

## Analysis of Stress Distribution in a Curved Structure Using Photoelastic and Finite Element Method

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**Abstract:** Stress concentration adversely affects the structural properties and hence the knowledge of stress distribution becomes essential. The reduction in stress concentration improves the fatigue life of a component. This paper deals with the investigation of stress distribution in the curved structure without and with stress reliever of type circular and elliptical hole subjected to uniaxial tension by using photoelastic technique. Photo stress method is whole field technique and evaluates precise and accurate values of stress regardless of geometric complexity and material composition. The ability of the Finite element Method to determine the stress field around stress concentration area is enhanced by proper selection of elements. It is found that the introduction of single elliptical stress reliever, at low stressed region rather than circular hole, can reduce stress concentration. It is found that intensity of stress decreases considerably at the inner boundary of the structure, when major axis of the ellipse normal to the load. The results were validated by Finite element method.

**Keywords:** Curved section, stress Intensity, Tilt angle, Photoelasticity, ANSYS, FEM

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### I. Introduction

The structural design is closely associated with the subject of stress analysis. All engineering structures and machines designed according to modern principles should be strong and sufficiently rigid while using minimum weight of materials. While designing such structures and machines various factors have to be taken into account. One of the important factors is stress concentration arising due to complicated cross section and discontinuities in structures. Curved beams are the parts of machine members found in C-clamps, crane hooks, frames machines, planers etc. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis of the section is shifted towards the centre of curvature of the beam causing a non linear [hyperbolic] distribution of stress. The neutral axis lies between the centroidal axis and the centre of curvature and will always be present within the curved beams. Since curvature of the beam is the source of the stress concentration, a curved beam subjected to bending will have a higher stress on the inside. The analysis of stress concentration of the curved beams enables one to obtain a better balanced design of the structure and machines. Distribution of stress in curved flexural member is determined by using the basic assumptions. In present work our **objective** is to study stress distribution in an initially curved

Structure subjected to tensile loading and to expose the experimental technique like photo-elasticity for analyzing stresses in machine components. Curved beam is one of the structural components, which can be analyzed for stresses. In this work we have concentrated study of stress distribution in a curved structure subjected to tensile loading. The intensity of stress is reduced by providing single circular or elliptical stress reliever at the region where less stress are present by which we can reduce the weight of the structure without affecting the strength. Results obtained by means of experimental technique will be validated with Finite element results using ANSYS software. Only finite element results are used to optimize or validate the values that are obtained by an experimental analysis. The experimental technique used for the analysis of two-dimensional curved structure can also be extended for the analysis of stresses in any curved structures irrespective of their materials including composite materials. It can also be used to study the stresses in three dimensional components having complicated geometry. The results obtained from a photo-elastic model can be translated to stress values in the metal prototype by applying the law of similarity even though model may differ from scale, thickness, applied loads and the elastic constants. This method has recently been extended to study the thermal stresses, dynamic stresses and it has been called Photo-thermo-elasticity, Dynamic photo-elasticity. The Polariscopes used in the study of static load is identical for the study of thermal loads. Hence, thermal and coupled analysis can be done for knowing the effect of temperature in the curved structure.

## II. Literature Survey

Extensive literature is available on stress concentration. Many authors have investigated the problems of stress concentration of a straight plate. Stress concentration around circular holes, semicircular holes, notches and holes of others shapes in case of tension, bending and torsion were investigated.

J. M. Dulieu-Barton [1] School of Engineering Sciences University of Southampton, explained the analysis of stress/strain by means of photoelasticity. This technique involves manufacturing a model of an artefact from a birefringent material, usually a transparent polymer, and applying a representative load to the model to get fringe pattern which are proportional to stress.

S.K.Chakrabarti,[2] A complete photoelastic stress analysis depends, in the majority of cases, on the accuracy of the fractional-fringe-order measurement. The error involved due to an error in the measurement of the isoclinic angle has been studied

A.J.Durelli,[3] Obtained stress distribution around an elliptical discontinuity in a 2- dimensional, uniform and axial system of combined stress. He compared experimental results from photoelastic and brittle coating tests with theoretical values.

