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# Reliability Prediction of Fixed Vanes based on Physics of Failures Approach

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***Abstract**: Physics of Failure models are mathematically derived, deterministic models based on knowledge of failure mechanisms and the root causes of failures. Failure rates are predicted based on stresses, material properties, individual use and environmental conditions. Fixed Vanes are primarily used in fluid machines (turbines, pumps, and windmills etc.) and in aircraft (control surfaces) apart from a variety of other uses. Reliability for vanes is considered to be static, i.e. the reliability function is invariant of time. Failures are defined to be those set of data points wherein the stress random variable experienced by the component exceeds the strength random variable of the material of the component. Static models are used in events where failures occur due to (nearly) instantaneous loads and are not a consequence of any previous effects/ history. This research text utilizes the Physics of Failures approach to quantitatively predict the Reliability of Fixed Vanes for a range of applications and environments. Only the primary/ dominant failure mode is targeted in this text.*
***Keywords**: Reliability, Physics of failure, Vanes, Normal Distribution, Bending stresses, Fixed, Curved vanes*

## I. Introduction

Reliability prediction is one of the most preliminary analyses carried out during design stage. It provides a rational basis for making design decisions such as choice between alternative concepts, choice of part quality levels, de-rating to be applied and use of proven versus state-of-the-art techniques amongst other factors. A properly performed reliability prediction is invaluable to those responsible for making program decisions regarding the feasibility and adequacy of a design approach. [1]

The classical approach towards predicting an item's reliability using a generic failure rate database (NPRD, MIL-HDBK 217 etc) is replete with the following problems: [2]

i) No distinction between design and manufacturing failures
ii) Failures due to overstresses
iii) Assumption of a constant failure rate
iv) Usage of average values, that are neither user nor device specific

Additionally, Cushing et al [3] theorized that traditional approaches miss out on crucial failure details such as failure site, mechanism, load history etc. which are not collected and addressed, because of which:

- The designer has no insight or control over actual causes of failure since the physics is not established.
- The design and usage parameters that greatly influence reliability are not addressed, thus the prediction may not be tailored for different applications.

PoF is an approach to the design and development of reliable product to prevent failure, based on the knowledge of root cause failure mechanisms. This concept is based on the understanding of the relationships between requirements and the physical characteristics of the product. Variation in the manufacturing processes, reaction of product elements and materials to loads (stressors), interaction under loads and their influence on the fitness for use with respect to the use conditions and time are also taken into consideration. [4]

This paper attempts to present a PoF based reliability prediction of fixed vanes. Only primary failure mode (bending) of the vanes (that account for the highest proportion of failures) have been considered for the analysis and prediction.

## II. Vanes

Vanes are aerodynamic devices that are used to either redirect the flow of a fluid (liquid/gas) in any direction, or to get aligned along the direction of flow. They can be classified on the basis of:

### 3.1 Position

**3.1.1 Fixed** – Their orientation does not change with changing directions of fluid flow. Their position can either be controlled using actuators or they remain in the same fixed positions throughout their functional life. E.g. control surfaces of aircraft, inlet/ outlet vanes of turbine/ pump vanes etc.

**3.1.2 Movable** – they change their orientation to align themselves along the direction of fluid flow. E.g. vane anemometers, angle of attack (aoa) sensors, weathervanes etc.

**3.2 Surface Contour**

**3.2.1 Flat vanes** – Their surface is completely flat, with no curvature. They are most used in systems where not a high deflection of the fluid flow is required. E.g. aircraft control surfaces, flow deflector plates etc.

**3.2.2 Curved vanes** – Their surface bears a curvature, and the Vane outlet & inlet curvature angles may or may not be equal. They are used in places where a higher deflection/complete reversal of fluid flow is required in a relatively smaller space. E.g. pump/turbine inlet/ outlet guide vanes etc.

**3.3 On working Medium**

**3.3.1 Liquid** – The working medium is a liquid (Water, oils, fuels etc). Water Turbines, pumps employ such vanes.

**3.3.2 Gas** – The working medium is a gas (Hot expanded fuel mixture gases, Air etc). They are employed in Gas turbine engines, control surfaces of aircraft etc.

**3.4 Sides facing flow**

**3.4.1 Single –side flow** – The vane experiences flow only on a single side when viewed sideways. E.g. Race- car spoilers, pump/turbine guide vanes etc.

