

A Analytical Model For Vibration Period With SSI Of R/C Structures

Turkia Haithem¹, Lahbari Nouredine²

^{1,2} Department of Civil Engineering, Institute of Civil Engineering, Hydraulique and Architecture, University of Batna, Algeria

Abstract: The effect of the fundamental period, the soil structure interaction SSI, and the site on the seismic behaviour of R/C structures is investigated using analytical model based on the Algerian seismic regulations. Hence the aim of this study is to formulate model covering the fundamental period of vibrations based on a system with continuous columns in which the deformations of structure and soil represent the degree of freedom. Shear and flexural deformation are appointed for the structure, whereas relative displacement of the foundation base and rocking are meant for the soil (isolated footings for stiff and medium soil - sites S_1, S_2 and mat for soft soil sites S_3 or S_4). Finite element method is used to analyse the response of various R/C frames (low, medium and high rise), assuming fixed and flexible base, (vertical, horizontal translations stiffness, rocking and torsional stiffness), and compared with Newmark Rosenblueth, Deleuze and Gazetas methods.

Keywords: soil structure interaction, foundation, R / C frame, Seismic response, effective periods.

I. Introduction

Often, seismic structural design is based on rigid base assumption, and interaction with the soil-foundation system is either ignored or carried out separately, whereas in reality these systems are coupled. Ignoring the SSI effects may lead to erroneous structural assessment and estimates of seismic demands.

This work presents a simplified and accurate formulation using finite element method applied to the analytical model based on the Algerian seismic regulation, a rapid assessment of the fundamental period of vibrations when SSI effects are accounted for. Furthermore, it investigates the importance of SSI phenomena on the response of frames as function of different parameters such as soil rigidity, foundation rigidity, foundation mass, and soil mass.

It is shown that for structures founded on soft soils with high relative rigidity, the SSI effects amplifies the dynamic response of the system. Also, it is not necessary to take into account SSI effects when designing a R/C building on stiff soil.

II. Formulation Of The Soil Structure Interaction - Description Of Model SSI

Based on the formules of the period of vibration given by THOMPSON [1] neglecting SSI effects, the new formulation taking into account SSI effects will be as follows:

$$T^* = \sqrt{(T_s^*)^2 + (T_b^*)^2} \quad (1)$$

2.1 Shear Mode with SSI (T_s^*)

In this case the bending deformation is negligible and the shear deformation is the determining factor in calculating the period. The fundamental period of vibration in Shear mode is given by:

$$\frac{T_s^*}{T_s} = \sqrt{\frac{W^*}{W_{St}} \times \frac{R_S}{R_S^*}} = \sqrt{mr} \quad (2)$$

Where:

T_s : Shear fundamental period without SSI is given by [2]:

$$T_s = \frac{2\pi}{\omega_s} = 4H \sqrt{\frac{\mu F}{AG}} = 4H \sqrt{\frac{\mu}{R_S}} = 4H \sqrt{\frac{W_{St}}{Hg} \times \frac{1}{R_S}} = 4 \times \sqrt{\frac{W_{St} H}{g R_S}} \quad (3)$$

W^* : Total mass calculated from structure, soil and foundation; W_{St} : Total mass calculated from structure; r : Ratio of shear stiffness equal to: R_S/R_S^*

R_S^* , R_S : Shear stiffness with and without SSI.

$$R_S = \frac{AG}{F_S} \quad \text{and} \quad R_S^* = \frac{AG}{F_S^*} \quad (4)$$

A: area of the section; G: Modulus of the reinforced concrete; F_s, F_s^* : security coefficient without and with SSI.
 m: Ratio of unit mass of building is given by:

$$m = \frac{W^*}{W_{St}} = \frac{W_{St} + W_F + W_S}{W_{St}} = 1 + \frac{W_F}{W_{St}} + \frac{W_S}{W_{St}} \quad (5)$$

Where:

W_F : foundation mass ; W_S : soil mass.

2.2 flexural Mode with SSI

In this case the shear deformation is negligible and the bending is the determining factor in calculating the

$$T_b^* = \frac{2\pi}{\omega_b^*} = 1.79H^2 \sqrt{\frac{\mu^*}{EI^*}} = 1.79H \sqrt{\frac{W^*H}{gEI^*}} \quad (6)$$

period.

T_b : Flexural fundamental period without SSI is given by [2]:

I^* : Total moment of inertia of the structure with SSI.

Consequently:

$$\frac{T_b^*}{T_b} = \sqrt{\frac{W^*}{W} \times \frac{I}{I^*}} = \sqrt{m\lambda} \quad (8)$$

$$T_b = \frac{2\pi}{\omega_b} = 1.79H^2 \sqrt{\frac{\mu}{EI_{St}}} = 1.79H \sqrt{\frac{W_{St}H}{gEI_{St}}} \quad (7)$$

$$\frac{1}{\lambda} = \frac{I^*}{I_{St}} = \frac{I_{St} + I_F}{I_{St}} = 1 + \frac{I_F}{I_{St}} \quad (9)$$

λ : Ratio of the moment of inertia with and without SSI given by:

Where:

I_F : Moment of inertia of the foundation.

