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Inverse Kinematics Solution for Biped Robot

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Abstract: *A biped is a multi-jointed mechanism that performs a human's motions. It seems more difficult to analyze the behavioral character of walking robot due to the complexity of mathematical description. This paper focuses on developing a methodology for deriving an inverse kinematic joint solution of a biped robot. This work aimed to build the lower side, the locomotion part of a biped robot. It couples a design considerations and simplicity of design to provide inverse kinematics analysis of 11 degree-of-freedom (DOF) biped robot. The model used consists of 5-links which are connected through revolute joints. The identical legs have hip joint, knee joints and ankle joint. This paper addresses symbolic formulation for reducing problem in solving univariate polynomial. An effective approach is developed for the solution of inverse kinematics task in analytical form for given end-effector position. This method presents a simple and efficient procedure for finding the joint solution of bipeds.*
Keywords: *Biped, Degree of freedom, Denavit-hartenberg parameters, Inverse kinematics.*

I. Introduction

From ancient times, man has tried to create the mechanism that resembles the human body. Bipedal locomotion involves a large number of degrees of freedom. Pieper stated two conditions to find a closed-form joint solution to a robot manipulator where there are three adjacent joint axes which are parallel to one another or they intersect at a same point [1]. This can be either prismatic or revolute joints .but this approach has difficulty in developing a consistent procedure for finding a closed-form joint solution for a humanoid robot and selecting one required solution from multiple solutions. Biped-robot researchers often use iterative methods for modeling humanoid robots. Some of the iterative methods makes use of the jacobian matrix [2]. Error in position can result because of velocity based nature of jacobian method due to the iterative nature of the algorithm. Computational complexity and singularity are the main shortcomings of using the inverse jacobian matrix method [3]. Inverse-transform technique was presented by Paul et al [4]. It leads to inverse kinematic joint solution of a 6-DOF robot manipulator. Common method for deriving inverse kinematics used is the geometric method [5]. Geometric method requires trigonometric expertise in solving the joint solution of a biped. This method is difficult to adopt when joint solution for more than four or five joints are involved. A closed-form joint solution for a 6-DOF humanoid robot arm was derived by Cui et al [6]. But singularities were not considered.

In this paper, we proposed a inverse method by viewing the kinematic chain of a leg of a biped in reverse order and finally employing the inverse technique in deriving a joint solution for the Biped robot. This paper presents the inverse kinematics of biped robot. This paper is organized as follows: An outline of the mechanical design of the developed biped robot is given in Section 2. The kinematical model which is prerequisite for inverse kinematic is described in Section 3.In Section 4, Inverse kinematic approach is explained. Finally, Section 5 contains conclusion.

II. Mechanical Design

The design of biped is based on human body in terms of ratios, body proportions, and range of motion. This paper propose to have sufficient DOF to imitate human motion. The model used consists of 5-links which are connected through revolute joints, 2-links for each leg and 1-link for torso. It is considered as a robot with waist or torso, linking two legs which are linked together through hip joints to emulate a human's activities. The identical legs have hip joint between torso and thigh, knee joints between the thigh and shank, ankle joint between shank and foot, and a rigid body forms the torso. The joint structure of the biped has eleven degrees of freedom, 5 DOF for each leg and 1 DOF for waist or torso. DOF for waist is shared between legs. The Hip joint has 2-DOF, which allows it motion in the sagittal and the lateral plane. The range of motion in the sagittal plane is between +70º to -50º and +50º and -60º in the lateral plane. The Knee joint has 1-DOF, which allows it motion in the sagittal and the lateral plane. The mobility for the knee joint is +140º in the sagittal plane. The ankle joint has 2-DOF, which allows it motion in the sagittal and the lateral plane. The range of motion in the sagittal plane is between +70º to -50º and +50º and -60º in the lateral plane.

Servos are mounted on the biped robot serves as actuators for the system. One servo is attached to torso. On each leg, two servos are attached to the hip, one servo is attached to the knee and two servos are attached to the ankle. The mechanical design of the bipedal robot is modular, making it easy to change and replace parts. The frameworks of biped will be fabricated from acrylic in order to obtain light weight, and a wide range of motion.