**3.4.2 Double –side flow** – The vane experiences fluid flow in both sides, when viewed sideways. E.g. Control Surfaces of aircraft, AoA Sensors etc.

## III. Failure Modes Of Vanes

Some of the generic failure modes for vanes are:

**4.1 Bending**

This failure mode occurs when deflection due to aerodynamic/ fluid force acting on the vane exceeds that value of deflection at which the vane loses its functionality due the change in the contour of the vane. The tolerance of the structural deflections of the vane within which it functions normally and can be 'accepted' must be defined. Any deflections beyond this tolerance would be considered as a failure. This failure mode is only encountered by fixed vanes and especially occurs on Aircraft control surfaces, deflector plates etc. that bend due to the fluid force when actuated to divert air flow.

**4.2 Flutter**

This failure mode occurs when the vane enters a state of dynamic instability in twisting/ longitudinal direction caused by positive feedback between fluid-flow force and vane deflection (aero-elastic flutter). It is caused due to the excess velocity of fluid flow and intrinsic undesirable contour of the vane. Thus is encountered only by fixed vanes, E.g. Aircraft wings/ control surfaces, fins etc.

**4.3 Shear of vane**

This failure mode occurs when the fluid force exceeds the ultimate strength of the vane material leading to its complete shear-off. This failure mode is encountered only by fixed vanes.

**4.4 Icing**

The probability of aircraft icing at a specified altitude in the atmosphere is the chance that an aircraft at that specified altitude will in fact experience icing. Estimates of icing probabilities are based on climatological estimates of the parameters involved. For vanes functioning in high altitudes, cold locations or other similar environments, icing of vanes may cause a distortion in the surface contour of the vane, leading to failure to vane to function desirably. AoA Sensor vanes, Windmills etc have a high probability of failure due to icing. [5]

**4.5 Wear**

Due to excessive usage over time, or cavitation of the fluid, the surface of the vane wears off over time. This may lead to distortion in the contour, rendering the vane unable to perform its required functions. [6]

## IV. Pof Model Development Approach

The flowchart for failure model development using PoF approach involves the steps as shown in Fig 1. [3]

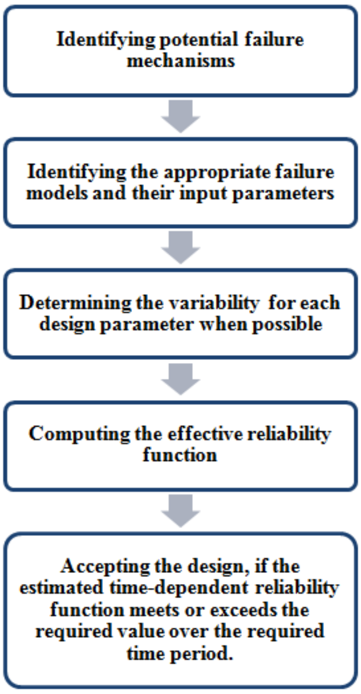



Figure 1 – Flow chart for processes involved in PoF model development

## V. Failure Rate Model

One of the most predominant failure modes for vanes is bending/ physical deformation. This paper is a study on the reliability parameters predicted considering bending to be the only failure mode. This failure is observed in applications where subtle variations in contour/ surface lead to complete vane failure. A common example of this is observed in high speed aircraft, where the wings bend sufficiently to cause a complete control surface role reversal. E.g. Ailerons actuated to roll the aircraft in clockwise directions instead cause an anti-clockwise rolling of aircraft. Only fixed vanes are affected by this failure mode. [7]

The following text would elucidate the theory for the failure rate modeling for 3 different configurations of vanes:

### 6.1 Case-1

Vane Position – Fixed
Vane Surface – Flat
Side of flow – Flow on Single/Both sides of vane
Fluid Medium – Both liquid and gases
Force on a flat plate due to impingement of a jet is given by (1).

$$F_{Total} = F_{Reaction} + F_{Coanda} \qquad (1)$$

Where,

$F_{Total}$= Total force acting on the vane
$F_{Reaction}$ = Reaction force due to change in velocity of the fluid
$F_{Coanda}$ = Force generated by fluid deflected due to Coanda Effect

#### 6.1.1 Fluid force

As far as the scope of this research text is concerned, $F_{Coanda}$ (Force due to Coanda Effect) will be ignored for the analysis and only $F_{Reaction}$ , which is the force generated by the changing velocity of the fluid due to Newton's third Law of Motion, will be considered. Refer Fig 2. [8]. Thus, equation 1 may be reduced to, $F_{Total} = F_{Reaction}$.