I_{St} : Moment of inertia calculated of the structure only without SSI.

2.3 Rocking mode

2.3.1 The case of isolated footings (stiff and medium soil - site S1, S2)

Using the simplified method from VELETOS [3,4,5], the expression of rocking stiffness K_ϕ from vertical and rocking stiffnesses of the soil is:

$$K_\phi = \sum K_{\phi i} + \sum K_{vi} y_i^2 \quad (10)$$

With:

K_{vi} and $K_{\phi i}$ the corresponding vertical and rocking stiffnesses respectively.

Y_i represents the normal distance from the centroid of the i^{th} footing to the rocking axis of the foundation.

The vertical and rocking stiffnesses of the i^{th} footing are defined by the following relations [3]:

With r_{ai} and r_{mi} are given as follows [3, 6]

$$K_{vi} = \frac{4Gr_{ai}}{1-\mu} \left[1 + \frac{2d_i}{5r_{ai}} \right] \quad (11)$$

$$K_{\phi i} = \frac{8G_i r_{mi}^3}{3(1-\mu)} \left[1 + 2 \frac{d_i}{r_{mi}} \right] \quad (12)$$

r_{ai} : Radius of a circular footing that has the area of the i^{th} footing; d_i : Depth of effective embedment for the i^{th}

$$r_{ai} = \sqrt{\frac{A_F}{\pi}} \quad (13)$$

$$r_{mi} = \sqrt[4]{\frac{4I_F}{\pi}} \quad (14)$$

footing.

2.3.2 The case of mat (soft soil - site S3, S4)

From the simplified method of VELETOS [4,5] described by SOULUMIAC [7] and based on the text of ATC-3 [8]; The expression of rocking stiffness K_{Φ} of rectangular footing can be expressed by [3]:

$$K_{\Phi} = \frac{8Gr_m^3}{3(1-\mu)} \quad (15)$$

Where: K_{Φ} : Rocking stiffness of rectangular footing; G : Shear modulus of soil beneath the i^{th} footing; μ : The Poisson's ratio of soil.

r_m : Radius of the circle of the equivalent foundation calculated as follows:

$$r_m = \sqrt{\frac{A_F I_F}{\pi}} \quad (16)$$

With:

A_F : Area of the section of the foundation.

Then:

$$K_{\Phi} = \frac{8G}{3(1-\mu)} \sqrt[3]{\frac{A_F I_F}{\pi}} \quad (17)$$

The final formulation for the fundamental period of vibration, taking into account SSI effect will be:

$$T^* = \sqrt{mrT_s^2 + m\lambda T_b^2} \quad (18)$$

III. Formulation Of SSI To Reinforced Concrete Frames

In the following, an approximate formula for the lateral drift of the frame is determined by considering the interaction of soil structure. The assumptions of the method of analysis of rigid frame were adopted [9]. Figure 1 shows the frame after deflection under lateral forces. The total lateral displacement of a level U^* is equal to the sum of the displacement without SSI and the displacement due to the interaction U_r [2, 10].

$$U^* = U + U_R \quad (19)$$

Where:

U^* : The total lateral displacement with SSI; U : The displacement at the n^{th} floor of the built structure without SSI.

$$U = U_C + U_g \quad (20)$$

U_C, U_g : The displacement of the columns due to the bending mode and the displacement of the beams due to the shear mode.

U_R : The displacement due to the translation and rocking of the foundation [11, 12, 13].

The lateral displacement U without SSI is calculated by [2]:

$$U = \frac{Vh^2[(N_C - 1)K_g + N_C K_C]}{12EN_C(N_C - 1)K_g K_C} \quad (21)$$

The linear rigidities of columns and beams are:

$$K_C = \frac{I_C}{h} \quad \text{and} \quad K_g = \frac{I_g}{L} \quad (22)$$

V : shear force at the base of the structure without SSI.

I_C, I_g : moment of inertia of the columns and beams respectively

N_C : Number of columns.

E : Modulus of elasticity of the concrete.

h : story height.

L : length of bay.

The total displacement taking into account the SSI is calculated by simplified method from VELETOS [3] as:

$$U^* = \frac{V^*}{V} \left[\frac{M_0 H}{K_{\Phi}} + U \right] \quad (23)$$

Where:

V^* : The reduced shear force corresponding to the soil structure interaction (with SSI).

M_0 : The moment due to the lateral forces without SSI is: $(2HV)/3$

K_ϕ : The rocking stiffness of the foundation with SSI.

H: total height of the structure.

