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Vibration Analysis of Rotating Composite Beam with Dynamic Stiffness Matrix Method

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Abstract*: The turbine, propeller, helicopter blades are idealized as rotating cantilever beams in the analysis of its different characteristics. The motive of this paper is to find the natural frequency of rotating composite beams. In the present work a rotating composite beam is considered and the natural frequencies of the beam are determined using dynamic stiffness matrix method. The Dynamic Stiffness matrix method developed for the homogeneous cantilever beams is implemented to composite cantilever beams. First the effective young's modulus is determined for the composite material. The effective young's modulus is used to predict the frequency of rotating composite beam for various parameters. The results obtained, indicates how the natural frequency is influenced by various parameters such as speed, hub radius and number of layers in composite.*
Key words:*Dynamic stiffness matrix method, Effective young's modulus, Rotating Composite Beam, Various parameters.*

I. Introduction

The importance of cantilever beams as the application of engineering structures is mostly seen in gas and steam turbine blades, rotor blades of helicopter and spinning space crafts. The analysis of vibration characteristics provides useful information in designing and modeling of mechanisms. Abundance of literature is available in the free vibration of beams. Free vibration analysis of uniform beam are discussed in [1] by finite element method of formulations. Compared to the beams in the stationary state, the natural Frequencies and mode shapes vary significantly with the rotating speed caused by the additional bending stiffness of the beam. The development of methods in free vibration from the past are [2] deals with the Rayleigh-Ritz method, [3] Sinc Galerkin method, [4] Hamilton's principle and Lagranges method,[5] method of Carrera Unified Formulations , [6] Newmark direct integration method, [7] Variational Iteration Method (VIM) and Parameterized Perturbation Method (PPM), in[9,10,11,12] presented various approaches to DSM among these methods Dynamic Stiffness Matrix Method is considered as advanced and elegant method as compared to finite element formulations because it gives exact results for all natural frequencies and mode shapes, without making any approximation enroute andthe results are independent of the number of elements used in the analysis. It appears to be no work was done on rotating composite beams based on the formulations of Dynamic Stiffness Matrix method and the paper helps to fill the gap and extend the further scope of research. The prime motive of this paper is to apply the dynamic stiffness matrix method for composite beams by using the same formulations developed for homogeneous materials without any necessity of separate formulations for composite structures.

To derive the dynamic stiffness matrix of a rotating Bernoulli-Euler beam Analytical and computational efforts are required. Starting from the basic governing deferential equations in free vibration, the dynamics stiffness matrix of a uniform rotating Bernoulli-Euler beam [9] is derived in the paper with the effects of hub radius. The vibrational characteristics of static composite beams are discussed [13] by mixed finite element method, [14, 17] studies CUS (circumferentially uniform stiffness), CAS (circumferentially asymmetric stiffness) and synergistic effects by ANSYS, and It is known that the Young's modulus of a multi-layered composite beam with respect to fiber orientation may be obtained by measuring the moduli in three basic modes of deformation: longitudinal, transverse and longitudinal shear. In practical applications, most of the laminas are sufficiently thin to assume that a state of plane stress exists within each lamina. However, significant inter laminar shearing occurs in the flexure of thick laminates. For this reason, formulations have been developed for inter laminar shearing, while keeping with the assumption that the state of stress within the lamina remains that of plane stress. Effective flexural modulus obtained for the composite beams[18] is used in the dynamic stiffness matrix method for the prediction of effective young's modulus of composite by the in-plane flexural stiffness coefficients $D_{ij}$and the vibrational response of the rotating composite beams are obtained. The derived formulations are validated for cantilever beam results with reference [1] and the frequencies for three composites (boron epoxy, glass epoxy and graphite epoxy) are presented with respect to variation in speed, hub radius and number of layers.

II. Theoretical Formulations

From the usual assumption of plane stress conditions for a thin composite beam, it follows that the normal to the undeformed planes in the beam would remain normal and undeformed in the deformed planes with the implication that the in-plane strains in the plate are linear functions of thickness. Q_{ij}refers to the reduced stiffness matrix of the k[th] layer; and h is the thickness of the beam. The matrices A_{ij}, B_{ij} and $D_{ij}$are referred to as the in-plane modulus, bending-stretching coupling modulus and the flexural modulus, respectively The kth layer stiffness matrix, Q_{ij} , is a function of both material properties and ply orientation.