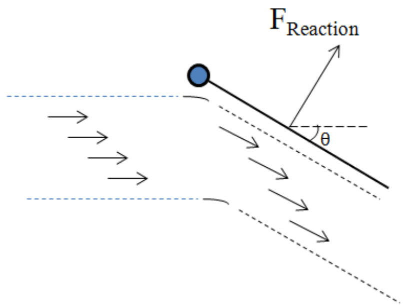



Figure 2 – Schematic of a fixed flat vane diverting fluid flow

The forces are generated only in the plane normal to the vane is equal to the reaction force generated by the fluid, and may be given by (2).

$$F_N = F_{Reaction} = ꞵ*A*V^{2*}\cos\theta \quad (2)$$

Where, $F_N$ = Force acting normal to the plane of the vane
$F_{Reaction}$ = Reaction force of the fluid onto the vane
ꞵ = Density of the fluid
A = Area of the vane immersed in the fluid flow
V = Velocity of fluid
θ = Angle that the vane makes with the horizontal plane.

The entire normal force contributes to the bending loads. Thus, the bending force, '$F_B$' is given by (2).

### 6.1.2 Bending Stress For Flat Vanes

The assumptions made to arrive at the Bending Stress equation are as follows:

- The load is assumed to be of a uniformly distributed nature.
- Poisson effect is ignored
- The section of the vane is rectangular and remains same across its entire length despite loading
- Loading is of the cantilever form

For such sections, the moment of inertia for solid and hollow sections is given by (3) and (4). [9] Refer fig 3 & 4.

$$\text{Moment of inertia, } I = \frac{BH^3}{12} \text{ (For solid sections)} \quad (3)$$

$$\text{Moment of inertia, } I = \frac{BH^3 - bh^3}{12} \text{ (For hollow sections)} \quad (4)$$

Where,
B= length of the base of the section
H= Length of the height of the section

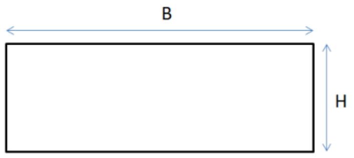



Figure 3 – Solid rectangular Section

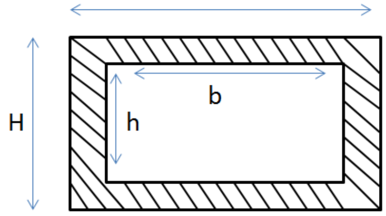



Figure 4 – Hollow rectangular Section

For uniform rectangular section, the distance from the Neutral Axis to the Outermost fibre may be given by (5). Refer fig 5. This equation holds for both solid and hollow sections.

$$L = \frac{\sqrt{B^2 + H^2}}{4} \quad (5)$$

Where, L = distance from the Neutral Axis to the Outermost fibre

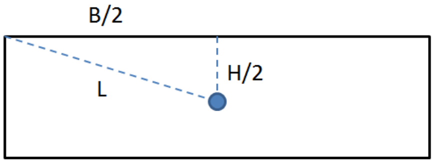



Figure 5- The distance from Neutral axis to outer most fibre for a solid section

Given all the above variables, and assuming a case of simple bending, the equation for bending may be given by (6) using Euler- Bernouli bending theory. [9]

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \quad (6)$$

Where,

M = Bending moment acting on the vane
I = Moment of inertia against bending
σ = Bending stress developed in the vane
y = Distance of neutral axis from the outermost fibre=L
E = Modulus of elasticity of material of vane
R = Radius of curvature of vane due to bending

From equations (6), the stress due to bending may be given by (7).

$$\sigma = \frac{My}{I} \quad (7)$$

### 6.1.3 Bending moment

The loading of the vane is cantilevered, as can be seen from force modeled in Fig (6). For a cantilever beam arrangement, the moment generated due to a Uniformly Distributed Fluid force, can be given by (8). The force is considered to be uniform in nature because a reaction force is generated at every point where fluid impinges the vane, which in case is throughout. Also, the only force causing a bending tendency onto the vane is the normal force, thus, $F_N = F_B$, where $F_B$ is the bending force.