Then the shear deformation due to the lateral displacement will be:

$$\gamma^* = \frac{U^*}{H} = \frac{V^*}{V} \left[\frac{M_O}{K_\phi} + \gamma \right] \quad (24)$$

Where:

γ : The shear deformation without SSI calculated by: $\gamma = U / h$

Hence the shear stiffness with SSI is:

$$R_S^* = \frac{V^*}{\gamma^*} = \frac{V}{\frac{M_O}{K_\phi} + \gamma} = \left(\frac{3K_\phi}{3K_\phi + 2HR_S} \right) R_S \quad (25)$$

The Shear stiffness without SSI [2] is:

$$R_S = \frac{12EN_C K_C (N_c - 1) K_g}{h[(N_C - 1)K_g + N_C K_C]} \quad (26)$$

And consequently the stiffness ratio r is:

$$r = \frac{R_S}{R_S^*} = \frac{2HR_S + 3K_\phi}{3K_\phi} = 1 + \frac{2HR_S}{3K_\phi} = 1 + \theta \quad (27)$$

θ Is the stiffness corrector ratio with effects SSI

IV. Numerical Application

4.1 Characteristics of dimensionless parameters

The characteristic parameters of the interaction model are defined as well as the intervals of typical values for building structures as follows [14]:

- Ratio of the foundation mass to the structure mass: $0 \leq W_F / W_{St} \leq 0.5$
- Ratio of the moment of inertia of the foundation to the mass moment of inertia of the structure: $0 \leq I_F / I_{Str} \leq 0.1$
- Damping ratio for the fixed-base structure and the soil $\phi = 0.07$, which is a conventional value adopted for the most buildings and soils (SSI effects are not sensitive to the. Fixed base structural damping ratio [23])
- Poisson's ratio for the soil: $\mu = 0.20, 0.25$ and 0.4 which are representative values for stiff, medium and soft soils, respectively.
- Ratio of the shear stiffness: $1 \leq r \leq 1.1$
- Relative mass density between the structure and the soil: $2 \leq W_S / W_{St} \leq 5$.
- Slenderness ratio of the structure: $H / R = 2$ to 5 .

4.2 Assumptions

In the case of structures without SSI, the assumption of fixed base is used to estimate the fundamental period of vibrations. This is assessed according to the RPA code [15] for different categories of sites. In the case of structures with SSI, the soil is modelled by springs: horizontal, vertical and rocking.

To determine the stiffness, the methods of NEWMARK - ROSENBLUETH, DELEUZE and GAZETAS [16, 17] are applied. The shear modulus of the soil G is given three values, the density of soil is set at $2t/m^3$ and the coefficient of critical damping is taken as $\xi = 7\%$; Table. 1 summarizes the different values.

V. Results

- Table 2 presents values of fundamental natural periods calculated by different methods: exact method, RPA code [15], ADELI model without soil-structure interaction.
- It shows a good correlation between exact solution and ADELI model [1] without SSI: a difference of 2.5% is observed in all sites for the ratio T_{exact} / T_{ADELI} with a deviation of 0.09, Fig. 2.
- It can be observed from the results that the interaction effects are negligible ($1/\sigma < 0.10$) in stiff soil and outstanding in medium and soft Soil ($1/\sigma > 0.10$), Table 3 (Fig.3a).These results are in good correlation with those obtained by MASSUMI, TABATABAIEFAR [18] and MICHAEL JAMES GIVENS [11].
- The incorporation of SSI and number of stories tends to increase the fundamental period by 26.3% in medium soil S_2 and 27.9% in soft soil $S_3 - S_4$ as showed in [19]

- The effect of soil-structure interaction is to be considered when the following criterion is satisfied: $H / (V_s \cdot T) > 0.10$.
- The values of factor $1/\sigma$ for the seismic behaviour of R/C structures, according to the Algerian code RPA2013 considering the SSI effect, are given only for soft soil site S_3 , (Fig.3b).
- Table 4 presents values of the natural periods with SSI obtained by the methods of NEWMARK - ROSENBLUETH, DELEUZE and GAZETAS and by the proposed model.
- The fundamental periods of vibration obtained by the proposed model gives good results compared to GAZETAS, DELEUZE and NEWMARK- ROSENBLUETH methods: 5.57% for site S_2 and 6.01% for site S_3 , (Fig.4).