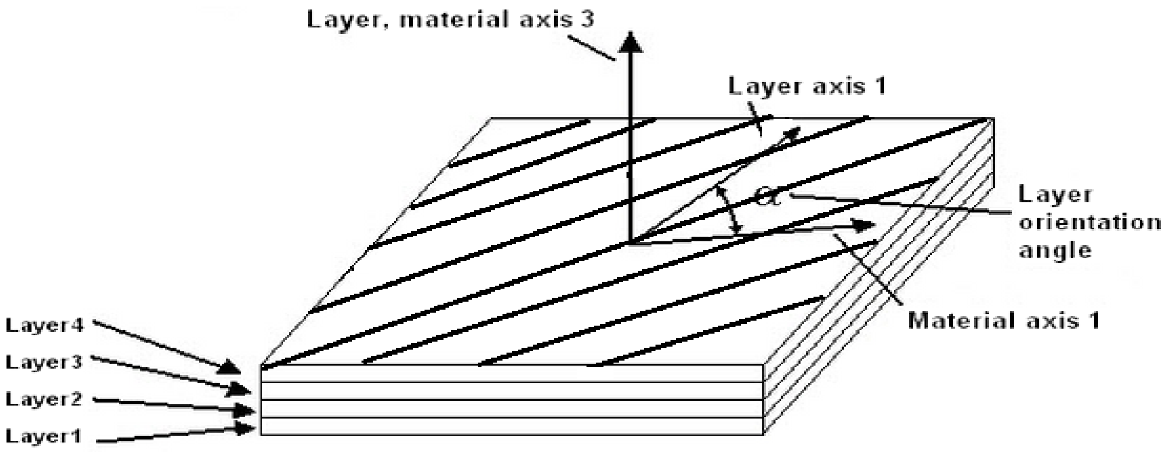


Figure 1 shows layered composite with orientation of fibers

$A_{ij} = \sum_{k=1}^{n} Q_{ij}^{k} * (Z_k - Z_{k-1})$………………………………..……… (a)

$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} Q_{ij}^{k} * (Z_k^2 - Z_{k-1}^2)$………………………………………… (b)

$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} Q_{ij}^{k} * (Z_k^3 - Z_{k-1}^3)$…………………………………………..(c)

$(Z_k - Z_{k-1})=d_k(i,j) = (x,y,s)$

Reduced stiffness matrix $Q_{ij}$is obtained from the given relation:

$$\begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & (m^2-n^2) \end{bmatrix} * \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & 2Q_{66} \end{bmatrix} * \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & (m^2-n^2) \end{bmatrix} = \begin{bmatrix} Q_{xx} & Q_{xy} & 2Q_{xs} \\ Q_{yx} & Q_{yy} & 2Q_{ys} \\ Q_{sx} & Q_{sy} & 2Q_{ss} \end{bmatrix} = [Q_{ij}]$$

……..…………………………………..……… (d)

$Q_{11}=\left(\frac{E_1}{1-\mu_{12}*\mu_{21}}\right)$

$Q_{12}=\left(\frac{E_2*\mu_{12}}{(1-\mu_{12}*\mu_{21})}\right)$

$Q_{66}=G_{12}$ conditions for ply stiffness of[isotropic plies]$_S$ laminates;

$Q_{16} = Q_{26} = 0$

$Q_{22}=\left(\frac{E_2}{1-\mu_{12}*\mu_{21}}\right)$

The flexural modulus, D_{ij}, is given by (c). In this equation, the reduced stiffness matrix Q_{ij}, is independent of thickness within the k[th] lamina. Where N is the total number of layers; and Z_k, Z_{k-1} are the upper and lower co-ordinates of the k[th] layer.For the case where the layers are of the same thickness, Z_0.

$D^* = inv(D)$……..……………………………………………………..……… (e)

Effective flexural modulus is given [18] as

$E_e = 12 \big/ \left(h^3 * D^*(1,1)\right)$…………………………….…………….………..…… (f)

Figure.2 shows the axis system of a typical Bernoulli-Euler beam element of length¸ with its left-hand end at a distance $r_i$from the axis of rotation. Note that $r_i$may or may not be equal to the hub radius $\mathbf{r}_h$, and also L may or may not be equal to the total length L_T shown in the figure. The beam is assumed to be rotating at a constant angular velocity Ω and has a doubly symmetric cross-section such as a rectangle or a circle so that thebending and torsional motions as well as the in-plane and out-of-plane motions areuncoupled. In the right-handed Cartesian co-ordinate system chosen, the origin is taken tobe at the left-hand end of the beam as shown the Y-axis coinciding with the neutral axisof the beam in the undeflected position. The Z-axis is taken to be parallel (but notcoincidental) with the axis of rotation while the X-axis lies in the plane of rotation. Theprincipal axes of the beam cross-section are, therefore, parallel to X and Z directions. Thesystem is able to flex in the Z direction (flapping) and in the X direction (lead-lag motion).These two motions can be coupled only through Coriolis forces, but for the system shownfor the present analysis this coupling is ignored.

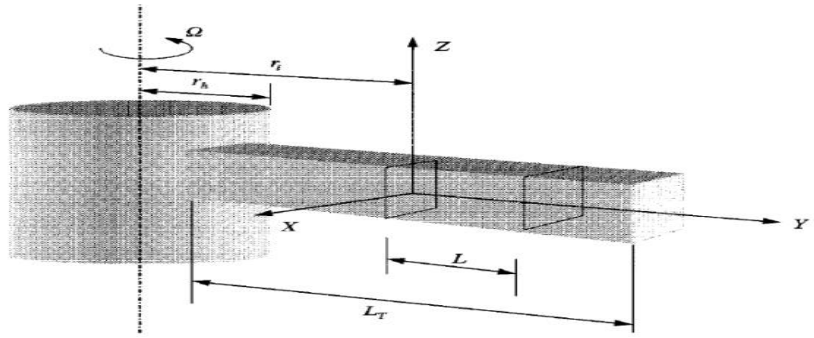


Figure 2 Co-ordinate system and notation for a rotating Bernoulli-Euler beam.