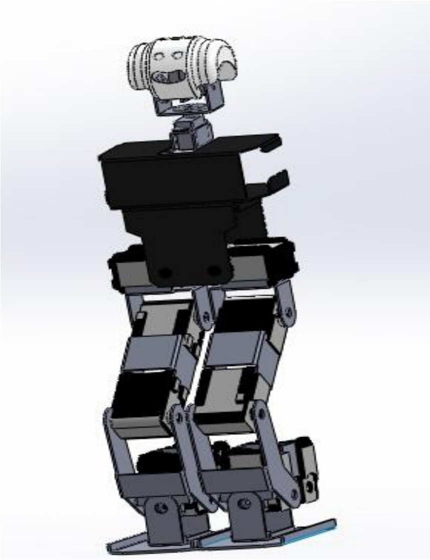

Fig.1: CAD model of biped.

III. Kinematic Model

Kinematic model depending upon above planned movements, can be formulated. Kinematic analysis is based on the basic equation of the geometric model that aids in determining the position and orientation of a foot with a reference to torso for known values of the joint variables of kinematic chain that compose the robot. Denavit-hartenberg formulation is used to model biped. Each part is considered as a link represented by a line along its joint axis and common normal to next joint axis. Coordinate system is attached to each link illustrating relative position amongst various links. A 4×4 transformation matrix relating i+1 frame to i frame is given by,

$$^{i-1}H_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_{i-1} & \sin\theta_i \sin\alpha_{i-1} & a_{i-1}\cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_{i-1} & -\cos\theta \sin\alpha_{i-1} & a_{i-1}\sin\theta_i \\ 0 & \sin\alpha_{i-1} & \cos\alpha_{i-1} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{........ (1)}$$

Where,

θ_i = Rotation angle is angle between X_{i-1} and X_i measured about Z_i.

α_{i-1} = Twist angle is angle between lines along joints i-1 and i measured about common perpendicular X_{i-1}.
a_{i-1} = link length is the distance between the lines along joints i-1 and i along common perpendicular.
d_i = link offset is distance along Z_i from line parallel to X_{i-1} to the line parallel to $X_i$and are called as Denavit-hartenberg(D-H)parameters.

Equation 1 is homogeneous transformation matrix indicating position and orientation of each joint. An origin(X_0, Z_0) is established at the torso and each joint has a coordinate frames are attached following D-H definition. For the biped robot with all revolute joints, we have formulated θ_i,α_{i-1},a_{i-1},d_i.Table 1 and Table 2 lists D-H parameter used to solve transformation matrix. Transformation matrix of each joint can be obtained by substituting D-H parameters into Equation 1.

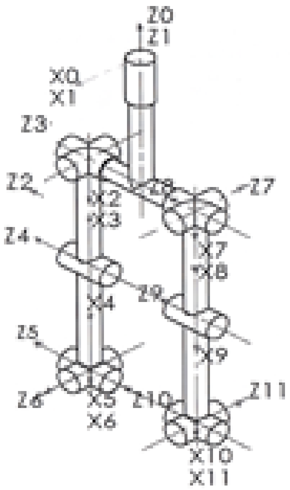

Fig. 2: Frame assignment

Table1: Denavait-Hatenberg parameters for left leg

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	l_2	0	θ_2+90
3	90	0	0	θ_3
4	0	l_4	0	θ_4
5	0	l_5	0	θ_5
6	-90	0	0	θ_6

Table2: Denavait-Hatenberg parameters for right leg

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
7	-90	l_7	0	θ_7+90
8	-90	0	0	θ_8
9	0	l_9	0	θ_9
10	0	l_{10}	0	θ_{10}
11	90	0	0	θ_{11}

