bottom discharge.

The load and bearing schedule is in fig. 6. It shall be consider a sigle beam finit element between two joints with length *l*

### *3.2. Earth (soil) and beam (conduit) parameters*

The conduit parameters are:
$A=5.36\ m^2$; $I_b=6.67\ m^4$; $E_b=26$ GPa (for C12/15 concrete)

The ground under conduit
Each node will be thought of as a spring with its elasticity determined according to Chung [ ] by :
$k_s = B \cdot k$ in which
$B = 3.2$ m is the width of the conduit

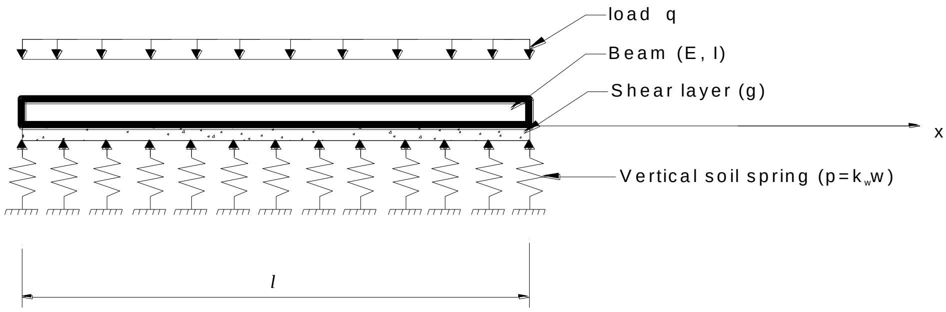


Fig. 6 Beam loading schedule

The marginal nodes will have the same coefficient of subgrade reaction as the other ones according to Bowles

Coefficient of subgrade reaction according to Vesić apud Bowles [5]

$$k = 0.65 \sqrt[12]{\frac{E_p B^4}{E_b I_b}} \frac{E_p}{B(1-\mu_p^2)} \tag{24}$$

Ground parameters are (silty clay):

$E_p$=35 MPa; $\mu_p$=0,35; $\gamma_p$=19 kN/m$^3$

$$k = 0.65 \sqrt[12]{\frac{35 \cdot 3.2^4}{26000 \cdot 6.67}} \frac{35}{3.2(1-0.35^2)} = 5875 \text{ kN/m}^3$$

$k_s$ = 3.2 · 5875 = 28 200 kN/m;

Shear modulus for shear layer in foundation is

$$g = \frac{E_p}{2(1+\mu_p)} = 13 \text{ Mpa} \tag{25}$$

$g_s = B \cdot g$

Foundation parameters *k* and *g* may be calculated according Horvath [7] with following relations

$$k = \frac{E_p}{H} \tag{26}$$

$$g = \frac{E_p}{2(1+\mu_p)} \frac{H}{2} \tag{27}$$

where H is depth to effective rigid base

The effective rigid base is defined as the depth at which settlements caused by the structure can be taken to be zero. For decades it has been assumed that the "depth of influence" for settlement equivalent conceptually to the effective depth to rigid base is twice the width of a square loaded area and four times the width of an infinite strip-Colasanti and Horvath [8]

With this assumptions H=6,4 m ; k=5468 kN/m$^2$; g=41,5 MPa

Earth load on conduit may be consider uniform distributed (crown width is 6 m and conduit beam length is 9 m).

Earth load together with self weight of conduit is q=826 kN/m

With this parameter it shall be calculate structure wich schedule is presented in fig 6

### *3.3. Solving equilibrium equation sistem*

Matrix equation is (8) $([S_e]+[S_w]+[S_g])\{d\} = \{q\}$ , whitch write like (1) is

$[S]\{D\} = \{Q\}$

in which members are:

$[S] = [S_e]+[S_w]+[S_g]$ = stiffness matrix

{D}={*d*} = displacement vector

{Q}={*q*} = load (force) vector.