The dynamic stiffness development which follows concerns the out-of-plane free vibration of the beam so that the displacements are confined only in the YZ-plane as shown in Figure 3 The beam element is assumed to be undergoing free natural vibration with circular (angular) frequency u in the YZ-plane, the derived dynamic stiffness matrix can be assembled to study the free-vibration characteristics of a beam with a uniform distribution of structural properties. In order to derive the equilibrium equations the forces acting on an incremental length d*y*at an instant of time *t* are shown in Figure 4. The senses shown for these forces constitutea positive sign definition in this paper for axial force (T), bending moment (*M*) and shearforce (S) respectively. The governing differential equations of motion of the beam element can now be derived using Newton's second law by considering the equilibrium of the infinitesimal length d*y*of the beam element shown in Figure 3.

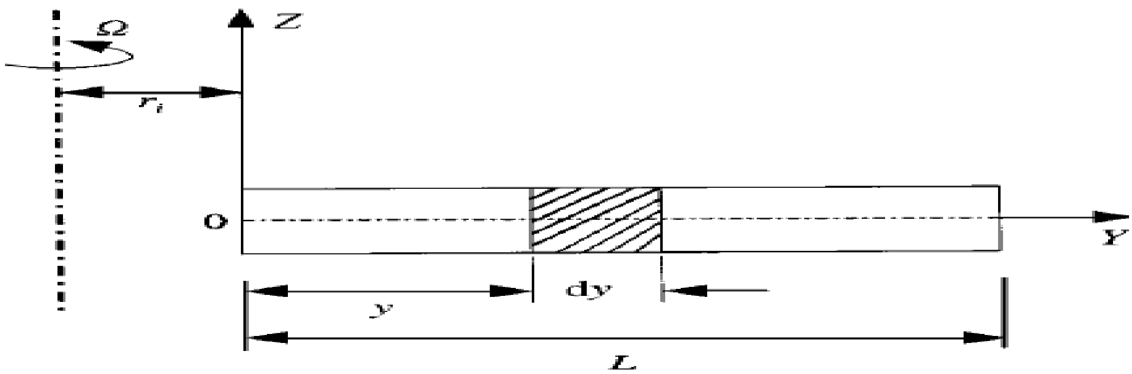


Figure 3 Out-of-plane vibration of a rotating beam element of length L [9]

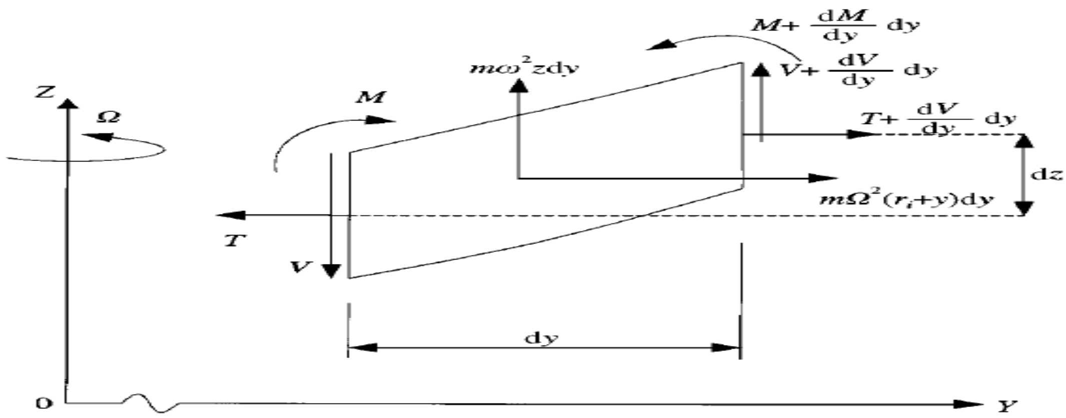


Figure 4 The forces acting in an incremental element d*y*during out-of-plane vibration. [9]

Referring to Figure 4, the centrifugal tension T(y) at a distance y from the origin is given by [9]
$T(y) = 0.5m\Omega^2(L^2 + 2Lr_i - 2r_i y - y^2)$………………………….……..… (1)

Where m is the mass per unit length of the beam and X is the rotational speed in radian per second. Consideration of equilibrium of an infinitesimal element shown in Figure 4 in the Y and Z directions gives
$\frac{dT}{dy} + m\Omega^2(r_i + y) = 0$…………………………..…………………… (2)

$\frac{dT}{dy} + m\omega^2 \mathrm{z(y)} = 0$……………………………………………………… (3)

Finally, rotational equilibrium of the element about the X-axis gives
$V + \frac{dM}{dy} \quad - \mathrm{T(y)}\frac{dz}{dy} = 0$………………………………………………. (4)

The Bernoulli-Euler bending moment equation is given by
$\mathrm{M}(y) = EI_{xx}\frac{d^2{}_z}{dy^2}$……………………………………………………… (5)

Where E is the Young's modulus of the beam material and $I_{xx}$is the second moment of area of the cross-section about the X-axis so that $EI_{xx}$is the flexural rigidity of the beam in the YZ plane. Equations (1-5) can be combined into one differential equation and can be expressed in
Non-dimensional form as follows:
$D^4\bar{h}(\varsigma) - \{0.5\nu^2(1 + 2\rho - 2\rho\varsigma - \varsigma^2) + \eta\} + D^2\bar{h}(\varsigma) + \nu^2(\rho + \varsigma)D\bar{h}(\varsigma) - \mu^2\bar{h}(\varsigma) = 0$.. (6)