VI. Discussions

- Influence of the ratio W_F / W_{St} : the period of vibrations increase with the increase of the ratio W_F/W_{St} , about 7.4% in soft and medium soil.
- Influence of the ratio I_F / I_{St} : no notification $< 1\%$ as showed by[35].
- Influence of the ratio W_S / W_{St} : increase of the period with the increase of the mass soil about 27% (Fig.5),[10] presents an increase of 20%
- Influence of the ratio D/R: no notification: $> 2\%$.
- The variation of lateral natural period due to incorporation SSI increases with the reduction in stiffness of soil. It is minimum in case of stiff soil (S_1) and maximum in soft soil (S_3 and S_4) about 75%. A maximum increase of more than about 78% is noted in [21] and 70% in [22]

VII. Conclusion

- When considering SSI effects, the soil flexibility and number of stories have an influence on the nature period.
- Natural period of R/C system including SSI effects increases when the ground is softer.
- It is not necessary to consider the effect of soil-structure interaction for seismic design of reinforced concrete frame buildings founded on stiff soil. Hence it is possible to include the soil-structure interaction effects in the analysis of multi-story building response by other means such as incorporating a few modifications to the fixed base condition. These modifications include mass of soil, inertia of foundation, ratio of shear stiffness and slenderness.
- As $1/\sigma$ increases, the significance of SSI effects increases.
- Finally, it is essential to consider the effect of soil-structure interactions for seismic design of reinforced concrete frame for: $1/\sigma > 0.10$

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Captions to tables

Table. 1: Geotechnical specification of the utilized soils in research.

Soil type	Elastic Module E (Kn/m ²)	Shear Module G (Kn/m ²)	Poisson Ratio μ	Mass Density ρ (Kn.S ² /m ⁴)	σ_{sol} (Bars)	Shear Wave Vs(m/s)
Stiff - Site S ₁	1640000	648000	0.28	1.8	2	600
Medium - Site S ₂	494500	180800	0.39	1.75	1.3	320
Soft - Site S ₃ and S ₄	93500	33500	0.4	1.50	0.6	150

Table. 2: Variation of fundamental lateral natural period without SSI

Dimensional specification of the studied frames					Fundamental natural periods without soil-structure interaction WSSI					
Soil Type	Number of Bay	Number of stories	Story Height (m)	Story Width (m)	T (s)			Comparison		
					Exact	RPA 2003	ADELI Model	$T_{exact} / T_{RPA2003}$	$T_{ADELI} / T_{RPA2003}$	T_{exact} / T_{ADELI}
S ₁ -S ₂ -S ₃ and S ₄	2b	2s	8	10	0.176	0.356	0.151	0.494	0.424	1.165
	6b	2s	8	30	0.178	0.356	0.157	0.500	0.441	1.133
	2b	3s	12	10	0.219	0.484	0.228	0.452	0.471	0.960
	6b	3s	12	30	0.222	0.484	0.236	0.458	0.487	0.940
	2b	4s	16	10	0.266	0.600	0.304	0.443	0.506	0.875
	6b	4s	16	30	0.270	0.600	0.315	0.450	0.525	0.857
	2b	5s	20	10	0.346	0.700	0.343	0.494	0.490	1.008
	6b	5s	20	30	0.348	0.700	0.354	0.497	0.505	0.983
	2b	6s	24	10	0.460	0.813	0.412	0.565	0.506	1.116
	2b	7s	28	10	0.583	0.912	0.481	0.639	0.527	1.212
					Mean = 0.499			Mean = 0.484		
					Devation =0.05			Devation=0.01		
								Mean = 1.025		
								Devation=0.09		

Table. 3: Factor of the relative stiffness between structure and soil.

$1/\sigma = H / Vs T$			
Soil Type	Frame Type	RPA 2003	Proposed Model
Stiff Soil - S1	2b 2s	0.037	0.088
	6b 2s	0.037	0.084
	2b 3s	0.041	0.087
	6b 3s	0.041	0.084
	2b 4s	0.044	0.087
	6b 4s	0.044	0.084
	2b 5s	0.047	0.097
	6b 5s	0.047	0.094
	2b 6s	0.049	0.097
	2b 7s	0.051	0.097
		Mean = 0.043	Mean = 0.089
Medium Soil - S2	2b 2s	0.070	0.165
	6b 2s	0.070	0.159
	2b 3s	0.077	0.164
	6b 3s	0.077	0.158
	2b 4s	0.083	0.164
	6b 4s	0.083	0.158
	2b 5s	0.089	0.182
	6b 5s	0.089	0.176
	2b 6s	0.092	0.182
	2b 7s	0.095	0.181
		Mean = 0.082	Mean = 0.168

Soft Soil - S3 and S4	2b 2s	0.149	0.353
	6b 2s	0.149	0.339
	2b 3s	0.165	0.350
	6b 3s	0.165	0.338
	2b 4s	0.177	0.350
	6b 4s	0.177	0.338
	2b 5s	0.190	0.388
	6b 5s	0.190	0.376
	2b 6s	0.196	0.388
	2b 7s	0.204	0.388
		Mean = 0.176	Mean = 0.360

Table. 4: Variation of fundamental lateral natural period with SSI.
(4a) WS = 2 WSt