J.B.Hanus,[4]Studied stress concentration factor, for elliptical holes at the edge in a finite plate in a tension. This work was an interaction between the holes and edges. The holes had the major axis normal to the direction of loading. The elliptically ratio and distance between edge and holes was varied. The maximum stress at each end of an ellipse increases as the ellipse gets closer to the edge of the plate.

Dr.U.C. Jindal,[5] studied SCF around a circular hole can be reduced by introducing smaller auxiliary holes on either side of the original hole. To achieve maximum possible reduction, these auxiliary holes have to be introduced very nearer to the original hole. By introducing auxiliary holes of equal diameter in the direction normal to the loading, on either sides of the original hole, the stress concentration at central hole, increases as the auxiliary holes come nearer to the original hole.

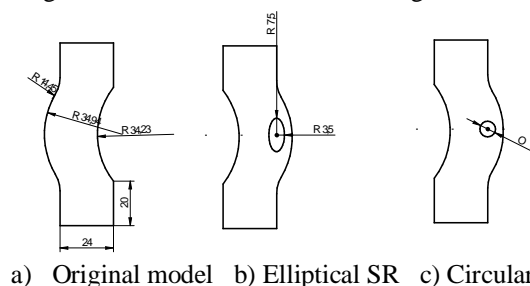
R.E. Peterson [6] studied the phenomena of SCF in fatigue of metals. He discussed the effects of the size of discontinuity in relation to the size of member in fatigue fracture of metals.

Ling Ling Zhang Jian Hou[7] studied the Stress and Strain Concentrations in Curved Beams of Finite Thickness with End Moments

## III. Present Investigation

Fig 1.1shows the cross section of part of an initially curved beam subjected to uniaxial tensile loading. This Initially curved beam was studied to find distribution of stress using Photoelasticity and Finite Element Analysis. Based on intensity of stress present in an initially curved beam, we removed the material by providing stress relievers in low stressed regions without affecting to the main structure. The stress intensity at the inner region of the beam is tried to minimise by providing a circular hole in low stressed region. Also investigation is done to find out intensity of stress distribution at the inner surface of the beam by providing auxiliary elliptical stress reliever in low stressed region in place of circular hole.

Fig 3.1(a),(b)and (c) shows the basic geometry of initially curved beam used for present investigation and it's modification by providing stress relievers at low stressed region.



a) Original model b) Elliptical SR c) Circular SR

Fig. 3.1 Models of initially curved beam

### 3.1 Experimental setup and Photoelastic Analysis

In the whole problem of photoelastic stress analysis, preparation of the model bears its own importance because it has got a major effect on the results obtained. Hence, models were prepared with maximum possible accuracy. Thickness of the model was kept very small as the solution of the problem is obtained under plane stress conditions. The Araldite hardener HY-951 and Araldite resin CT-231 supplied in liquid state by Huntsmen Pvt.Ltd were mixed in proper proportion of 10:100 by volume and a thin sheet of araldite, 6mm thick was cast between two 22mm thick acrylic plates. After complete polymerization i.e after 24 hours from the time of casting the photoelastic sheet of required dimensions was removed from molding box. By using profile cutter circular disc of 50 mm (D) diameter for calibration and curved structures of required curvature were cut from casted photoelastic sheet for further analysis. The stress distribution in a complex model is function of the load.