Where

$$D = d/d\varsigma\varsigma = y/L\bar{h}(\varsigma) = z/L\rho = r_i/L\mu^2 = \frac{m\omega^2L^4}{EI_{xx}} \ ,\nu^2 = \frac{m\Omega^2L^4}{EI_{xx}},\eta = \frac{FL^2}{EI_{xx}}$$

Thus, the dimensionless expressions for tension, bending moment and shear force are defined as

$$t(\varsigma) = \ T(y)L^2EI = 0.5\nu^2(1 + 2\rho - 2\rho\varsigma - \varsigma^2) + \eta$$
$$\bar{M}(\varsigma) = M(y)L/EI$$
$$\bar{S}(\varsigma) = S(y)L^2/EI$$

Using the Frobenius method, the solution is sought in the form of the following series
$\mathrm{f}(\varsigma, \mathrm{p}) = \ \sum_{\mathrm{n}=0}^{\infty}((\mathrm{a}_{\mathrm{n}+1}(\mathrm{p})\varsigma^{\mathrm{p}+\mathrm{n}})$………………………………………………………… (7)

Where a_{n+1} are the coefficients and p is an undetermined exponent.
By substituting (7) in (6) the following indicial equation obtains
$p(p - 1)(p - 2)(p - 3) = 0$…………………………………………………………. (8)

$$a_{n+5}(p) = \frac{\{0.5\nu^2(1 + 2\rho) + \eta\}}{(p + n + 4)(p + n + 3)}a_{n+3}(p) - \frac{\nu^2\rho(p + n + 1)}{(p + n + 4)(p + n + 3)(p + n + 2)}a_{n+1}(p)$$
$$- \frac{0.5\nu^2(p + n)(p + n + 1) - \mu^2}{(p + n + 4)(p + n + 3)(p + n + 2)(p + n + 1)}a_{n+1}(p)$$
$$a_1(p) = 1$$

$a_{2(p)} = 0a_3(p)=\frac{\{0.5\nu^2(1+2\rho)+\eta\}}{(p+2)(p+1)}a_4(p) = -\frac{\nu^2\rho p}{(p+3)(p+2)(p+1)}$
$f(\varsigma, 0) = 1 + \{0.5\nu^2(1 + 2\rho) + \eta\}\,\varsigma^2/2 + \ \sum_{n=0}^{\infty}((a_{n+5}(0)\varsigma^{n+4})$……….……………. (9)

$f(\varsigma, 1) = \varsigma + \{0.5\nu^2(1 + 2\rho) + \eta\}\,\varsigma^3/6 - \nu^2\rho\,\varsigma^4/24 + \ \sum_{n=0}^{\infty}((a_{n+5}(1)\varsigma^{n+5})$……… (10)

$f(\varsigma, 2) = \varsigma^2 + \{0.5\nu^2(1 + 2\rho) + \eta\}\,\varsigma^4/12 - \nu^2\rho\,\varsigma^5/30 + \ \sum_{n=0}^{\infty}(a_{n+5}(2)\varsigma^{n+6})$……. (11)

$f(\varsigma, 3) = \varsigma^3 + \{0.5\nu^2(1 + 2\rho) + \eta\}\varsigma^5/20 - \nu^2\rho\,\varsigma^6/40 + \ \sum_{n=0}^{\infty}(a_{n+5}(3)\varsigma^{n+7})$…...... (12)
The general solution of the deferential equation (6) can be written as
$\bar{g}(\varsigma) = e_1f(\varsigma, 0) + e_2f(\varsigma, 1) + e_3f(\varsigma, 2) + e_4f(\varsigma, 3)$………………………….. (13)

$\bar{y}(\varsigma) = \bar{h}' = e_1f'(\varsigma, 0) + e_2f'(\varsigma, 1) + e_3f'(\varsigma, 2) + e_4f'(\varsigma, 3)$…………………………... (14)

$\bar{M}(\varsigma) = \bar{h}'' = e_1 f''(\varsigma,0) + e_2 f''(\varsigma,1) + e_3 f''(\varsigma,2) + e_4 f''(\varsigma,3)$………………………..... (15)

$\bar{S}(\varsigma) = -\bar{h}'''(\varsigma) + t(\varsigma)\bar{h}'(\varsigma) = -\{e_1 f'''^{(\varsigma,0)} + e_2 f'''^{(\varsigma,1)} + e_3 f'''^{(\varsigma,2)} + e_4 f'''^{(\varsigma,3)}\} + t(\varsigma)\{ e_1 f'^{(\varsigma,0)} + e_2 f'^{(\varsigma,1)} + e_3 f'^{(\varsigma,2)} + e_4 f'^{(\varsigma,3)}\}$…………………………………………………………………..… (16)

The dynamic stiffness matrix which relates the amplitudes of harmonically varying forces to the corresponding harmonically varying displacement amplitudes at the ends of the element can now be derived by imposing the end conditions for displacements and forces. The end conditions for displacements and forces of the element (see Figure 5) are, respectively,

