

## Free Vibration Analysis of Inflatable Space Structure

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**Abstract:** This paper present the modal analysis for the predicating the behavior of inflatable membrane structure of general square shape with a thickness in millimeter using the various smart material which optimally within structural member subjected to edge- constrained rather than applying bending or moments. A numerical solution for membranes may also be found using the finite element method. In this paper flat thin membrane choose to analysis the behavioral effect of the multi-layered membranes using the properties of different smart material and compare their results in terms of frequency and out plane deflection with mode shape. This analysis makes more effective to select the smart materials in the space technology. Vibration analysis of arbitrary square shape membrane is also done using a finite element package, ANSYS APDL. The analysis shows good agreement in finite element solutions.

**Keywords:** Boundary condition, finite element, material property, membrane, mode shape, natural frequency, out plane displacement.

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### I. Introduction

In the field of Engineering and Architecture, membrane structures play a vital role in many ways. Examples include textile covers and roofs, aircraft and space structures, parachutes, automobile airbags, sails, windmills, human tissues and long span structures. They are typically built with very light materials which are optimally used. These structures are characterized because they are only subjected to in-plane axial forces. Even in the field of architectures and civil engineering, both pre-stressed membranes and cable networks constitute a very remarkable group. A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any compression at all. However, membrane theory accounts for tension and compression stresses, and the need for a computational procedure that takes into account tension stresses only is needed. In membrane theory only the in-plane stress resultants are taken into account. A numerical solution for membranes may be found using the finite element method [1–3].

The deployable space structures consist of thin polymer films that offer a wider range of packaging configurations than structures with traditional deployment mechanisms. Due to the flexibility of such deployable structure like shell or membrane shows greater importance for space application and hold great promise. The material constitutive behavior and the analytical tools to analyze them are required to make advances in building cheaper, lighter and more reliable structures. Many structures are in the developing stage and the materials that are meant to serve to make these applications possible are not yet within reach. Future missions depend much on new discoveries, mainly in material manufacturing. There are several different space applications in which the use of thin membrane structures are used or being considered. Due to their light weight, high strength-to-weight ratio and ease of stowing and deploying, membranes are especially attractive for space applications. Inflatable reflectors, space-based radar, space based communication systems such as antennae and solar power collection panels on spacecraft, etc are the examples included. [4–5].

The membrane material used in the numerical analysis was assumed inextensible and its weight was neglected in the determination of the equilibrium shape. They found that the membrane's mass density is of little influence on the computed natural frequencies. Other researchers used finite elements and boundary elements to model and compute natural frequencies and mode shapes of a single-anchor inflatable dam [6]. This study makes impact on finding the vibration aspect on the flat membrane using the various smart materials. The pressure in an inflatable structure can also play a critical role in the suppression of vibration [7]. Literature that exists on pure structural membrane components has concentrated mostly on inflated components such as beams [8], torus (Main), and inflated lenticular concentrators [9]. The dynamics of the membrane themselves are of great interest though, as it is the membrane itself that is performing the useful work, and in some applications they could be attached to more traditional aerospace structures. Therefore improving understanding the behavior of the membranes appears to be important. A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any Compression at all.

However, membrane theory accounts for tension and compression stresses. In membrane theory only the in-plane stress resultants are taken into account [10]. This paper present the modal analysis for the predicating the behavior of various inflatable membrane structure of general square shape with a thickness in millimeter using the various smart materials. A numerical solution for membranes may be found using the finite element method. Finite element analysis of membrane structures for small deformations can be found in [11] but

with only single material. In this paper, the square shaped general sketch of flat thin multi-layered membrane is chosen & analysis of the behavioral effect of the membranes using different properties for different smart material, Comparing the various parameters like frequency, Eigen values, displacement, etc. This analysis is more effective in future to select the suitable smart material in the design of the space technology. The geometrically non-linear vibration due to pre-stressed is modeled and analyzed using finite element package, ANSYS APDL.

## II. Membrane material Properties

Membrane structures consist of thin membrane or fabric as a major structural element. A membrane has no compression or bending stiffness, therefore it has to be pre-stressed to act as a structural element [13]. The analysis, design and construction of such structures are a field that has developed very considerably during the last 30 years. The type of structures that is of interest in the present study is high-precision deployable for spacecraft, where is a growing requirement for furlable reflecting surfaces for antennae, reflectors and solar arrays. The configurations that are being considered include at pre-stress membrane panels and paraboloidal pre-stressed membranes formed by contiguous cylindrical pieces [14].

The performance efficiency of these reflective surfaces depends not only on the geometric accuracy of the surface but also on its vibration characteristics. The vibrations of lightweight structures are afflicted considerably by the surrounding medium. Thus, spacecraft structures should be tested in a vacuum chamber, but this would be too costly for a large structure. The efficiency and stability of the membrane structures depends on their dynamic controls in the deployed configuration, thus it is necessary to have a detailed understanding of vibration characteristics of these membrane structures.