The continuous homogeneous transformation from ${}^0H_1$ to ${}^5H_6$ transform ankle coordinate to base torso coordinate, shown in equation 2.Pose of ankle with respect to torso is given by,

$${}^0H_6 = {}^0H_1 \cdot {}^1H_2 \cdot {}^2H_3 \cdot {}^3H_4 \cdot {}^4H_5 \cdot {}^5H_6 \quad \ldots\ldots (2)$$

$$P = {}^0H_6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & Px \\ r_{21} & r_{22} & r_{23} & Py \\ r_{31} & r_{32} & r_{33} & Pz \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots (3)$$

Equation 3 provides solution of forward kinematics with matrix P being result. The translation vector{Px, Py, Pz} gives position of foot and orientation matrix $\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ shows direction of foot.

IV. Inverse Kinematics

The inverse kinematics problem for biped is fundamental for controlling of robot. Given the pose of ankle, problem corresponds to finding joint configuration for that pose. The placement and orientation of the legs determines where the feet are placed and also orientation of the torso i.e. the posture of the robot. This necessities' for the inverse kinematics of the legs to include the orientation of the torso as well as the orientation of the feet. It is for calculated positions of ankle, finding of joint solutions for these positions. The problem of inverse kinematics corresponds to finding joint variables θ_1, θ_2, θ_3, θ_4, θ_5, θ_6 such that,
${}^0H_1 \bullet {}^1H_2 \bullet {}^2H_3 \bullet {}^3H_4 \bullet {}^4H_5 \bullet {}^5H_6 = {}^0H_6$.

Inverse kinematics can be solved by using geometric solution. We can draw following figure 3 by viewing biped in sagittal plane i.e. plane perpendicular to Z_3, Z_4, and Z_5 axes.

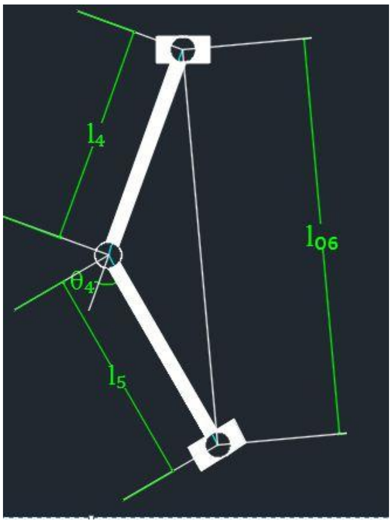

Fig. 3: Geometrical relationship amongst link in sagittal plane.

Distance from the hip to ankle is given by equation 7.

$l_{06} = \sqrt{Px^2 + Py^2 + Pz^2}$…….. (7)

The Knee ankle θ_4 can be calculated by law of cosine for known values of link length l_4, l_5 and distance l_{06} is given by equation 8.

$\theta_4 = \cos^{-1}\left(\frac{l_{06}^2 - l_4^2 - l_5^2}{2 l_4 l_5}\right)$…….. (8)

Now, If we viewed in frontal plane i.e. plane perpendicular to the Z_5 axis, we can draw following figure 4.

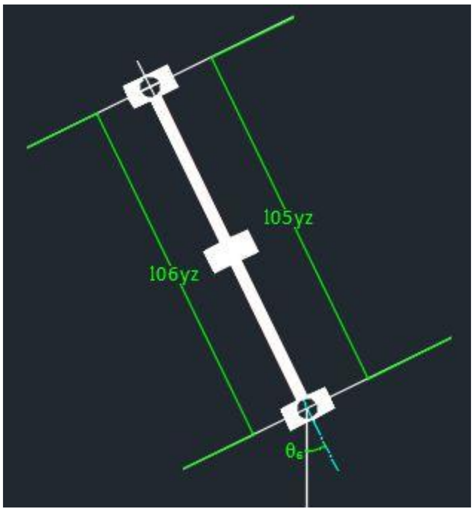

Fig. 4: Geometrical relationship amongst link in frontal plane.

From geometry of figure 4, distance l06yz and l05yz can be calculated using equation 9, 10, 11.