Soil Type	Frame Type b: bay and s:storey	Natural Periods with Soil-Structure Interaction SSI											
		Veletsos	Deleuze - Newmark	Gazetas	T* Proposed Model with Isolated footings								
					WS = 2 WSt								
					IF = 0			IF = 0.05 ISt			IF = 0.1 ISt		
WF = 0 WSt	WF = 0.25 WSt	WF = 0.5 WSt	WF = 0 WSt	WF = 0.25 WSt	WF = 0.5 WSt	WF = 0 WSt	WF = 0.25 WSt	WF = 0.5 WSt					
Medium Soil - S2, r = 1.05	2b 2s	0.374	0.24689	0.24781	0.268	0.278	0.289	0.268	0.278	0.289	0.268	0.278	0.289
	6b 2s	0.406	0.23622	0.23714	0.277	0.288	0.299	0.277	0.288	0.299	0.277	0.288	0.299
	2b 3s	0.491	0.36980	0.37030	0.404	0.421	0.436	0.404	0.420	0.436	0.404	0.420	0.436
	6b 3s	0.505	0.35029	0.35080	0.418	0.435	0.451	0.418	0.435	0.451	0.418	0.435	0.451
	2b 4s	0.603	0.49631	0.49659	0.541	0.563	0.585	0.541	0.563	0.585	0.541	0.563	0.584
	6b 4s	0.611	0.46697	0.46729	0.559	0.582	0.604	0.559	0.582	0.604	0.559	0.582	0.604
	2b 5s	0.702	0.62469	0.62484	0.612	0.637	0.661	0.612	0.637	0.661	0.611	0.636	0.661
	6b 5s	0.704	0.58452	0.58470	0.635	0.661	0.686	0.635	0.661	0.686	0.634	0.660	0.685
	2b 6s	0.814	0.75475	0.75482	0.737	0.767	0.796	0.737	0.767	0.796	0.736	0.766	0.795
2b 7s	0.913	0.88626	0.88626	0.863	0.899	0.932	0.863	0.898	0.932	0.862	0.898	0.931	
Soft Soil S3 and S4, r = 1.10	2b 2s	0.372	0.29092	0.29681	0.274	0.285	0.296	0.274	0.285	0.296	0.274	0.285	0.296
	6b 2s	0.402	0.27413	0.28013	0.284	0.295	0.306	0.284	0.295	0.306	0.284	0.295	0.306
	2b 3s	0.493	0.40929	0.41201	0.413	0.430	0.447	0.413	0.430	0.447	0.413	0.430	0.447
	6b 3s	0.511	0.37734	0.38056	0.428	0.445	0.462	0.428	0.445	0.462	0.428	0.445	0.462
	2b 4s	0.607	0.53954	0.54037	0.554	0.577	0.598	0.554	0.576	0.598	0.554	0.576	0.598
	6b 4s	0.621	0.49042	0.49209	0.572	0.596	0.618	0.572	0.595	0.618	0.572	0.595	0.618
	2b 5s	0.705	0.65970	0.65898	0.626	0.652	0.676	0.626	0.651	0.676	0.626	0.651	0.676
	6b 5s	0.714	0.59900	0.59908	0.650	0.676	0.702	0.649	0.676	0.701	0.649	0.676	0.701
	2b 6s	0.818	0.79931	0.79806	0.754	0.785	0.814	0.754	0.785	0.814	0.753	0.784	0.814
2b 7s	0.916	0.94120	0.93942	0.883	0.919	0.954	0.883	0.919	0.953	0.882	0.918	0.953	

