

## A Modal Pushover Analysis on Multi-Span Bridge to Estimate Inelastic Seismic Responses

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**Abstract:** Nonlinear static procedures are finding widespread use in performance based seismic design since it provides practitioners a relatively simple approach to estimate inelastic structural response measures. However, conventional Nonlinear static procedures using lateral load patterns recommended in FEMA-356 do not adequately represent the effects of varying dynamic characteristics during the inelastic response or the influence of higher modes. To overcome these drawbacks, some improved procedures have recently been proposed by several researchers. A method of modal combinations that implicitly accounts for higher mode effects is investigated in this paper. Modal pushover analysis method is based on invariant force distributions formed from the factored combination of independent modal contributions. The performance of Modal Pushover Analysis (MPA) in predicting the inelastic seismic response of multi-span concrete bridge is investigated. The bridge is subjected to lateral forces distributed proportionally over the span of the bridge in accordance to the product of mass and displaced shape. The bridge is pushed up to the target displacement determined from the peak displacement of the *n*th mode inelastic Single Degree of Freedom System.

**Keywords:** Modal pushover target displacement Nonlinear static single degree freedom system.

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### I. Introduction

A large number of bridges were designed and constructed at a time when bridge codes had no seismic design provisions, or when these provisions were insufficient according to current standards. Many of these bridges may suffer severe damage when struck by earthquakes, as evident by recent moderate earthquakes. Linearly elastic procedures are efficient as long as the structure behaves within elastic limits. If the structure responds beyond the elastic limits, linear analyses may indicate the location of first yielding but cannot predict failure mechanisms and account for redistribution of forces during progressive yielding. This fact makes the elastic procedures insufficient to perform assessment and retrofitting evaluation for those bridges in particular and structures in general. Nonlinear (static and dynamic) procedures can overcome this problem and show the performance level of the structures under any loading level.

Pushover analysis has been widely used for analyzing the seismic behavior of bridge structures. It can be used as a method for determining the capacity of a bridge structure neglecting the higher mode effects. This approach may produce an error for long or irregular bridges, especially in cases where the bridge has a large scattered mass distribution in the transverse direction. At the same time, many researchers reported the successful of pushover analysis on building structures especially for low to medium-rise building, which is typically dominated by the first mode. However, as the structure becomes higher, the participation of higher modes may increase. These higher mode effects may contribute to the structure's response significantly. In this case, the single invariant force distribution used by pushover analysis cannot represent the potential range of loading experienced in dynamic response. Therefore several new analysis methods have been developed to overcome the limitations of conventional pushover analysis. One of them is to perform pushover analysis using an invariant lateral force distribution for each mode independently, to consider the contribution of higher modes. The peak responses determined from every mode are combined using square-root of sum-of-square combinations. This procedure is termed as Modal Pushover Analysis (MPA) Chopra and Goel [1] claimed that as an improved pushover analysis, MPA offers conceptual simplicity but provides superior accuracy compared to the conventional pushover analysis in estimating seismic demands on bridge.

For elastic range, MPA has been proven consistent with Response History Analysis. The following discussion will be drawn based on the investigation of MPA on multi-span concrete bridge especially in the inelastic range.

### II. Modal Pushover Analysis

The governing equilibrium equations of the *N* degree of freedom (*N*-DOF) system shown in Fig 1 to horizontal earthquake ground motion  $\ddot{u}_g(t)$  are as follows[2]:

$$m\ddot{u} + c\dot{u} + ku = mi\ddot{u}_g(t) \tag{1}$$

where,  $\mathbf{u}$  is the vector of  $N$  lateral displacements relative to the ground;  $\mathbf{m}$ ,  $\mathbf{c}$  and  $\mathbf{k}$  are the mass, damping, and lateral stiffness matrices of the system respectively; where  $i$  is an influence vector with every member equal to unity.