$l06yz = \sqrt{\left(P^{-1}(2,4)\right)^2 + \left(P^{-1}(3,4)\right)^2}$…….. (9)

$l06yz = ((l_4\cos(\theta_4)\ \cos(\theta_5)\ \sin(\theta_3) - l_4\ \sin(\theta_4)\ \sin(\theta_5)\ \sin(\theta_3) + l_5\cos(\theta_4)^2\ \cos(\theta_5)\ \sin(\theta_3) + l_5\cos(\theta_5)\ \sin(\theta_4)^2\ \sin(\theta_3))^2/(\cos(\theta_4)^2\ \cos(\theta_5)^2\ \cos(\theta_3)^2 + \cos(\theta_4)^2\ \cos(\theta_5)^2\sin(\theta_3)^2 + \cos(\theta_4)^2\ \cos(\theta_3)^2\ \sin(\theta_5)^2 + \cos(\theta_4)^2\sin(\theta_5)^2\ \sin(\theta_3)^2 + \cos(\theta_5)^2\ \cos(\theta_3)^2\ \sin(\theta_4)^2 + \cos(\theta_5)^2\ \sin(\theta_4)^2\ \sin(\theta_3)^2 + \cos(\theta_3)^2\ \sin(\theta_4)^2\ \sin(\theta_5)^2 + \sin(\theta_4)^2\ \sin(\theta_5)^2$

$\sin(\theta_3)^2)^2 + (l_5 \sin(\theta_5) \cos(\theta_4)^2 + l_4 \sin(\theta_5) \cos(\theta_4) + l_5 \sin(\theta_5) \sin(\theta_4)^2 + l_4\cos(\theta_5) \sin(\theta_4))^2/(\cos(\theta_4)^2 \cos(\theta_5)^2 + \cos(\theta_4)^2 \sin(\theta_5)^2 + \cos(\theta_5)^2 \sin(\theta_4)^2 + \sin(\theta_4)^2 \sin(\theta_5)^2)^2)^{½}$…….. (10)

$l05yz =((l_4\cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - l_4 \sin(\theta_4) \sin(\theta_5) \sin(\theta_3) + l_5\cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) + l_5\cos(\theta_5) \sin(\theta_4)^2 \sin(\theta_3))^2/(\cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_3)^2 + \cos(\theta_4)^2 \cos(\theta_5)^2\sin(\theta_3)^2 + \cos(\theta_4)^2 \cos(\theta_3)^2 \sin(\theta_5)^2 + \cos(\theta_4)^2\sin(\theta_5)^2 \sin(\theta_3)^2 + \cos(\theta_5)^2 \cos(\theta_3)^2 \sin(\theta_4)^2 + \cos(\theta_5)^2 \sin(\theta_4)^2 \sin(\theta_3)^2 + \cos(\theta_3)^2 \sin(\theta_4)^2 \sin(\theta_5)^2 + \sin(\theta_4)^2 \sin(\theta_5)^2 \sin(\theta_3)^2)^2 + (l_5 \sin(\theta_5) \cos(\theta_4)^2 + l_4 \sin(\theta_5) \cos(\theta_4) + l_5 \sin(\theta_5) \sin(\theta_4)^2 + l_4\cos(\theta_5) \sin(\theta_4))^2/(\cos(\theta_4)^2 \cos(\theta_5)^2 + \cos(\theta_4)^2 \sin(\theta_5)^2 + \cos(\theta_5)^2 \sin(\theta_4)^2 + \sin(\theta_4)^2 \sin(\theta_5)^2)^2)^{½}$…….. (11)

The law of cosine can be used to calculate ankle angle as,

$\theta_6 = \mathrm{sign}\big(P^{-1}(2,4)\big) \cos^{-1}\left(\frac{l06yz^2 - l05yz^2 - l_6}{2\, l05yz\; l_6}\right)$…….. (12)