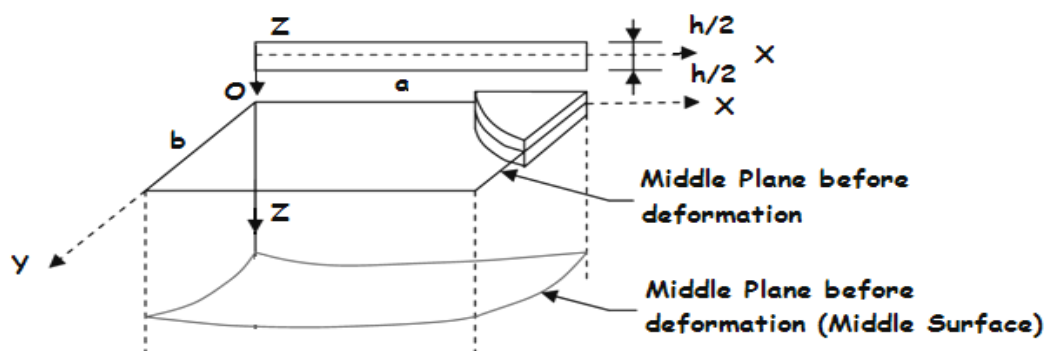
**Table 1:** Membrane material properties

Material	Density(Kg/m <sup>3</sup> )	Young's Modulus(GPa)	Poisson's Ratio
Mylar	1390	8.81	0.35
Kevlar	790	11.9	0.3
Kapton	1420	2.5	0.34

## III. Governing Equation

The plate in which the ratio  $a/h \geq 80-100$ , where "a" is a typical dimension of a plate in a plane and "h" is the plate thickness is maintained such plates are referred to as membranes and they are devoid of flexural rigidity. Membranes carry the lateral loads by axial tensile forces N (and shear forces) acting in the plate middle surface as shown in Fig. 1. These forces are called membrane forces; they produce projection on a vertical axis and thus balance a lateral load applied to the plate membrane. The fundamental assumptions of the linear, elastic, small-deflection for thin membrane structure may be stated as the material of the plate is elastic, homogeneous, and isotropic and initially remain flat. The deflection of the mid-plane is very small compared to that of membrane thickness. Middle surface remains unstrained even after bending, since the deflection is too small.

To derive the equation of motion of a membrane, consider the membrane to be bounded by a plane curve S in the XY plane. Let  $f(x, y, t)$  denote the pressure loading acting in the Z direction And P the intensity of tension at a point that is equal to product of tensile stress and thickness of the membrane. The magnitude of P is usually constant throughout the membrane. If we consider an elemental area  $dx dy$ , Forces of magnitude  $P dx$  and  $P dy$  act parallel to the Y and X axes respectively.



**Figure 1:** A load free membrane

The net forces acting along Z direction due to these forces are:

$$P \frac{d^2 w}{dy^2} dx dy \quad \text{And} \quad P \frac{d^2 w}{dx^2} dx dy$$

The pressure forces along the Z direction is  $f(x, y, t) dx dy$  and the net inertia force is:

$$\rho(x, y) \frac{d^2 w}{dt^2} dx dy$$

Where  $\rho(x, y)$  is the mass per unit area. The equation of motion (1) for free transverse vibration can be obtained as:

$$P \left( \frac{d^2 w}{dx^2} + \frac{d^2 w}{dy^2} \right) = \rho \frac{d^2 w}{dt^2} \dots\dots\dots (1)$$

The above equation can be expressed as:

$$P \nabla^2 w = \rho \frac{d^2 w}{dt^2}$$

Where,  $\nabla^2$  is the Laplacian operator

Since the equation of motion (1) contains second derivative with respect to t, x and y. We need to specify two initial conditions and four boundary conditions to find a unique solution of the problem. Usually the

displacement and velocity of the membrane at  $t=0$  is specified as  $w_0(x, y)$  and  $\dot{w}(x, y)$ . Hence the initial conditions (2-3) are given by:

$$W(x, y, 0) = w_0(x, y) \dots\dots\dots (2)$$

$$\frac{dw}{dt}(x, y, 0) = \dot{w}(x, y) \dots\dots\dots (3)$$

The boundary conditions are as follows:

1. membrane is fixed at any point  $(x_1, y_1)$  on the boundary, we have

$$W(x_1, y, t) = 0 \quad t \geq 0 \dots\dots\dots (4)$$

2. If the membrane is free to deflect transversely (in the z direction) at a different point  $(x_2, y_2)$  of the boundary, Then the force component in Z direction must be zero.