Solving ecuation (1) is by partitioning matrix S; D and Q whereby it separate out free displacement for degree of freedom (2 and 4) by degree of freedom with elastic bearings (1 and 3)- Jerca [9], see Fig 7

$$\begin{bmatrix} S_{nn} & S_{nr} \\ S_{rn} & S_{rr} \end{bmatrix} \begin{Bmatrix} D_n \\ D_r \end{Bmatrix} = \begin{Bmatrix} Q_n \\ Q_r \end{Bmatrix} + \begin{Bmatrix} R_n \\ R_r \end{Bmatrix} \quad (28)$$

$$S_{nn} D_n + S_{nr} D_r = Q_n + R_n$$
$$S_{rn} D_n + S_{rr} D_r = Q_r + R_r \quad (29)$$

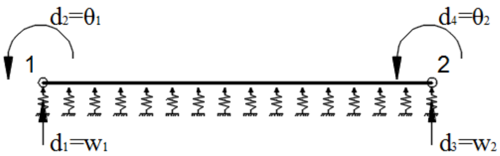

Fig. 7 Beam displacements (degrees of freedom)

$D_n = \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$; $D_r = \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix}$ are displacement vectors

$Q_n = q \begin{Bmatrix} -\frac{l^2}{12} \\ \frac{l^2}{12} \end{Bmatrix}$ $\quad Q_r = q \begin{Bmatrix} -\frac{l}{2} \\ -\frac{l}{2} \end{Bmatrix}$ are load vectors

$$R_n = \begin{Bmatrix} R_2 \\ R_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}; \quad R_r = \begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = k_s \begin{Bmatrix} d_1 \\ d_3 \end{Bmatrix} = k_s D_r \quad (30)$$

are reaction in degree of freedom directions

or (with the same result) $R_r = \begin{Bmatrix} R_1 \\ R_3 \end{Bmatrix} = \begin{bmatrix} k_s & 0 \\ 0 & k_s \end{bmatrix} \begin{Bmatrix} d_1 \\ d_3 \end{Bmatrix} = [k_s] D_r$ (31)

Replacing eq (30) writen like $D_r = \frac{1}{k_s} R_r$, in eq (29) obtain

$$S_{nn} D_n + S_{nr} \frac{1}{k_s} R_r = Q_n$$
$$S_{rn} D_n + (S_{rr} \frac{1}{k_s} - I) R_r = Q_r \quad (32)$$

From last ecuation result

$$R_r = (\frac{1}{k_s} S_{rr} - I)^{-1} (Q_r - S_{rn} D_n) \quad (33)$$

wich indroducing in first one, guide to displacement calculation

$$S_{nn} D_n + S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} (Q_r - S_{rn} D_n) = Q_n; \quad S_{nn} D_n + S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} Q_r - S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} S_{rn} D_n = Q_n$$

$$[S_{nn} - S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} S_{rn}] D_n = Q_n + S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} Q_r$$

Displacement in free(no bearing) degree of freedom directions(2 and 4, Fig. 6) are

$D_n = \left(S^*_{nn}\right)^{-1} Q^*_n$ in wich (34)

$$S^*_{nn} = S_{nn} - S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} S_{rn}; \quad Q^*_n = Q_n + S_{nr} \frac{1}{k_s} (\frac{1}{k_s} S_{rr} - I)^{-1} Q_r$$

After ends of beam displacement calculation it shall be calculated middle of the beam displacement with next relation:

$w_e(x=l/2) = N_1(x)w_1+N_2(x)\theta_1+N_3(x)w_2+N_4(x)\theta_2$ (35)

which in matrix shape is:

$w = [N]\{d\}$ (36)

in which shape function for x=$l$/2 are (4 equations)

$$N_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} = 0.5 \quad ; \quad N_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2} = 1.125$$
$$N_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} = 0.5 \quad ; \quad N_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2} = -1.125 \qquad (37)$$

*3.4. Results*

a) Continuum bearing and soil stiffness considering (Pasternak)