(4b) $W_S = 5 W_{St}$

Soil Type	Frame Type b: bay and s:storey	Natural Periods with Soil-Structure Interaction SSI											
		Veletsos	Deleuze - Newmark	Gazetas	T [*] Proposed Model with Isolated footings								
					$W_S = 5 W_{St}$								
					$I_F = 0$			$I_F = 0.05 I_{St}$			$I_F = 0.1 I_{St}$		
$W_F = 0 W_{St}$	$W_F = 0.25 W_{St}$	$W_F = 0.5 W_{St}$	$W_F = 0 W_{St}$	$W_F = 0.25 W_{St}$	$W_F = 0.5 W_{St}$	$W_F = 0 W_{St}$	$W_F = 0.25 W_{St}$	$W_F = 0.5 W_{St}$					
Medium Soil- S2 $r = 1.05$	2b 2s	0.374	0.24689	0.24781	0.379	0.386	0.394	0.379	0.386	0.394	0.379	0.386	0.394
	6b 2s	0.406	0.23622	0.23714	0.392	0.400	0.408	0.392	0.400	0.408	0.392	0.400	0.408
	2b 3s	0.491	0.36980	0.37030	0.572	0.583	0.595	0.572	0.583	0.595	0.571	0.583	0.595
	6b 3s	0.505	0.35029	0.35080	0.591	0.603	0.615	0.591	0.603	0.615	0.591	0.603	0.615
	2b 4s	0.603	0.49631	0.49659	0.766	0.782	0.797	0.766	0.781	0.797	0.765	0.781	0.797
	6b 4s	0.611	0.46697	0.46729	0.791	0.807	0.823	0.791	0.807	0.823	0.791	0.807	0.823
	2b 5s	0.702	0.62469	0.62484	0.865	0.883	0.901	0.865	0.883	0.901	0.865	0.883	0.901
	6b 5s	0.704	0.58452	0.58470	0.898	0.917	0.935	0.898	0.916	0.935	0.897	0.916	0.934
	2b 6s	0.814	0.75475	0.75482	1.042	1.064	1.085	1.042	1.064	1.085	1.042	1.063	1.084
2b 7s	0.913	0.88626	0.88626	1.221	1.246	1.271	1.220	1.246	1.270	1.220	1.245	1.270	
Soft Soil S3 and S4 $r = 1.10$	2b 2s	0.372	0.29092	0.29681	0.387	0.395	0.403	0.387	0.395	0.403	0.387	0.395	0.403
	6b 2s	0.402	0.27413	0.28013	0.401	0.410	0.418	0.401	0.410	0.418	0.401	0.410	0.418
	2b 3s	0.493	0.40929	0.41201	0.585	0.597	0.609	0.585	0.597	0.609	0.585	0.597	0.609
	6b 3s	0.511	0.37734	0.38056	0.605	0.618	0.630	0.605	0.618	0.630	0.605	0.618	0.630
	2b 4s	0.607	0.53954	0.54037	0.784	0.800	0.816	0.783	0.800	0.815	0.783	0.799	0.815
	6b 4s	0.621	0.49042	0.49209	0.809	0.826	0.842	0.809	0.826	0.842	0.809	0.826	0.842
	2b 5s	0.705	0.65970	0.65898	0.886	0.904	0.922	0.885	0.904	0.921	0.885	0.903	0.921
	6b 5s	0.714	0.59900	0.59908	0.919	0.938	0.957	0.919	0.937	0.956	0.918	0.937	0.956
	2b 6s	0.818	0.79931	0.79806	1.067	1.089	1.110	1.066	1.088	1.110	1.066	1.088	1.109
2b 7s	0.916	0.94120	0.93942	1.249	1.275	1.300	1.248	1.274	1.299	1.248	1.274	1.299	

9. Captions to figures

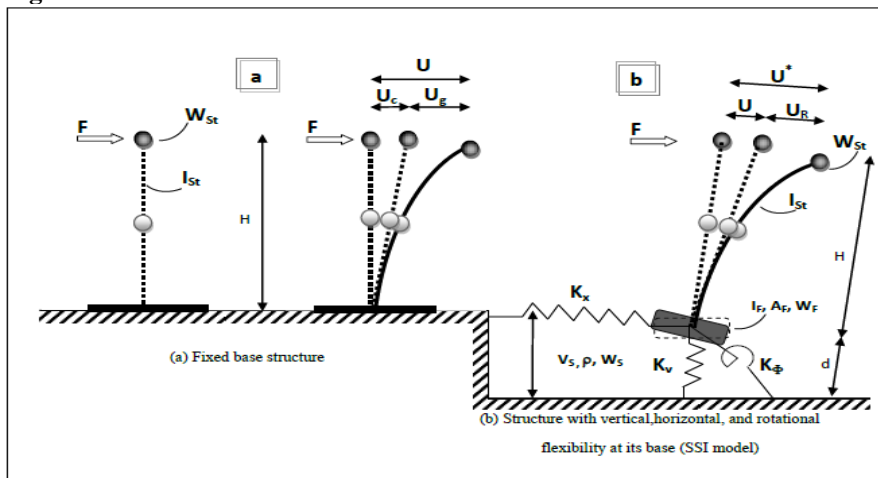


Fig. 1: Schematic illustration of SSI Model.

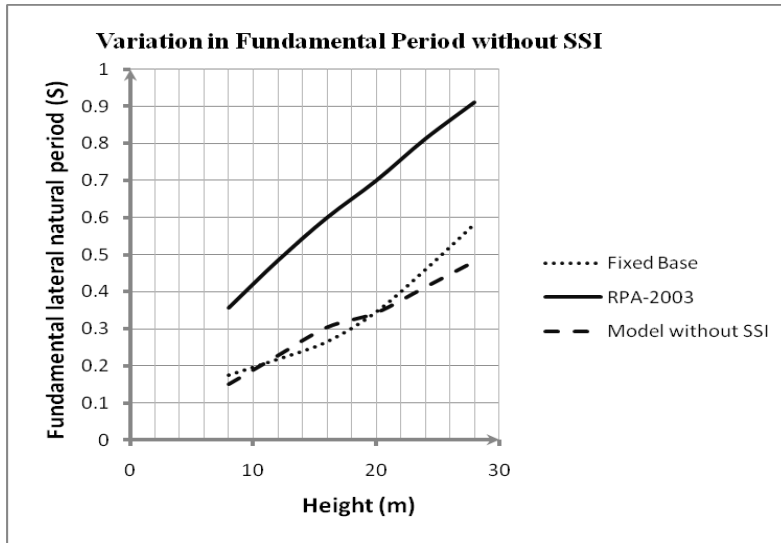


Fig. 2: Variation of change in period of vibration without SSI.