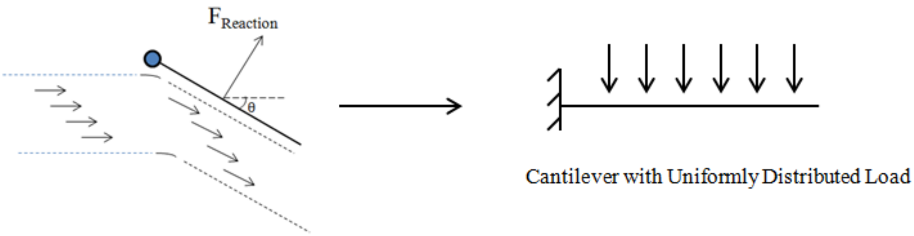



Figure 6 – Illustration of loading of a fixed vane

$$M = \frac{F_B l}{2} \quad (8)$$

Considering equation (3), (5) and (7), the stress in the vane with a (solid rectangular section) due to bending may be given by (9).

$$\sigma_L = \frac{3F_B l\sqrt{b^2+h^2}}{2hb^3} \quad (9)$$

### 6.1.4 Strength

The vane may fail within the elastic limit of the material if the vane is sufficiently distorted such that its functionality is lost. Thus, yield strength of the vane material is not an appropriate selection of strength. Instead, a new parameter of strength can be defined, which relates to that amount of stress which results in the deflection beyond the defined design tolerances and hence resulting in vane failure.

Assuming maximum deflection '$\delta_M$' is the highest limit of deflection of vane at the tip in the aforementioned cantilevered configuration, as allowed by design; its expression is given by (10). [10]

$$\delta_M = \frac{F_B l^3}{8EI} \quad (10)$$

Rearranging (10), considering (3) and (5) for a vane with solid rectangular section, the magnitude of strength can be given by (12).

$$\sigma_S = \frac{E\delta_M\sqrt{b^2+h^2}}{l^2} \quad (12)$$

Where,

$\sigma_S$ = for given section

Of all the above parameters, only 'E', i.e. the modulus of elasticity is a material property of the vane, the others being design properties. Thus, any variability in the magnitude of 'E' will directly affect the 'Interpreted material Strength of vane material'.

It can also be inferred that keeping the design properties constant, the stress induced for any particular deflection is a function of the material elasticity.

**6.2 Case 2**

Vane Position – Fixed
Vane Surface – Curved
Side of flow – Flow on Single/ both sides of vane
Fluid medium – Both liquid and gases
Flow – Direction of incoming flow is tangential, and along the inlet angle
Force on a curved vane due to impingement of a jet is given by (1).

$$F_{Total} = F_{Reaction} + F_{Coanda}$$

Where,

$F_{Total}$= Total force acting on the vane
$F_{Reaction}$ = Reaction force due to change in velocity of the fluid
$F_{Coanda}$ = Force generated by air deflected due to Coanda Effect

**6.2.1 Fluid Force**

$F_{Reaction}$ is the force generated by the changing velocity of the fluid, due to Newton's 3rd Law of Motion. Refer Fig 7.

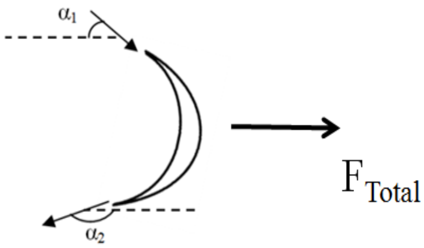



Figure 7 – Schematic of a fixed curved vane with tangential inlet flow diverting fluid flow

The forces generated in 'X' and 'Y' axes are given by (13) and (14). The total bending force is given by (15) are generated only in the plane normal to the vane. [8]

$$F_x = ꞵ*A*V^2(\cos\alpha_1+\cos\alpha_2) \quad (13)$$

$$F_y = ꞵ*A*V^2(\sin\alpha_1-\sin\alpha_2) \quad (14)$$

$$\text{Total force on vane, } F_{Total} = \sqrt{Fx^2 + Fy^2} \quad (15)$$

Where,

$\alpha_1$ = Inlet angle of fluid flow/ Vane
$\alpha_2$ = Exit angle of fluid flow/ Vane

**6.3**

**6.4 Case 3**

Vane Position – Fixed
Vane Surface – Curved
Side of flow – Flow on Single/ both sides of vane
Medium – Both liquid and gases
Flow – Direction of incoming flow is NOT along the inlet angle of the vane
Force on a curved vane due to impingement of a jet is given by (1).