To determine this stress distribution accurately requires the careful calibration of the material fringe value. For the present investigation a circular disc is loaded in diametric compression was employed for the calibration purpose. Material fringe value was estimated using following relation

$$\text{Material fringe value } (f_{\sigma}) = 8 \times P / \pi \times D \times N$$

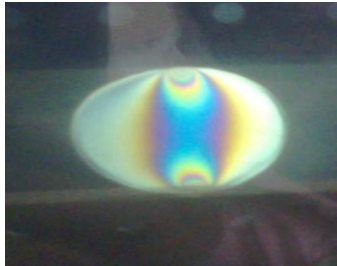


Fig 3.1 (a) Circular disc loaded with white light



Fig 3.2(b) Circular disc loaded with monochromatic light

**Table 3.1** Material fringe value (Calibration) estimated by Tarday’s method of compensation

Sl No.	Load at the end of the lever (W kgs)	Actual load on the specimen (P kgs)	Fringe order at the center of the disc (N)	Material fringe value $f_{\sigma}$ Kg/cm	Average fringe value $f_{\sigma}$ Kg/cm
1.	3	9.375	0.511	8.07	9.84 Kg/cm
2.	4	12.5	0.611	9.6	
3.	8	25	1.188	10.72	
4.	10	31.25	1.45	10.97	

**3.2 Stress analysis of curved structure**

For each photoelastic model, subjected to uniaxial tensile loading, the isochromatic fringe order at the point of maximum stress was carefully observed using Tardy’s method of compensation, as shown in fig 3.2(a)-fig 3.2(e). Stress in the model is calculated using the relation  $\sigma_m = N * f_{\sigma} / h$ . And these, stress are transferred over prototype using scaling relation ship or laws of similarity (By goodier)

$$\sigma_p = \{(P_p / P_m) * (L_m / L_p) * (t_m / t_p)\} * \sigma_m$$

Readings are tabulated in Table 3.2

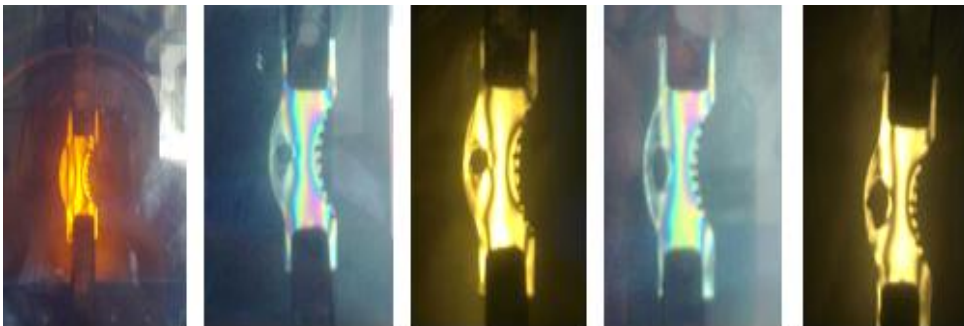


Fig 3.2(a)

Fig 3.2(b)

Fig 3.2(c)

Fig 3.2(d)

Fig 3.2 (e)

Fig 3.3(a) – (e) Represents the Isochromatic and isoclinic fringe patterns of the curved structure without and with stress relievers

**Table 3.2** Experimental results of curved structure

Points Over the inner surface	Model without out stress reliever			Model with circular stress relievers			Model with elliptical stress relievers		
	Fringe order (N)	$\sigma_m$ N/mm <sup>2</sup>	$\sigma_p$ N/mm <sup>2</sup>	Fringe order (N)	$\sigma_m$ N/mm <sup>2</sup>	$\sigma_p$ N/mm <sup>2</sup>	Fringe order (N)	$\sigma_m$ N/mm <sup>2</sup>	$\sigma_p$ N/mm <sup>2</sup>
01	0.991	1.594	15.94	0.847	1.363	13.63	0.675	1.085	10.86
02	1.091	1.755	17.55	0.933	1.4	14	0.769	1.237	12.37
03	1.156	1.8598	18.598	1.004	1.616	16.16	0.864	1.39	13.9
04	1.0146	1.6323	16.323	1.067	1.716	17.16	0.949	1.527	15.27
05	1.074	1.728	17.28	1.092	1.757	17.57	1.093	1.634	16.34
06	1.055	1.697	16.97	1.064	1.712	17.12	1.088	1.759	17.59
07	0.96	1.544	15.44	1.016	1.634	16.34	1.053	1.75	17.5
08	0.89	1.437	14.37	0.942	1.516	15.16	1.019	1.694	16.94
09	-	-	-	0.868	1.397	13.97	0.969	1.64	16.4