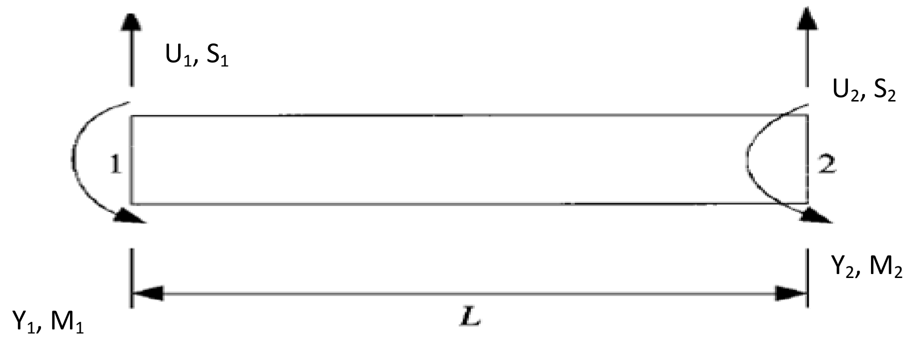


Figure 5 End conditions for displacements and forces of the beam element.

Displacements:
At end 1($\varsigma = 0$): $\bar{h} = \overline{U_1}, \bar{y} = \overline{Y_1}$
At end 2($\varsigma = 1$): $\bar{h} = \overline{U_2}, \bar{y} = \overline{Y_2}$ ..……………………….. (17)

Forces:
At end 1($\varsigma = 0$): $\bar{S} = -\overline{S_1}, \bar{M} = -\overline{M_1}$
At end 2($\varsigma = 1$): $\bar{S} = \overline{S_2}, \bar{M} = \overline{M_2}$ ……………………….. (18)

$$f(0,0) = 0\,, f(0,1) = 1, f(0,2) = 0, f(0,3) = 0$$
$$f'(0,0) = 0\,, f'(0,1) = 1, f'(0,2) = 0, f'(0,3) = 0$$

By substituting 17&18 in equations in 13, 14, 15&16

$$\begin{bmatrix} \overline{U_1} \\ \overline{Y_1} \\ \overline{U_2} \\ \overline{Y_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad => \bar{U} = Ce \text{…………………. (19)}$$

$$c_{31} = f(1,0), c_{32} = f(1,1), c_{33} = f(1,2), c_{34} = f(1,3),$$
$$c_{41} = f'(1,0), c_{42} = f'(1,1), c_{43} = f'(1,2), c_{44} = f'(1,3),$$
$$f'''(0,0) = 0, f'''(0,1) = \{0.5\nu^2(1+2\rho) + \eta\}, f'''(0,2) = 0, f'''(0,3) = 6, t(0) = 0.5\nu^2(1+2\rho) + \eta f''(0,1) = 0, f''(0,2) = 2, f''(0,3) = 0, t(0) = \eta$$

$$\begin{bmatrix} \overline{S_1} \\ \overline{M_1} \\ \overline{S_2} \\ \overline{M_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 6 \\ d_{21} & 0 & -2 & 0 \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_3 \end{bmatrix} \quad => \bar{F} = De \text{………………… (20)}$$

$$d_{21} = -\{0.5\nu^2(1+2\rho) + \eta\}, d_{31} = \eta f'(1,0) - f'''(1,0), d_{32} = \eta f'(1,1) - f'''(1,1), d_{33} = \eta f'(1,2) - f'''(1,2), d_{34} = \eta f'(1,3) - f'''(1,3)$$
$$d_{41} = f''(1,0), d_{42} = f''(1,1), d_{43} = f''(1,2), d_{44} = f''(1,3)$$

The dynamic stiffness matrix $\bar{K}$can be obtained by eliminating the constant vector e from equations (19) and (20) to give the force- displacement relationship as follows:

$$\bar{F} = \bar{K}\bar{U} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(21)}$$

$$\begin{bmatrix} \overline{S_1} \\ \overline{M_1} \\ \overline{S_2} \\ \overline{M_2} \end{bmatrix} = \begin{bmatrix} \bar{k}_{11} & \bar{k}_{12} & \bar{k}_{13} & \bar{k}_{14} \\ \bar{k}_{21} & \bar{k}_{22} & \bar{k}_{23} & \bar{k}_{24} \\ \bar{k}_{31} & \bar{k}_{32} & \bar{k}_{33} & \bar{k}_{34} \\ \bar{k}_{41} & \bar{k}_{42} & \bar{k}_{43} & \bar{k}_{44} \end{bmatrix} \begin{bmatrix} \overline{U_1} \\ \overline{Y_1} \\ \overline{U_2} \\ \overline{Y_2} \end{bmatrix}$$

$$\bar{K} = DB^{-1}$$

$\bar{K}_{11}$=6$(c_{31}c_{44} - c_{33}c_{42})/\Delta$,$\bar{K}_{12}$=6$(c_{32}c_{43} - c_{33}c_{42})/\Delta$,$\bar{K}_{13}$= -6$(c_{33})/\Delta$,$\bar{K}_{14}$= 6$(c_{33})/\Delta$,
$\bar{K}_{22}$=6$(c_{32}c_{44} - c_{34}c_{42})/\Delta$,$\bar{K}_{23}$= -2$(c_{44})/\Delta$,$\bar{K}_{24}$= 2$(c_{34})/\Delta$,$\bar{K}_{33}$=$(c_{44}d_{33} - c_{43}d_{34})/\Delta$,
$\bar{K}_{34}$=$(c_{33}d_{34} - c_{34}d_{33})/\Delta$,$\bar{K}_{44}$=$(c_{33}d_{44} - c_{34}d_{43})/\Delta$ and $\Delta = (c_{33}c_{44} - c_{34}c_{43})$