$\theta_6 = (\pi \sin((l_4\cos(\theta_4) \cos(\theta_5) \sin(\theta_3) - l_4 \sin(\theta_4) \sin(\theta_5) \sin(\theta_3) + l_5\cos(\theta_4)^2 \cos(\theta_5) \sin(\theta_3) + l_5\cos(\theta_5) \sin(\theta_3)^2 \sin(\theta_3))/(\cos(\theta_4)^2 \cos(\theta_5)^2 \cos(\theta_3)^2 + \cos(\theta_4)^2 \cos(\theta_5)^2 \sin(\theta_3)^2 + \cos(\theta_4)^2 \cos(\theta_3)^2 \sin(\theta_5)^2 + \cos(\theta_4)^2 \sin(\theta_5)^2 \sin(\theta_3)^2 + \cos(\theta_5)^2 \cos(\theta_3)^2 \sin(\theta_4)^2 + \cos(\theta_5)^2 \sin(\theta_4)^2 \sin(\theta_3)^2 + \cos(\theta_3)^2 \sin(\theta_4)^2 \sin(\theta_5)^2 + \sin(\theta_4)^2 \sin(\theta_5)^2 \sin(\theta_3)^2)))/2$…….. (13)

If we inverse ${}^5H_6$ and multiply it by P, we get equation 14,

${}^0H_1 \cdot {}^1H_2 \cdot {}^2H_3 \cdot {}^3H_4 \cdot {}^4H_5 = P\,({}^5H_6)^{-1}$ …….. (14)

From element (3, 3), Hip angle can be calculated as,

$\theta_2 = \sin^{-1}((\cos(\theta_6) (\cos(\theta_6) \sin(\theta_5) + \sin(\theta_6) (\cos(\theta_5) (\cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \cos(\theta_5) \cos(\theta_3) \cos(\theta_4)) + \sin(\theta_5) (\cos(\theta_5) \cos(\theta_3) \sin(\theta_4) + \cos(\theta_5) \cos(\theta_4) \sin(\theta_3)))))/(\cos(\theta_6)^2 - \sin(\theta_6)^2) - (\sin(\theta_6) (\sin(\theta_5) \sin(\theta_6) + \cos(\theta_6) (\cos(\theta_5)(\cos(\theta_5) \sin(\theta_3) \sin(\theta_4) - \cos(\theta_5) \cos(\theta_3) \cos(\theta_4)) + \sin(\theta_5) (\cos(\theta_5) \cos(\theta_3) \sin(\theta_4) + \cos(\theta_5) \cos(\theta_4) \sin(\theta_3)))))/(\cos(\theta_6)^2 - \sin(\theta_6)^2))$…….. (15)

From element (1, 3) and (2, 3) torso angle can be calculated by equation,

$\theta_1 = \cos^{-1}((\sin(\theta_6) (\cos(\theta_6) (\cos(\theta_5) (\cos(\theta_4) (\sin(\theta_4) \sin(\theta_3) - \cos(\theta_4) \cos(\theta_3) \sin(\theta_2)) + \sin(\theta_4))(\cos(\theta_3) \sin(\theta_4) + \cos(\theta_4) \sin(\theta_2) \sin(\theta_3))) + \sin(\theta_5) (\cos(\theta_4) (\cos(\theta_3) \sin(\theta_4) + \cos(\theta_4) \sin(\theta_2) \sin(\theta_3)) - \sin(\theta_4) (\sin(\theta_4) \sin(\theta_3) - \cos(\theta_4) \cos(\theta_3) \sin(\theta_2)))) - \cos(\theta_4) \cos(\theta_2) \sin(\theta_6)))/(\cos(\theta_6)^2 - \sin(\theta_6)^2) - (\cos(\theta_6) (\sin(\theta_6) (\cos(\theta_5) (\cos(\theta_4) (\sin(\theta_4) \sin(\theta_3) - \cos(\theta_4) \cos(\theta_3) \sin(\theta_2)) + \sin(\theta_4) (\cos(\theta_3) \sin(\theta_4) + \cos(\theta_4) \sin(\theta_2) \sin(\theta_3))) + \sin(\theta_5) (\cos(\theta_4) (\cos(\theta_3) \sin(\theta_4) + \cos(\theta_4) \sin(\theta_2) \sin(\theta_3)) - \sin(\theta_4) (\sin(\theta_4) \sin(\theta_3) - \cos(\theta_4) \cos(\theta_3) \sin(\theta_2)))) - \cos(\theta_4) \cos(\theta_2) \cos(\theta_6)))/(\cos(\theta_6)^2 - \sin(\theta_6)^2)) / \cos(\theta_2)$……… (16)