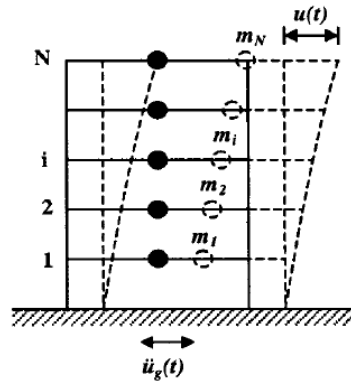


Figure 1. N-DOF System Under Ground Motion

In inelastic system, the relations between lateral forces  $f_s$  and the lateral displacements  $u$  are not single-valued, but depend on the history of the displacements:

$$f_s = f_s(u, \dot{u}) \tag{2}$$

Therefore for inelastic system Equation (1) can be rewritten as follows:

$$m\ddot{u} + c\dot{u} + f_s(u, \dot{u}) = mi\ddot{u}_g(t) \tag{3}$$

Equation (3) consists of coupled equations. Solving these coupled equations directly, leads to the Nonlinear Time History Analysis (NLTHA).

In developing MPA for inelastic structures, Equation (3) will be transformed to the modal coordinates of the corresponding linear system. Although it is not proper because modal analysis is not valid for inelastic system, it can be assumed that at initial state of inelastic condition, the inelastic system has the same properties (e.g. stiffness, mass, and damping) with the elastic system. Expanding the displacements of the inelastic system in terms of the natural vibration modes of the corresponding linear system one will obtain:

$$u(t) \cong \sum_{n=1}^N \phi_n q_n(t) \tag{4}$$

where,  $\phi_n$  and  $q_n(t)$  are the  $n^{\text{th}}$  natural vibration mode of the structure, and the modal coordinate respectively. Then, using Equation (4) and pre multiplying by  $\phi_n^T$ , Equation (3) can be rewritten as [8]:

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \frac{F_{sn}}{M_n} = -\Gamma_n \ddot{u}_g(t) \quad \text{for } n=1,2,3,\dots,N \tag{5}$$

Where:

$$\Gamma_n = \frac{L_n}{M_n} \qquad L_n = \phi_n^T m i \qquad M_n = \phi_n^T m \phi_n$$

in which  $\omega_n$  is the natural circular frequency and  $\zeta_n$  is the damping ratio for the  $n^{\text{th}}$  mode. The solution  $q_n(t)$  can readily be obtained by comparing Equation (5) to the equation of motion for the  $n^{\text{th}}$  mode elastic SDF system subjected to  $\ddot{u}_g(t)$ :

$$\ddot{D}_n + 2\zeta_n \omega_n \dot{D}_n + \omega_n^2 D_n(t) = \ddot{u}_g(t) \tag{6}$$

Comparing Equation (5) and (7) gives:

$$q_n(t) = -\Gamma_n \phi_n(t) \tag{7}$$

and substituting in Equation (4) gives the floor displacements:

$$u_n(t) = \Gamma_n \phi_n D_n(t) \tag{8}$$

The preliminary step in developing modal pushover analysis for inelastic systems is performing uncoupled modal response history analysis (UMRHA). The UMRHA neglects the coupling of the  $N$ -equations in modal coordinates in Equation (5) to obtain the maximum displacement (Equation (9)) in each mode in the modal coordinate.

To represent the relation between lateral forces  $f_s$  and the lateral displacements  $u$  (Equation 2), structure is pushed to a maximum value determined in Equation (9) using lateral forces distributed over the building height in accordance to  $s_n^*$ :

$$S_n^* = m\phi_n \tag{9}$$

The base shear  $V_{bn}$  can be plotted against displacement  $u_n$ . A bilinear idealization of this pushover curve for the  $n$ th mode is shown in Fig 2(a). The relation between forces and displacement follows [8]:

$$F_{sn} = \frac{V_{bn}}{\Gamma_n} , \quad D_n = \frac{u_{rn}}{\Gamma_n \phi_{rn}}$$

By these relationships, pushover curve can be converted into the  $F_{sn}/L_n - D_n$  relation as shown in Fig 2(b). The yield value of  $F_{sn}/L_n$  and  $D_n$  are:

$$\frac{F_{sny}}{L_n} = \frac{V_{bny}}{M_n} , \quad D_{ny} = \frac{u_{rny}}{\Gamma_n \phi_{rn}} \tag{10}$$