The elements of the dimensional dynamic stiffness matrix K can now be recovered from the elements of $\bar{K}$ so that

$$\begin{bmatrix} S_1 \\ M_1 \\ S_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} U_1 \\ Y_1 \\ U_2 \\ Y_2 \end{bmatrix}$$

$k_{11} = X_3\bar{k}_{11}, k_{12} = X_2\bar{k}_{12}, k_{13} = X_3\bar{k}_{13}, k_{14} = X_2\bar{k}_{14}, k_{22} = X_1\bar{k}_{22}, k_{23} = X_2\bar{k}_{23}, k_{24} = X_1\bar{k}_{24}, k_{34} = X_2\bar{k}_{34}, k_{33} = X_3\bar{k}_{33}, k_{33} = X_3\bar{k}_{33}, k_{44} = X_1\bar{k}_{44}$ …………………………………………… (22)

where

$$X_1 = \frac{E*I_{xx}}{L}\; X_2 = \frac{E*I_{xx}}{L^2}\; X_3 = \frac{E*I_{xx}}{L^3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(23)}$$

In above equations value of E is taken as the Effective flexural modulus (E_e) value which was calculated from equation (f) for the concerned composite material.

III. Numerical Results

To validate the present work it is compared with previous literature. The natural frequencies are obtained by taking the dimensions and the material properties for a uniform fixed free beam (cantilever beam) studied in [1] are: Material of beam = Al, Total length (L) = 0.5 m, width (B) = 0.045 m, height (H) = 0.005 m, Young's Modulus (E) = 70 x 10^9, mass density = 2700kg/ m^3. Table 1 shows the comparison of frequencies and it shows a good agreement with numerical results presented by Chopade [1].

Table 1 comparison of frequencies to reference paper with present work results

Mode	Reference [1] (Hz)	Present work (Hz)
1	16.45	16.45
2	103.06	103.09
3	288.52	288.68

Problem Definition:

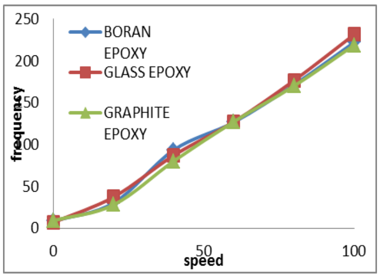
To determine the natural frequency of rotating composite beam the following dimensions are assumed. Total length (L) = 1000 mm, width (B) = 20 mm, height (H) = 20 mm, A Composite beam with four layers and the stacking Sequence is [45/-45/-45/45] is considered. The table 2 represents the material properties.

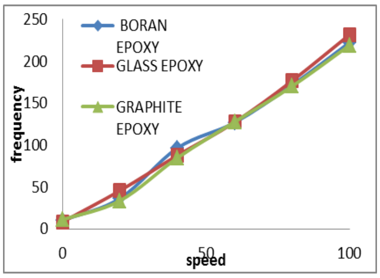
Table 2: Material Properties of Composite Materials

Properties	Boron Epoxy (B5.6/5505)	Glass Epoxy (E-glass)	Graphite Epoxy (Gy-70/934)
Young's modulus in longitudinal direction E1 (GPa)	201	41	294
Young's modulus in transverse direction E2 (GPa)	21.7	10.4	6.4
Density ρ (Kg/m3)	2030	1970	1590
Modulus of rigidity G12(GPa)	5.4	4.3	4.9
Poisson's ratio	0.17	0.28	0.23

IV. Results and Discussion

The present work deals with the effect of Rotating Speed, hub radius ratio and number of layers of the composite on Natural frequency of the rotating Composite beam. The effect of rotating speed on natural frequency of the rotating composite beam of various material are obtained and first three natural frequencies are plotted in Fig 6, Fig 7 and Fig 8. From these figures it is concluded that the increase in speed leads to increase in frequency. This is due to increase of stiffness (centrifugal force) with rotating speed.





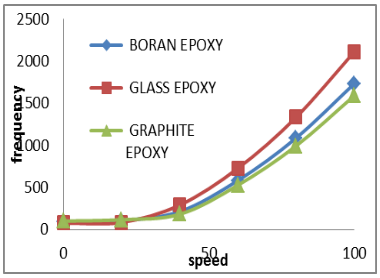
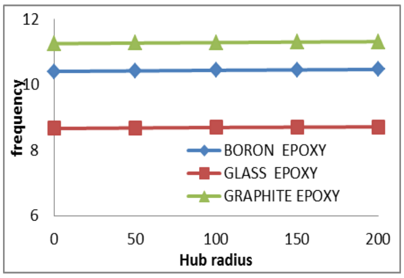
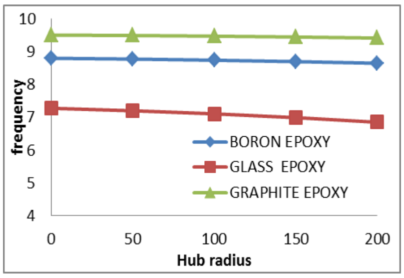