#### IV. Finite Element Method

Finite element method is a numerical method , gives approximate solutions for any engineering problems .Optimisation using FEM is less time consuming compare to experimental technique.In present work authore used ANSYS 10 to optimise the position,shape and orientation of the of the stress reliver and also to validate experimental results .This curved structure is descetreised in to finite elements,which are Solid,Qusd 4-noded Plane -42 isoparametric elements .To reduce the weight of the structure ,initially circular hole has created at the low stressed region i.e near the edge of the outer boundary of the curved structure and studied for stress intensity.Investigation is done to optimise stress reliver position and orientation .

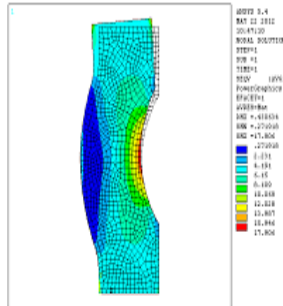


Fig 4.1 (a)

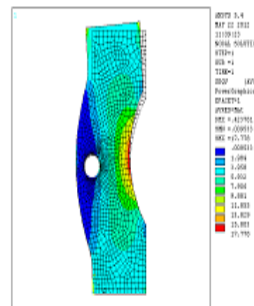


Fig 4.1(b)

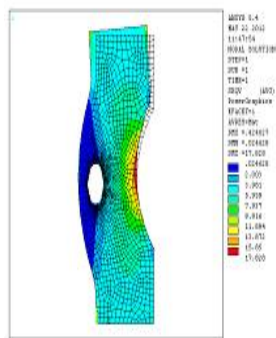


Fig 4.1(c)

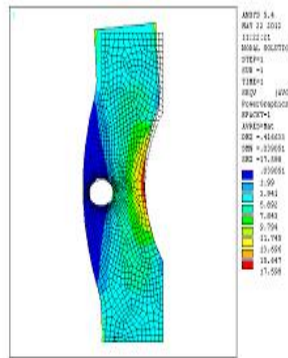


Fig 4.1 (d)

Fig 4.1 (a) – (d) Represents the F E M results of a curved structure with and without stress relievers

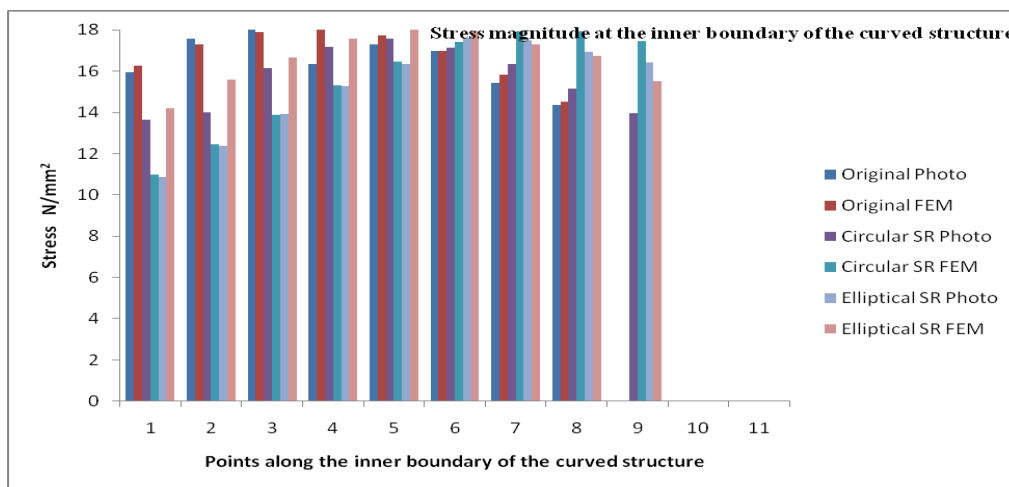


Fig 4.2Comparison of stress intensity at various pont of interest at the inner boundary of structure