Stiffness matrix of structure is: (obtained with Mathcad software)

$$[S] = [S_e] + [S_w] + [S_g]$$

$$S_e = \begin{pmatrix} 2.855\times10^6 & 1.285\times10^7 & -2.855\times10^6 & 1.285\times10^7 \\ 1.285\times10^7 & 7.708\times10^7 & -1.285\times10^7 & 3.854\times10^7 \\ -2.855\times10^6 & -1.285\times10^7 & 2.855\times10^6 & -1.285\times10^7 \\ 1.285\times10^7 & 3.854\times10^7 & -1.285\times10^7 & 7.708\times10^7 \end{pmatrix}$$

$$S_w = \begin{pmatrix} 6.285\times10^4 & 7.977\times10^4 & 2.175\times10^4 & -4.713\times10^4 \\ 7.977\times10^4 & 1.305\times10^5 & 4.713\times10^4 & -9.789\times10^4 \\ 2.175\times10^4 & 4.713\times10^4 & 6.285\times10^4 & -7.977\times10^4 \\ -4.713\times10^4 & -9.789\times10^4 & -7.977\times10^4 & 1.305\times10^5 \end{pmatrix}$$

$$S_g = \begin{pmatrix} 5.531\times10^3 & 4.148\times10^3 & -5.531\times10^3 & 4.148\times10^3 \\ 4.148\times10^3 & 4.978\times10^4 & -4.148\times10^3 & -1.244\times10^4 \\ -5.531\times10^3 & -4.148\times10^3 & 5.531\times10^3 & -4.148\times10^3 \\ 4.148\times10^3 & -1.244\times10^4 & -4.148\times10^3 & 4.978\times10^4 \end{pmatrix}$$

$$S = \begin{pmatrix} 2.923\times10^6 & 1.293\times10^7 & -2.838\times10^6 & 1.28\times10^7 \\ 1.293\times10^7 & 7.726\times10^7 & -1.28\times10^7 & 3.843\times10^7 \\ -2.838\times10^6 & -1.28\times10^7 & 2.923\times10^6 & -1.293\times10^7 \\ 1.28\times10^7 & 3.843\times10^7 & -1.293\times10^7 & 7.726\times10^7 \end{pmatrix}$$

End of beam displacements are (calculated with eq. 34; 33 and 30)

$$D := \begin{pmatrix} -0.0555 \\ -3.338\times10^{-4} \\ -0.0555 \\ 3.338\times10^{-4} \end{pmatrix} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} \qquad \begin{bmatrix} m \\ rad \\ m \\ rad \end{bmatrix}$$

Middle of the beam displacement, calculated with eq. 36 for $x=l/2$, is: w= - 0.0563 m

b) Continuum bearing and without soil stiffness considering (Winkler)

Stiffness matrix are obtained with Mathcad software:

$$[S] = [S_e] + [S_w]$$

$$S_e = \begin{pmatrix} 2.855\times10^6 & 1.285\times10^7 & -2.855\times10^6 & 1.285\times10^7 \\ 1.285\times10^7 & 7.708\times10^7 & -1.285\times10^7 & 3.854\times10^7 \\ -2.855\times10^6 & -1.285\times10^7 & 2.855\times10^6 & -1.285\times10^7 \\ 1.285\times10^7 & 3.854\times10^7 & -1.285\times10^7 & 7.708\times10^7 \end{pmatrix}$$

$$S_W = \begin{pmatrix} 6.285\times10^4 & 7.977\times10^4 & 2.175\times10^4 & -4.713\times10^4 \\ 7.977\times10^4 & 1.305\times10^5 & 4.713\times10^4 & -9.789\times10^4 \\ 2.175\times10^4 & 4.713\times10^4 & 6.285\times10^4 & -7.977\times10^4 \\ -4.713\times10^4 & -9.789\times10^4 & -7.977\times10^4 & 1.305\times10^5 \end{pmatrix}$$

$$S = \begin{pmatrix} 2.917\times10^6 & 1.293\times10^7 & -2.833\times10^6 & 1.28\times10^7 \\ 1.293\times10^7 & 7.721\times10^7 & -1.28\times10^7 & 3.844\times10^7 \\ -2.833\times10^6 & -1.28\times10^7 & 2.917\times10^6 & -1.293\times10^7 \\ 1.28\times10^7 & 3.844\times10^7 & -1.293\times10^7 & 7.721\times10^7 \end{pmatrix}$$