or

$\theta_1 = \sin^{-1}(\cos(\theta_6) (\sin(\theta_6) (\cos(\theta_5) (\cos(\theta_4) (\cos(\theta_4) \sin(\theta_3) + \cos(\theta_3) \sin(\theta_4) \sin(\theta_2)) + \sin(\theta_4) (\cos(\theta_4) \cos(\theta_3) - \sin(\theta_4) \sin(\theta_2) \sin(\theta_3))) + \sin(\theta_5) (\cos(\theta_4) (\cos(\theta_4) \cos(\theta_3) - \sin(\theta_4) \sin(\theta_2) \sin(\theta_3)) - \sin(\theta_4) (\cos(\theta_4) \sin(\theta_3) + \cos(\theta_3) \sin(\theta_4) \sin(\theta_2)))) + \cos(\theta_2) \cos(\theta_6) \sin(\theta_4)))/(\cos(\theta_6)^2 - \sin(\theta_6)^2) - (\sin(\theta_6) (\cos(\theta_6) (\cos(\theta_5) (\cos(\theta_4) (\cos(\theta_4) \sin(\theta_3) + \cos(\theta_3) \sin(\theta_4) \sin(\theta_2)) + \sin(\theta_4) (\cos(\theta_4) \cos(\theta_3) - \sin(\theta_4) \sin(\theta_2) \sin(\theta_3))) + \sin(\theta_5) (\cos(\theta_4) (\cos(\theta_4) \cos(\theta_3) - \sin(\theta_4) \sin(\theta_2) \sin(\theta_3)) - \sin(\theta_4) (\cos(\theta_4) \sin(\theta_3) + \cos(\theta_3) \sin(\theta_4) \sin(\theta_2)))) + \cos(\theta_2) \sin(\theta_4) \sin(\theta_6)))/(\cos(\theta_6)^2 - \sin(\theta_6)^2)/ \cos(\theta_2))$……… (17)

If we inverse ${}^0H_1 \cdot {}^1H_2$ and multiply it by P then we get equation 18.

${}^3H_4 \cdot {}^4H_5 \cdot {}^5H_6 = ({}^0H_1 \cdot {}^1H_2)^{-1} P$…….. (18)

From element (1, 4) and (2, 4), Roll angle of hip is given by equation 19, 20.

$\theta_3 = \cos^{-1}(((l_5 (\cos(\theta_5) \cos(\theta_4) (\cos(\theta_1) \sin(\theta_2) + \cos(\theta_2) \sin(\theta_1)) - \sin(\theta_5) \sin(\theta_4) (\cos(\theta_1) \sin(\theta_2) + \cos(\theta_2) \sin(\theta_1))) + l_4 \cos(\theta_5) (\cos(\theta_1) \sin(\theta_2) + \cos(\theta_2) \sin(\theta_1))) (\cos(\theta_1) \sin(\theta_2) + \cos(\theta_2) \sin(\theta_1)))/(\cos(\theta_1)^2 \cos(\theta_2)^2 + \cos(\theta_1)^2 \sin(\theta_2)^2 + \cos(\theta_2)^2 \sin(\theta_1)^2 + \sin(\theta_1)^2 \sin(\theta_2)^2) + ((l_5 (\cos(\theta_5) \cos(\theta_4) (\cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2)) - \sin(\theta_5) \sin(\theta_4) (\cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2))) + l_4 \cos(\theta_5) (\cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2))) (\cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2)))/(\cos(\theta_1)^2 \cos(\theta_2)^2 + \cos(\theta_1)^2 \sin(\theta_2)^2 + \cos(\theta_2)^2 \sin(\theta_1)^2 + \sin(\theta_1)^2 \sin(\theta_2)^2)-l_4 /l_5)$…….. (19)