Fig. 6 Variation of first natural Frequency with respect to speed
Fig. 7 Variation of Second natural Frequency with respect to speed
Fig. 8 Variation of Third natural Frequency with respect to speed

To study the effect of hub radius on natural frequency of the rotating composite beam of the three composites the natural frequencies are obtained by taking hub radius from 0 to 200 with step increament of 50 at speeds 5, 50 and 100 Rps. First three natural frequencies obtained at 5 Rps speed are plotted in Fig 9, Fig 10 and Fig 11. The First three natural frequencies of composite beam at moderate speed (50 Rps) are plotted in Fig 12, Fig 13 and Fig 14. The First three natural frequencies of composite beam at higher speed (100 Rps) are obtained and plotted in Fig 15, Fig 16 and Fig 17. From these figures it is concluded that the hub radius affects the frequency in medium and higher speeds. This is due to increase centrifugal force is very less in lower speeds irrespective of hub radius ratio. At higher speeds increase in hub radius ratio leads to increase in centrifugal force causes increment in frequency.





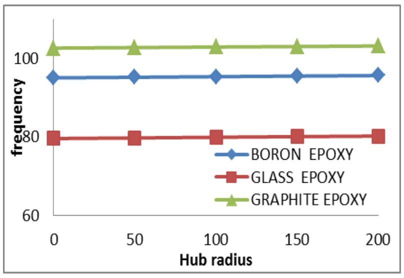
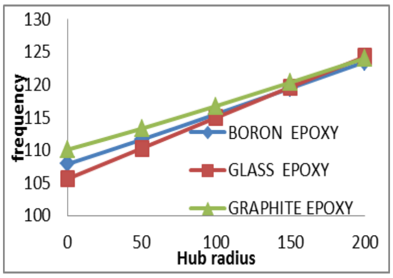
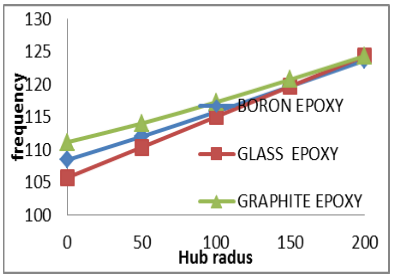


Fig 9 Variation of first natural frequency with respect to hub radius ratio at 5 Rps
Fig 10 Variation of Second natural frequency with respect to hub radius ratio at 5 Rps
Fig 11 Variation of Third natural frequency with respect to hub radius ratio at 5 Rps





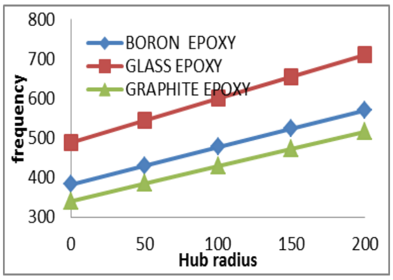
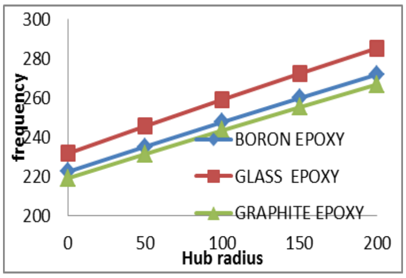
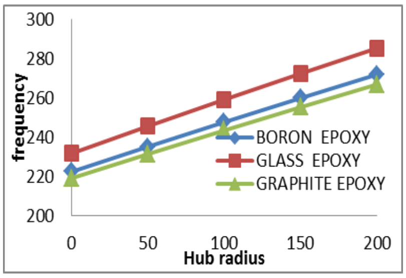


Fig 12 Variation of first natural frequency with respect to hub radius ratio at 50 Rps.
Fig 13 Variation of Second natural frequency with Respect to hub radius at 50Rps
Fig 14 Variation of Third natural frequency with respect to hub radius ratio at 50 Rps





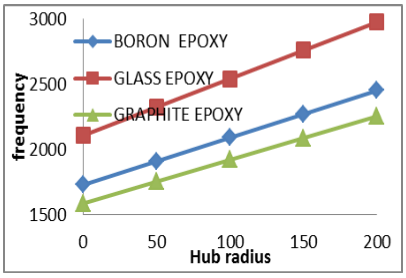
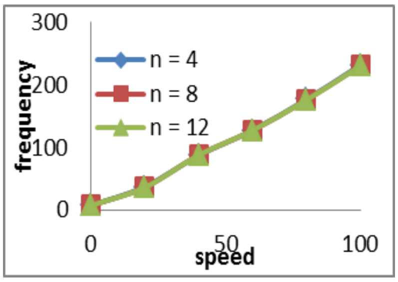
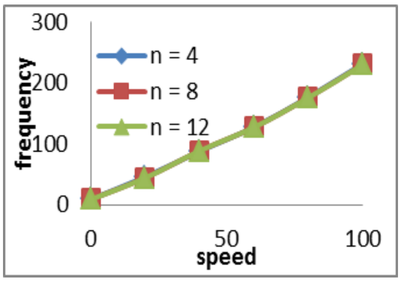


Fig 15 Variation of first natural frequency with respect to hub radius ratio at 50 Rps
Fig16 Variation of Secondnatural frequency with respect to hub radius ratio at 50 Rps
Fig 17 Variation of Third natural frequency with respect to hub radius ratio at 50 Rps