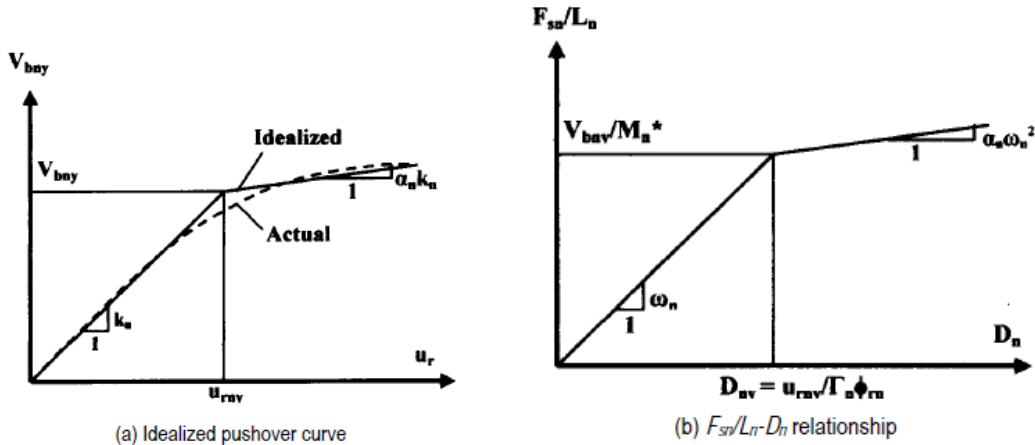


Figure 2. Properties of the  $n$ th-mode inelastic SDOF system

from pushover curve

in which  $n \ n \ n \ M^* = L \ \Gamma$  is the effective modal mass.

The two equations are related through

$$\frac{F_{sny}}{L_n} = \omega_n^2 D_{ny} \tag{11}$$

The peak displacement for each mode is given by:

$$u_{rno} = \Gamma_n \phi_n D_n \tag{12}$$

where  $D_n$ , the peak value of  $D_n(t)$  can be determined by solving Equation (7) or from the inelastic response (or design) spectrum. The other peak response (e.g. shear, moment, etc.) can be derived statically from this pushover analysis. The peak modal responses are combined according to the square-root-of-sum-of-squares (SRSS) rule. Then, the SRSS rule provides an estimate of the peak value of the total response:

$$r_o \approx [\sum_{n=1}^N r_{no}^2]^{1/2} \tag{13}$$

### III. Bridge Selection And Modeling

A multi-span concrete bridge in Badnera area is chosen as the study case. The bridge deck is supported by a single-span prestressed concrete girders. The girders are placed on the concrete pier head through the bearing and locked in the transverse direction. The supporting piers are in

various heights, but in this study equal height of 8 m is selected. The width of the bridge is 17.2 m with 24 m length of equal span. The bridge is considered is 13 span bridge which is to be able to represent the behavior of multi-span bridge as the whole.

The deck is supported by eight prestressed girders, connected. The deck structure is modeled as an assemblage of linear elements in SAP2000 Nonlinear program. The deck is assumed to be rigid in  $x$ - and  $y$ -direction. All node lies at the same elevation in which at the centre of gravity of the girder and stringers, and mass is lumped at both ends of element. Each pier is modeled as an element with an elastic plastic behavior. It is assumed that the piers will fail in flexural mode where a plastic hinge formed at the pier's base. The moment-rotation capacity of the plastic hinge is defined based on the stress-strain relationship of the section considering the confinement effect from transverse reinforcement. Bearing is modeled using link element in SAP2000 Nonlinear program. To accommodate the soil-structure interaction, each pile is modeled as spring with six degree of freedom to represent translational and rotational support.

To perform analysis of structure, the next step after modeling is applying loads. Design response spectrum should be available in order to perform pushover analysis. This bridge is to be built in a seismic zone with an acceleration coefficient of  $PGA = 0.3g$ .