or

$\theta_3 = \sin^{-1}((((l_5\ (\cos(\theta_5)\cos(\theta_4)\ (\cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)) - \sin(\theta_5)\sin(\theta_4)\ (\cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1))) + l_4\cos(\theta_5)\ (\cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)))\ (\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)))/(\cos(\theta_1)^2\cos(\theta_2)^2 + \cos(\theta_1)^2\sin(\theta_2)^2 + \cos(\theta_2)^2\sin(\theta_1)^2 + \sin(\theta_1)^2\sin(\theta_2)^2) - ((l_5\ (\cos(\theta_5)\cos(\theta_4)\ (\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)) - \sin(\theta_5)\sin(\theta_4)\ (\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2))) + l_4\cos(\theta_5)\ (\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)))\ (\cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)))/(\cos(\theta_1)^2\cos(\theta_2)^2 + \cos(\theta_1)^2\sin(\theta_2)^2 + \cos(\theta_2)^2\sin(\theta_1)^2 + \sin(\theta_1)^2\sin(\theta_2)^2)/\ l_5\ )$…….. (20)

The squat down motion only involves movement of pitch angle of hip, knee and ankle. The analysis is simplified as 3R manipulator with three joints are parallel with each other. To ensure that robot will not fall down, the origin of biped always lie along the x axis of reference frame during the motion shown in figure 5.

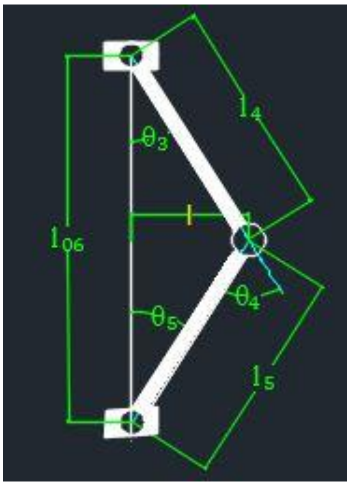

Fig. 5: Leg chain in squat down motion.

$l=\sqrt{l_4^{\ 2} - l_{0\ 6}^{\ 2}} + \sqrt{l_5^{\ 2} - l_{0\ 6}^{\ 2}}$

By using sine law, Pitch angle of knee is given by,

$\theta_5 = \sin^{-1}\left(\frac{l}{l_4}\right)$

V. Conclusion

This paper presented an easy way to visualize movement of a 5-link biped .It supports design of the associated complex mathematical models using inverse kinematics have been presented. Complexity of inverse kinematics is due to geometry of biped. There can be difficulty in finding all possible solutions.

References

[1]. D. L. Pieper, "The kinematics of manipulators under computer control," Ph.D. dissertation, Stanford Univ., 1968.

[2]. G. Tevatia and S. Schaal, "Inverse kinematics for humanoid robots," in Proc. IEEE International Conference on Robotics and Automation (ICRA), vol. 1, 2000, pp. 294–299.

[3]. R. Muller-Cajar and R. Mukundan, "A new algorithm for inverse kinematics," in Proc. of the Image and Vision Computing New Zealand, Dec. 2007, pp. 181–186.

[4]. K. S. Fu, R. C. Gonzalez, and C. S. G. Lee, Robotics: Control, Sensing, Vision, and Intelligence. McGraw-Hill, 1987.

[5]. R. P. Paul, B. E. Shimano, and G. Mayer, "Kinematic control equations for simple manipulators," in IEEE Transactions on Systems, Man and Cybernetics, vol. 11, 1981, pp. 449–455.

[6]. Y. Cui, P. Shi, and J. Rua, "Kinematics analysis and simulation of a 6-dof humanoid robot manipulator," in Proc. of the IEEE Intl.

[7]. Asia Conf. on Informatics in Control, Automation and Robotics (CAR), vol. 2, Mar 2010, pp. 246–249.