To determine the effect of number of layers on natural frequency of the rotating composite beam the frequencies are obtained for 4 layers, 8 layers and 12 layers with same total thickness for various materials with respect to speed. Fig. 18, Fig 19 and Fig 20 shows the variation of first three natural frequency of Glass Epoxy Beam with respect to speed for different number of layers. Fig. 21, Fig 22 and Fig 23 shows the variation of first three natural frequency of Boron Epoxy Beam with respect to speed for different number layers. Fig. 24, Fig 25 and Fig 26 shows the variation of first three natural frequency of Graphite Epoxy Beam with respect to speed for different number of layers. From these figures it is observed that frequency does not affect with respect to increase in layers. This is due to total thickness is constant for all cases. Considering various composite materials graphite epoxy exhibits the higher natural frequency compared with other materials due to high stiffness.





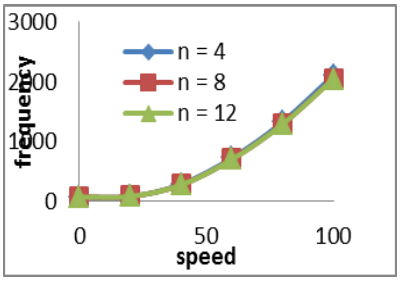
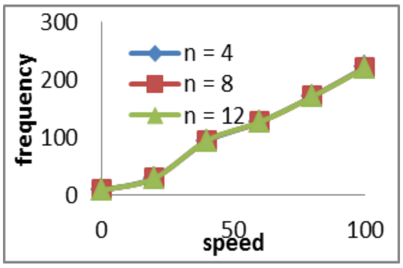
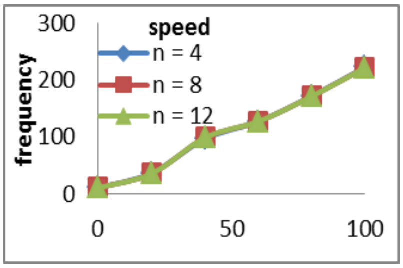


Fig 18 Variation of First natural frequency with respect to speed at different layers of E Glass Epoxy Beam
Fig 19 Variation of Second natural frequency with respect to speed at different layers of E Glass Epoxy Beam
Fig 20 Variation of Third natural frequency with respect to speed at different layers of E Glass Epoxy Beam





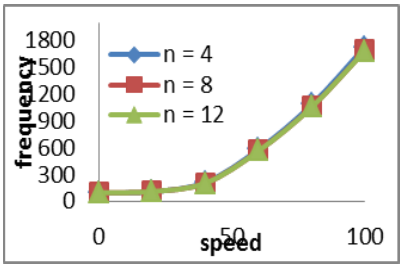


Fig 21 Variation of First natural frequency with respect to speed at different layers of Boron Epoxy Beam
Fig 22 Variation of Second natural frequency with respect to speed at different layers of Boron Epoxy Beam
Fig 23 Variationof Third natural frequency with respect to speed at different layers of Boron Epoxy Beam

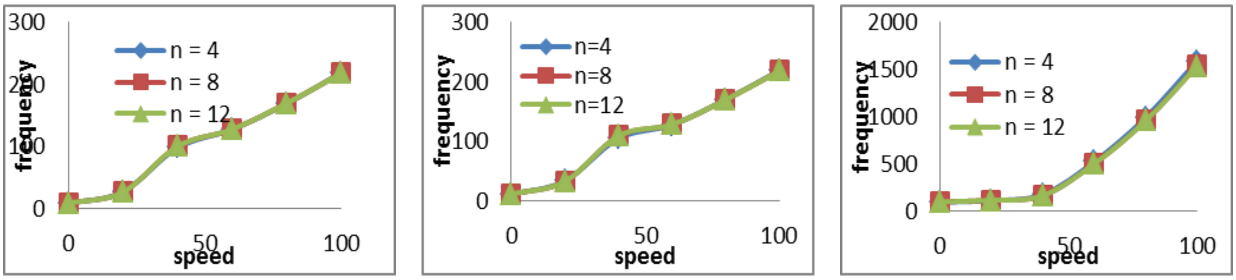


Fig 24 Variation of First natural frequency with respect to speed at different layers of Graphite Epoxy Beam
Fig 25 Variation of Second natural frequency with respect to speed at different layers of Graphite Epoxy Beam
Fig 26 Variation of Third natural frequency with respect to speed at different layers of Graphite Epoxy Beam

V. Conclusion

In this study, free vibration analysis of rotating composite beam is performed. The effective young's modulus for composite beam is obtained and natural frequencies are presented using dynamic stiffness matrix method. The present model is validated with available literature and results shows good agreement. In the present work the application of the dynamic stiffness matrix for the rotating composite beam and focussed on the three composite materials. The effect of rotating speed, hub radius ratio and number of layers on natural frequencies are investigated. Results show that the natural frequencies are increased with increase of rotating speed due to increased centrifugal force. The hub radius affects the natural frequencies only at higher speeds. It is observed that there is no influence of increasing number of layers on natural frequency due to constant total thickness. Compared with different composites, graphite epoxy exhibit higher natural frequencies due to high stiffness.

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