#### IV. Results And Discussion

##### 4.1 Pushover Curves

Applying the modal load pattern of the 4<sup>th</sup>, 18<sup>th</sup>, 19<sup>th</sup> and 31<sup>th</sup> modes in the transverse direction of bridge, the corresponding pushover curves were derived with respect to the deck displacement shown in fig.3. It is noted that these curves are not necessarily representative of the actual response of structural member of bridge. For example, the capacity curves corresponding to modes 4 is rather linear, hence conveying the impression that the bridge does not enter the inelastic range when subjected to 4<sup>th</sup> modal load pattern. In reality, it is only the central pier region that responds elastically in that case, whereas the edge pier do enter the inelastic range; this is due to the form of those higher modal load patterns which are not critical for the central region of the bridge.

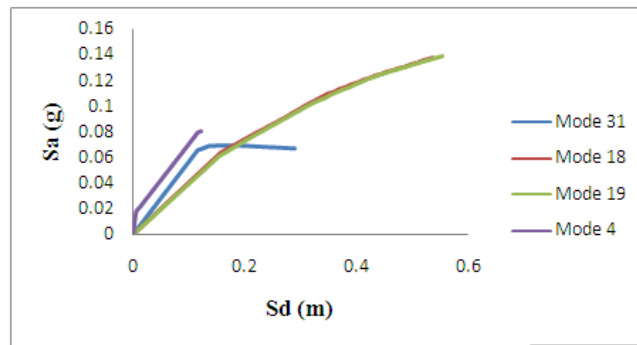


Fig.3: Capacity curves derived with respect to deck displacement.

By comparing the capacity curves constructed with respect to the different control load locations, it is clear that capacity curve produced using the most critical pier location are more representative of the actual behavior of bridge, since they indicated that at some stage of the response one or more piers of the structure yield. In the studied bridge, the capacity curves of Fig.3 using the most critical pier indicated that yielding of structure will initiate from its response to the fundamental transverse mode (4<sup>th</sup> mode) followed by yielding due to the 18<sup>th</sup> mode then 31<sup>th</sup> mode.

##### 4.2 Evaluation of NL-THA and MPA procedure

Results of the modal pushover approaches were evaluated by comparing them with those from the NL-THA. To this effect, a time acceleration records compatible with the design spectrum was used in the NL-THA analyses. The deck displacements determined from MPA analyses with respect to the control point of the most critical pier were compared with those from NL-THA for increasing levels of earthquake excitation, as shown in Fig 4. It is noted that the deck displacements shown in the figures as the THA case are the average of the peak displacements recorded in the structure during time history analysis. MPA procedure which accounts for four transverse modes predicts well the deck displacements of the bridge. On the other hand, the MPA procedure is much closer to NL-THA and gave better predictions at the end areas of the bridge. As the level of excitation increases and higher mode contributions become more significant.

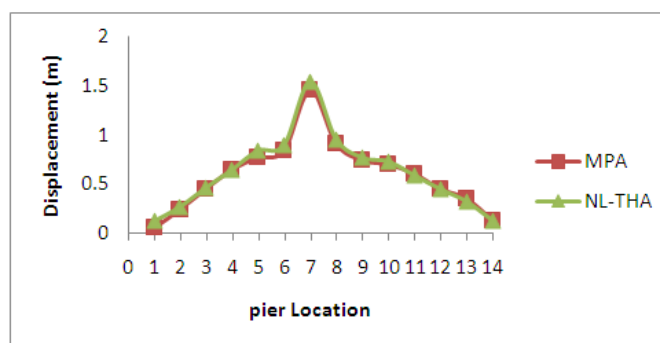


Fig.4 Deck displacement at the pier location

The displacement profile derived by the MPA method tends to match that obtained by the NL-THA, whereas predictions from SPA become less accurate as the level of inelasticity increases. The consideration of higher modes and the correction made to the target displacement significantly improve the accuracy of the predicted deck displacements.