**6.4.1 Fluid Force**

$F_{Reaction}$ is the force generated by the changing velocity of the fluid, due to Newton's 3rd Law of Motion. Refer Fig 8.

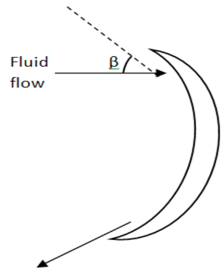



Figure 8 – Schematic of a fixed curved vane with non-tangential inlet flow diverting fluid flow

The forces generated in 'X' and 'Y' axes are given by (16) and (17). The total bending force is given by (18) are generated only in the plane normal to the vane. [11]

$$F_x = \beta * A * V^2 (\cos\alpha_1 + \cos\alpha_2) \cos^2\beta \qquad (16)$$

$$F_y = \beta * A * V^2 (\sin\alpha_1 - \sin\alpha_2) \cos^2\beta \qquad (17)$$

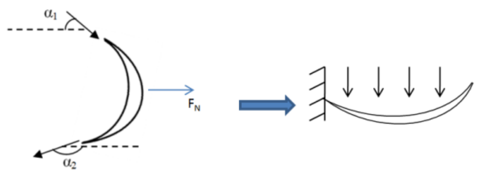
$$F_{Total} = \sqrt{Fx^2 + Fy^2} \qquad (18)$$

Where,

β= Angle between the inlet of vane and direction of fluid flow

### 6.5 Bending Stresses For Curved Vanes Of Case 2 & Case 3

$F_{Total}$ is the force that causes bending tendencies in the vane, and thus, $F_{Total} = F_B$, where '$F_B$' is the Bending force and it can be calculated from the equations presented before. Also, the loading of the vane is cantilevered. Refer Fig 9.




The assumptions made to arrive at the Bending Stress equation are as follows:

- The load is assumed to be of a uniformly distributed nature.
- Poisson effect is ignored
- Only the stress at inner fibre is considered, as it experiences the highest stresses
- The section of the vane is rectangular

The expression of bending stress moment for curved beams is given by (19). [12]

$$\sigma_{max,\ inner} = \frac{M_b C_i}{Ae(r_n - c_i)} \qquad (19)$$

Where,

$c_i$ = distance between Neutral axis and Inner fiber
$r_n$ = radius of curvature of neutral axis
e = distance between centroidal and neutral axis
A = Area of vane
h = Thickness of vane
$r_o$ = radius of curvature of outer fibre
$r_i$ = radius of curvature of inner fibre

The value of '$r_n$' for rectangular section may be given by (20). [12]

$$R_n = \frac{h}{ln(\frac{r_o}{r_i})} \qquad (20)$$

### 6.6 Strength

Similar to flat vanes, assuming maximum allowable radial deflection of inner fibre of curved vane to be '$r_i$' in the aforementioned cantilevered configuration, as allowed by design; the expression for corresponding stress for a given bending moment (8) is given by (21).

$$\sigma_S = \frac{F_B l c_i}{2her_i(r_o - r_i)} \qquad (21)$$

## VI. Prediction Of Reliability

The vane may be considered as a structural unit for the purpose of computation of reliability, because of its primary failure mode being bending, which is typical for structural items. In this case, Reliability is the probability that the strength random variable exceeds the stress random variable. The interference area between the probability Density functions (PDF) of S and L gives a measure of the Probability of failure. Refer Fig 9. The strength based reliability of the mentioned vanes may be calculated using the interference theory. [13]