End of beam displacements are

$$D := \begin{pmatrix} -0.0563 \\ -3.372\times10^{-4} \\ -0.0563 \\ 3.372\times10^{-4} \end{pmatrix} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} \begin{bmatrix} m \\ rad \\ m \\ rad \end{bmatrix}$$

Middle of the beam displacement is: w = -0.0571 m

## IV. Conclusions

Displacements in those two calculus hypothesis are very close (2% difference )

Shear stiffness of the soil considering in Pasternak hypothesis is inconsequent because of structure particulars. This is possible due to the overall rigidity of the structure. The rigidity of one section is according to Gorbunov Posadov [10] Paulos [11]

$$t = \frac{1-\mu_b^2}{1-\mu_p^2}\frac{E_p}{E_b}\frac{\pi(B/2)^3}{4I} \cong 10\frac{E_p}{E_b}\frac{(B/2)^3}{h^3}$$

Given this data, we have t=0,0007<<1 so the conduit is (very)rigid.

The explanation lies in the rigidity of the bottom-discharge conduit structure. Thus the elastic stiffness matrix of the structure is a little modified of rigidity matrix resulted by taking into consideration the specific earth stiffness of the Pasternak model.

So for rigid structures, earth stiffness change (increase) settled by Pasternak hypotthesis and many other researchers (Thangaraj [12], Tiwari [13]) is not suitable.

## References


[1] Horvath John S., Soil-Structure Interaction Research Project (2002) www.engineering.manhattan.edu/civil/CGT.html.

[2] Cook R.,D., Finite Element Modeling for Stress Analysis, John Wiley & Sons, Inc. 1995

[3] Chang-Yu Ou, Deep Excavation. Theory and practice. Taylor & Francis/Balkena Leiden 2006

[4] Teodoru I.,B., Musat V., Soil structure interaction numerical modelling. Foundation beams Editura Politehnium Iasi 2009

[5] Bowles Joseph E., Foundation Analysis And Design 5th ed, McGraw-Hill Book Co - Singapore 1996

[6] Chung Jae H., Finite Element Analysis of Elastic Settlement of Spreadfootings Founded in Soil, University of Florida, Gainesville, FL, USA; http://bsi.ce.ufl.edu/Downloads/Files/newsletter-shallow-foundation.pdf 2011

[7] Horvath John S., Regis J. Colasanti Practical Subgrade Model for Improved Soil-Structure Interaction Analysis: Model Development International Journal of Geomechanics, Vol. 11, No. 1, February 1, 2011

[8] Regis J. Colasanti, John S. Horvath, Practical Subgrade Model for Improved Soil-Structure Interaction Analysis: Software Implementation Practice Periodical on Structural Design and Construction, Vol. 15, No. 4, November 1, 2010. ©ASCE

[9] Jerca S.,H., Ungureanu N., Diaconu D., Numerical methods in construction design (in Romanian) UT Gh. Asachi Iasi, Romania 1997

[10] M.I. Gorbunov Posadov On Elastic soil construction design (in romanian) Editura Tehnica Bucuresti 1960

[11] Poulos H.G., Davis E.H., Elastic Solutions For Soil And Rock Mechanics Centre For Geotechnical Research University Of Sydney 1991

[12] Thangaraj D.,D., Ilamparuthi K., Parametric Study on the Performance of Raft Foundation with Interaction of Frame The Electronic Journal of Geotechnical Engineering Vol. 15 [2010], Bund. H

[13] Tiwari K., Kuppa R., Overview of Methods of Analysis of Beams on Elastic Foundation IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) Volume 11, Issue 5 Ver. VI (Sep-Oct. 2014), PP 22-29