**Table No 4.2 Comparison between FEM & Experimental Results**

PARTICULARS	Deformation mm	Max Stress N/mm <sup>2</sup>	Max Stress N/mm <sup>2</sup>
	FEM	FEM	Photoelasticity
Initial curved beam without stress reliever	0.28	17.906	18.598
Curved beam with circular stress reliever	0.279	17.78	17.57
Curved beam with elliptical hole (Major axis is parallel to load)	0.298	19.165	-
Curved beam with modified elliptical hole (Major axis is parallel to load)	0.278	17.828	17.59
Curved beam with elliptical hole.(Major axis is normal to load)	0.2778	17.598	-

**V. Results And Discussion**

In the present investigation , curved structure subjected to uniaxial tension is considered.The investigation is carried out to find the stress distribution in the curved structure as shown in fig 3.0(a).To,reduce weight of the structure without affecting strength,single stress reliver was introduced at the low stress area .Initially circular stress reliver of diameter 14mm was introduced near the outer edge which is subjected to compression,and studied the stress at the inner boundary with photoelastic technique and compare the results with FEM.Results of both the techniques were considerable and encourased for further modification of the stress reliver to reduce stress intensity of the structure furthr.Finite element analysis was carried out using “ANSYS-10” to optimise geometry of stress reliver which is cost effective and saved time compared to photoelastic method.At the same location elliptical stress reliver was introduced keeping major axis parallel to the axis of load and investigated the stress.intensity .It is observrd that,magnitude of stress was increased with elliptical stress reliver by 7.5%.Next investigation was carried out by modifying the elliptical hole with the same loading condition and was observed 0.3% reduction in stress compared to circular stress reliver.further investigation was carried by rotating the ellipse by 90 degrees and the major axis position was normal to the axial loading.Intensity of stress was reduced by 1.01% compared to circular stress reliver.Finite element Method results show the same trend as compared to Experimental results.The Experimental values are about 1.2% lower than the FEM values.Photoelastic stress analysis depends on the accuracy of the fractional-fringe-order and measurement of isoclinic angle.

**VI. Conclusion**

The maximum stress for curved structure subjected to uniaxial tension occurs at the inner fiber of the beam.This stress has been reeduced 2% by introducing elliptical stress reliver at low stress region with it’s major axis normal to the loading position.Results obtained by numerical method is well matched with Experimental results.It is concluded that,both the techniques can be used effectively to study the stress in curved structure subjected to uniaxial tensile loading.

**Reference**

[1]. S.N.Akour,j.F.Nayfeh and D.W Nicholson “J Strain Analysis”Vol.38 PP507-517  
 [2]. S.K. Chakrabarti and K.E. Mechin “Accuracy of compensation I any two-dimensional uniform and axial system of combined stress”SESA,Vol 1 PP429-431  
 [3]. A.J Durelli”stress distribution around an elliptical discontinuity in any two-dimensional uniform and axial system of combined system”SESA Vol1 PP19-31  
 [4]. J.B. Hanus and C.P. Burger,”Stress concentration factor for elliptical hole Near an edge”experimentalMechanics,PP336-340  
 [5]. U. C.Jindal”ReductionofstressConcentration around a hole in a uniaxially loaded plate””Journal –ME-PP66-70  
 [6]. S.A.Meguid “Finite Element Analysis of defence hole system for the reduction of stress concentration in a uniaxially – loaded plate with two coaxial holes””Engineering Fracture Mechanics”PP403-413  
 [7]. Dally and Riley “Experimental Stress Analysis”