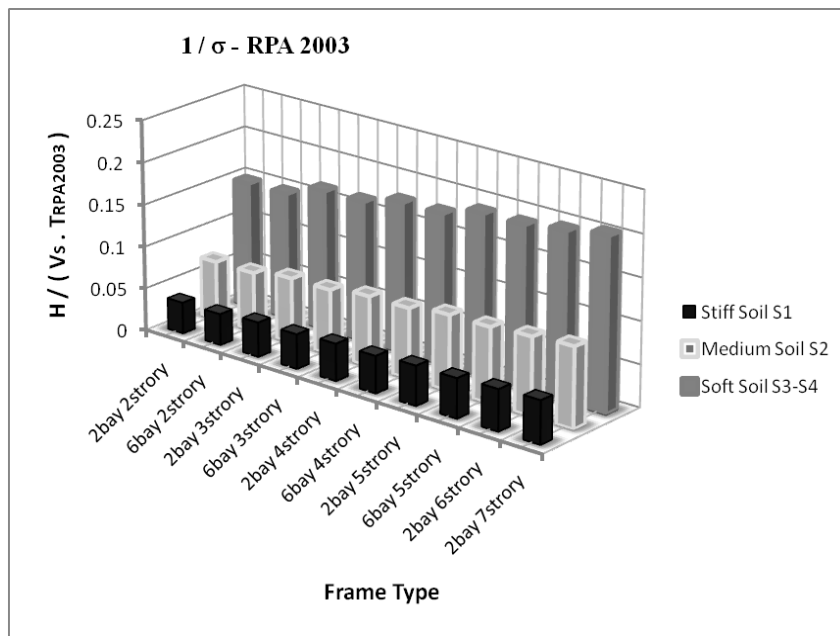
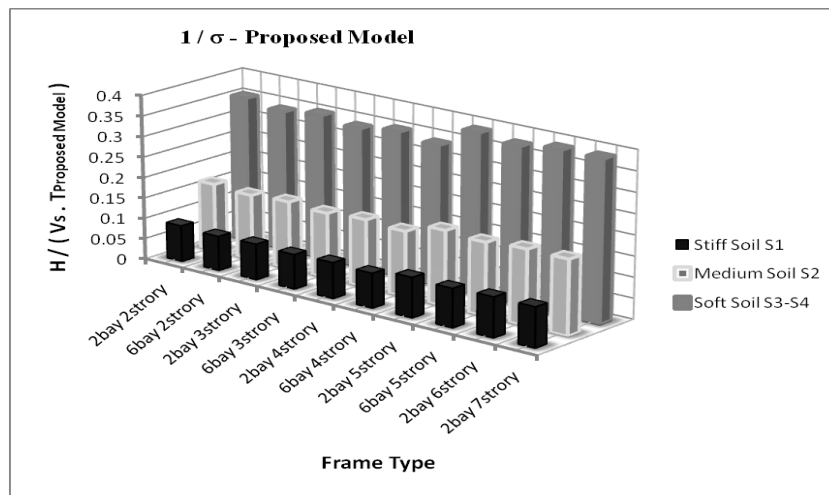


Fig. 3: Variation of the relative stiffness between structure and soil.

(3a)



(3b)