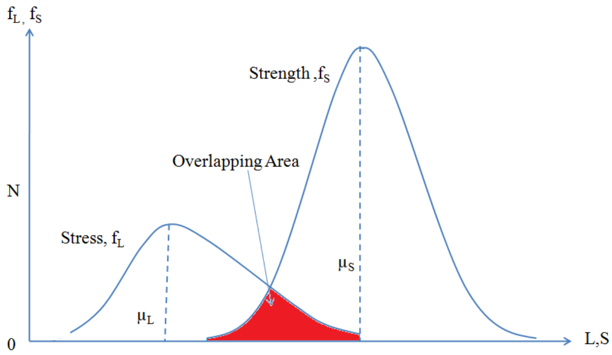



Figure 9 – Illustration for Interference Area between PDF of Strength and Load Random Variables

### 7.1 Assumptions

The following assumptions are made to arrive at an expression of reliability of structural items:

i) The strength Variable (S) is completely independent of load variable (L)

ii) The Probability Density functions of the Modulus of elasticity and the Load follow Normal distribution.

iii) Any variability in the magnitude of 'E' can be directly interpreted to be variability in the value of strength, and shall be further described as 'Strength variable' only, as in article 6.1.4.

iv) Both stress and strength variables follow normal distributions

The density functions may be given by (22) and (23). [14]

$$F_s = \frac{1}{\sigma_S\sqrt{2\pi}} e^{-1/2\{(s-\mu_s)/\sigma_s\}^2}; \qquad -\infty < s < \infty \qquad (22)$$

$$f_l = \frac{1}{\sigma_L\sqrt{2\pi}} e^{-1/2\{(l-\mu_L)/\sigma_L\}^2}; \qquad -\infty < l < \infty \qquad (23)$$

Where,

$\mu_S$ = Mean value of stress

$\mu_L$ = Mean value of strength

$\sigma s$ = Standard deviation of strength random variable

$\sigma_L$ = Standard deviation of load random variable

An expression for reliability may be defined by (24).

$$R = P(S - L \geq 0) = P(X \geq 0) \qquad (24)$$

Where, X = S – L

Here, X is a new random variable. 'X' is a linear function of normally distributed random variables S and L, thus, X also follows normal distribution.

The PDF, mean and Standard Deviation of X, may be given by (25), (26) and (27).

$$F_X = \frac{1}{\sigma_x\sqrt{2\pi}} e^{-1/2\{(x-\mu_x)/\sigma_x\}^2}; \qquad -\infty < l < \infty \qquad (25)$$

$$\mu_x = \mu_S - \mu_L \qquad (26)$$

$$\sigma_x = (\sigma^2 s + \sigma^2{}_L)^{1/2} \qquad (27)$$

The expression for Reliability can be rewritten as (28).

$$R = \frac{1}{\sigma_x\sqrt{2\pi}} \int_0^\infty e^{-1/2\{(x-\mu_x)/\sigma_x\}^2} \qquad (28)$$

The expression may be simplified by employing the standard normal variate z as given in (29).

$$z = \frac{x - \mu_X}{\sigma_x} \qquad (29)$$

The value of 'z' corresponding to lower limit of integration (reliability), and the lower limit of Reliability function is given as (30) and (31)

$$z_1 = \frac{-(\mu_S - \mu_L)}{\sqrt{\sigma^2 s + \sigma^2 L}} \qquad (30)$$

$$R = = \frac{1}{\sqrt{2\pi}} \int_{z=z_1}^{\infty} e^{-1/2z^2} \qquad (31)$$

Calculating the value of $Z_1$ using known mean and standard deviation of S and L, the corresponding component of reliability may either be calculated using standard normal tables.

The data regarding the standard deviation and mean values of strength of the materials may be obtained from the manufacturing QA control.

The values of standard deviation and mean values of stress may be obtained from actual stress data and monitoring during operations.

## VII. Conclusions

The presented approach would be advantageous to teams/ organizations involved in reliability-driven product programs, because the inputs required in this approach are usually the practical inputs available at the time for product design and very limited failure models exist for analyzing failure mechanisms exclusively for vanes

The technique described in this research text presents a crisp approach for the reliability analysis for a number of applications of vanes, which is used in a variety of applications throughout a large spectrum of complex systems.

Special Care must be taken while using this technique as only bending (primary failure mode) is considered. Also, the configuration of vane onto which this technique is intended to apply by a hypothetical user must strictly match with that of the vane described in this text.

This analysis can possibly be used to analyze the reliability of aircraft control surfaces, Inlet/Outlet Guide vanes of turbines and compressor units and blades of axial flow turbine and pumps, amongst a large number of possible applications.

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