**Table 1 Deck displacement of bridge from various methods**

Deck Location	THA	MPA	Diff %	Deck Location	THA	MPA	Diff %
P1	0.121	0.053527	-56	P8	0.954	0.91	-5
P2	0.268	0.243	-9	P9	0.768	0.742	-3
P3	0.464	0.45	-3	P10	0.732	0.704	-4
P4	0.65	0.648	0	P11	0.589	0.609	3
P5	0.842	0.768	-9	P12	0.444	0.452	2
P6	0.896	0.844	-6	P13	0.321	0.354	10
P7	1.54	1.457	-5	P14	0.129	0.126	-2

$$Diff (\%) = \frac{\delta_{po} - \delta_{Th}}{\delta_{Th}}$$

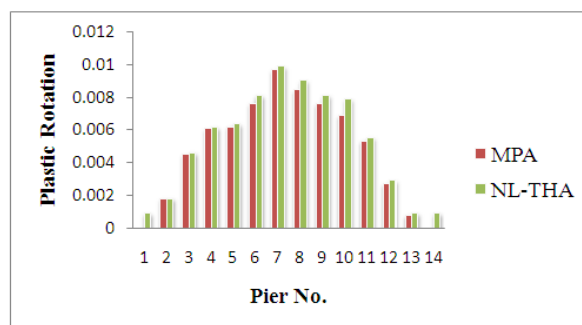
Table 1 lists the deck displacement of bridge calculated using Modal pushover analyses as well as the NLTHA as the benchmark to compare with others cases. As shown in the table, MPA procedure provided the best estimate of deck displacement. The difference between the maximum displacement calculated using the MPA (at pier no. 7) and that of the NL-THA is 5% and the MPA displacement profile is closely matching that profile derived from NL-THA with differences ranging from 3% at pier no. 11 to 9% at pier no.5.

**4.3 Total Base Shear and Plastic Rotations**

In order to further evaluate the results obtained from the MPA analysis, comparison is also performed for total base shear and plastic hinges’ rotations at the bottom of piers between results from MPA with corresponding values from the NL-THA procedure for increasing levels of earthquake excitation. MPA gives a better results and underestimates the base shear by only 21%. It is observed that MPA provided better predictions with differences range between 2% to 14%. Another significant advantage of the MPA method is that it is able to capture the plastic hinge development at P2 and P11, hence, the overall degree of agreement between MPA and NL-THA is deemed quite satisfactory.

**Table 2 Base shear and Plastic rotation by various methods**

	THA	MPA	Diff.%	
Base Shear	19514	15432	-21	
Plastic Rotation	P1	0.000927	0	
	P2	0.001837	0.001817	-1
	P3	0.004641	0.004541	-2
	P4	0.006218	0.006118	-2
	P5	0.006428	0.006245	-3
	P6	0.008176	0.007676	-6
	P7	0.009938	0.009738	-2
	P8	0.009086	0.008486	-7
	P9	0.008176	0.007676	s-6
	P10	0.007941	0.006941	-13
	P11	0.005578	0.005378	-4
	P12	0.002945	0.002745	-7
	P13	0.000927	0.00003	-14
	P14	0.000927	0	-



**Fig. 5 Rotation of plastic hinges at bottom of piers.**

## **V. Conclusion**

Bridges extends horizontally with its two ends restrained and that makes the dynamic characteristics of bridges different from buildings. By analyzing the structure using Modal pushover analysis and Non linear time history analysis method, it was concluded that:

- 1- On the basis of the results obtained, MPA seems to be a promising approach that yields more accurate results compared to the standard pushover, without requiring the higher modeling effort and computational cost, as well as the other complications involved in NL-THA.
- 2- The difference between the maximum displacement calculated using the MPA (at pier no. 7) and that of the NL-THA is 5% and the MPA displacement profile is closely matching that profile derived from NL-THA with differences ranging from 3% at pier no. 11 to 9% at pier no.5.
- 3- The MPA procedure introduced was found to yield better results when the level of earthquake excitation was increased and more inelastically developed in the structure.
- 4- On the contrary MPA provided a significantly improved estimate with respect to maximum displacement pattern reasonably matching the more refined NL-THA method, even for increasing level of earthquake loading that triggers increased contribution of higher modes.

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