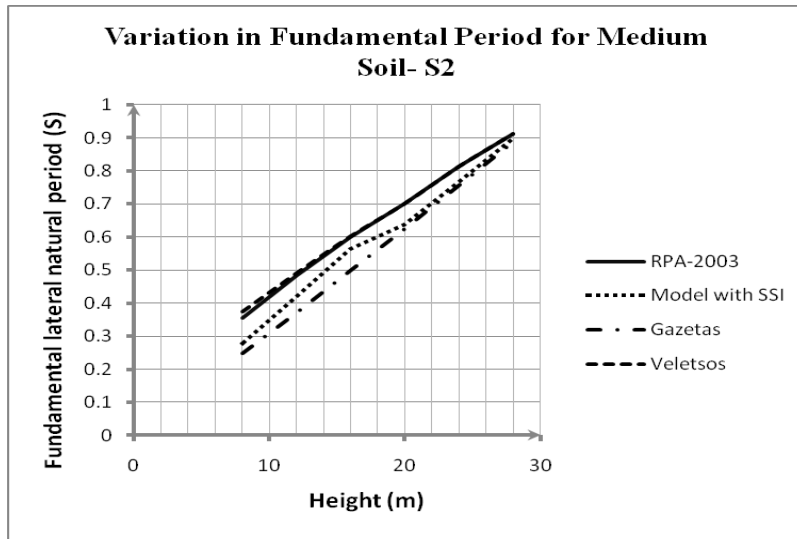
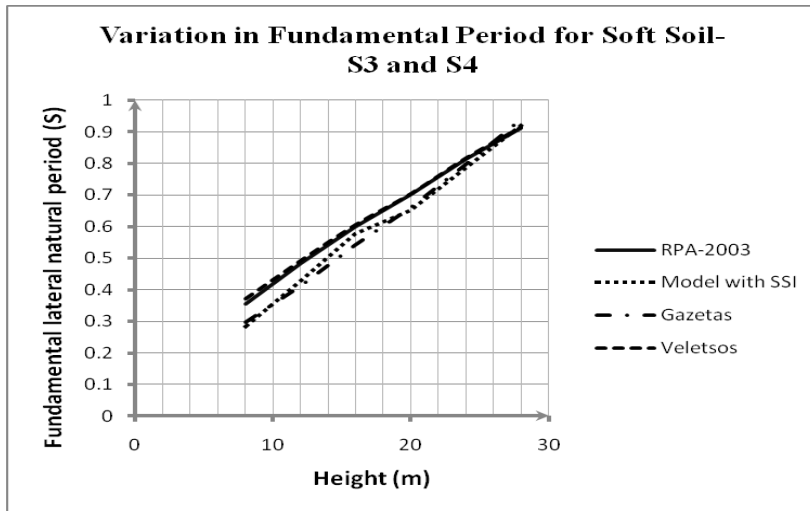


Fig. 4: Variation of change in period of vibration with SSI considering $W_s = 2W_{St}$, $I_f = 0.05 I_{St}$ and $W_f = 0.25 W_{St}$.

(4a)



(4b)

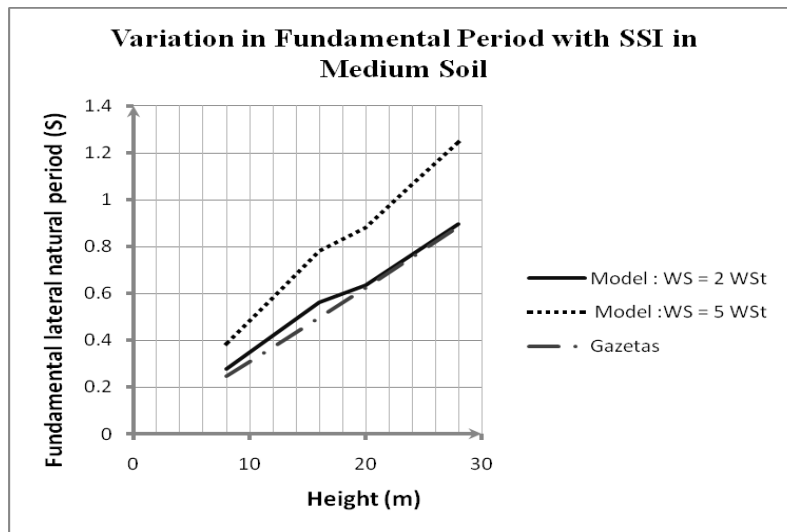
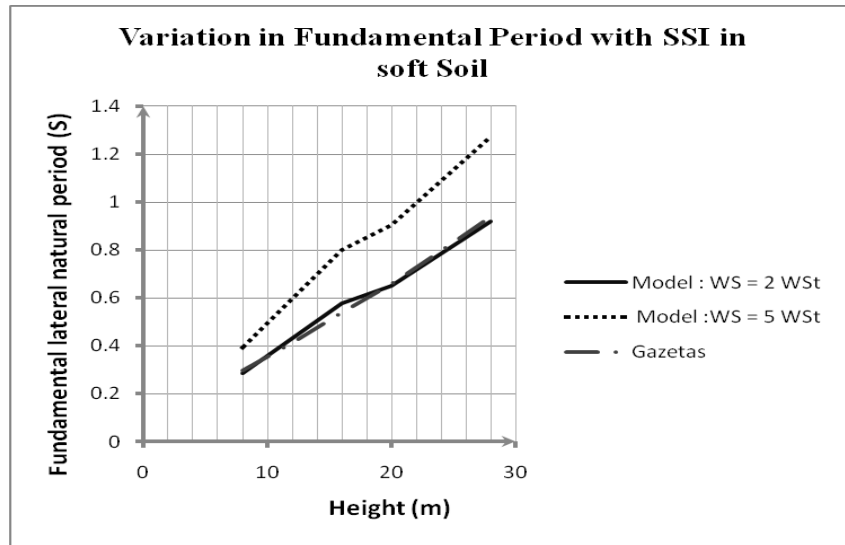


Fig. 5: Periods of soil-structure systems for various soil mass considering $W_f = 0.25W_{St}$, $I_f = 0.05 I_{St}$.



(5b)