$$P \frac{d w}{dn}(x_2, y_2, t) = 0 \quad t \geq 0 \dots\dots\dots (5)$$

Where  $\frac{dw}{dn}$  represents the derivative of w with respect to a direction n normal to the boundary point  $(x_2, y_2)$

The free vibration solution of the thin flat membrane can be used by using the method of separation of variable  $w(x, y, t)$  as can be assumed as:

$$W(x, y, t) = W(x, y) T(t) = X(x)Y(y)T(t)$$

By using the equation of motion we obtain:

$$\frac{d^2 X(x)}{dx^2} + \alpha^2 X(x) = 0 \dots\dots\dots (6)$$

$$\frac{d^2 Y(y)}{dy^2} + \beta^2 Y(y) = 0 \dots\dots\dots (7)$$

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0 \dots\dots\dots (8)$$

Where  $\alpha^2$  and  $\beta^2$  are the constants related to  $\omega^2$  as follows:

$$\beta^2 = \frac{\omega^2}{c^2} - \alpha^2 \quad \text{And} \quad C^2 = \frac{P}{\rho}$$

The solutions of the above equations are given by:

$$X(x) = C_1 \cos \alpha x + C_2 \sin \alpha x \dots\dots\dots (9)$$

$$Y(y) = C_3 \cos \beta y + C_4 \sin \beta y \dots\dots\dots (10)$$

$$T(t) = A \cos \omega t + B \sin \omega t \dots\dots\dots (11)$$

Where, the constants  $C_1, C_2, C_3, C_4, A$  and  $B$  in (9-11) can be obtained by Boundary Conditions.

#### IV. Geometry and Boundary condition

Modal Analysis was carried out in ANSYS on multilayer Kapton, Kevlar and Mylar sheets joined by epoxy. The properties of epoxy are A) Density  $58\text{Kg/m}^3$  B) Poisson Ratio 0.34 C) Stiffness =  $1.5\text{Gpa}$

In the boundary conditions, element type Shell-91 is used and all edges are fixed. Each layer of membrane is discretized in total 225 elements. Hexahedral elements with a size ratio of 3 are used. Mode shapes up to 6 are extracted using Block Lanczos mode extraction method with lumped mass approximation.

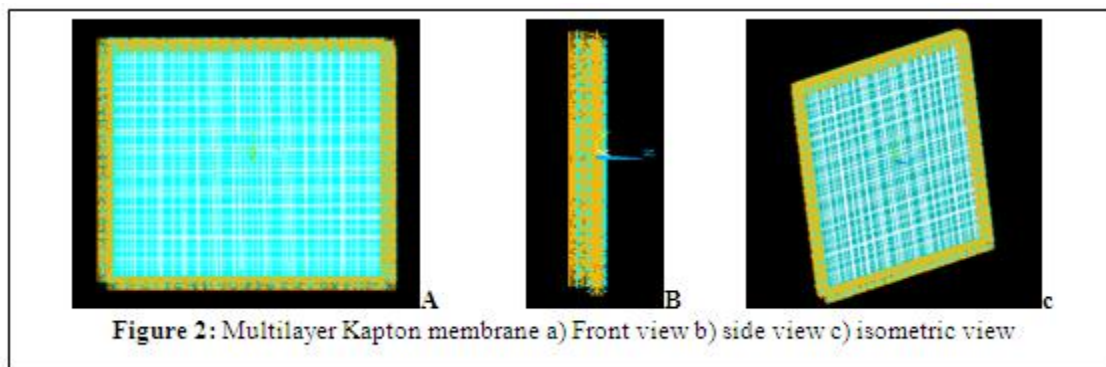


Fig.2 depicts the geometry, meshing and the boundary conditions of the multi-layered membrane used for analysis. The size of the membrane used is  $3\text{m} \times 3\text{m}$ . The thickness of each layer has been taken  $0.256\text{mm}$  which have been attached by epoxy layer. The boundary conditions are depicted by the orange color in Fig.2.

#### V. Results and Discussions

Natural Frequencies and out plane displacements of single layered smart material are obtained in the first 6 mode shapes, similarly in all the cases frequencies are determined from single layer to six layers joined by epoxy. The natural frequencies are plotted from single layer to six layers for kapton, Kevlar and Mylar material and a general pattern of variations has been noted. Similarly Graphs of natural frequencies and out plane deformations are obtained for Kapton, Kevlar and Mylar and the patterns are noted.

**Table 2:** Vibration Analysis of Kapton Inflatable Membrane

Mode shape	Natural Frequencies (in Hz) in different layers					
	Single	Double	Triple	Quad	Penta	Hexa
1	14.84	14.84	14.84	14.84	14.84	14.84
2	30.26	14.84	14.84	14.84	14.84	14.84
3	30.26	30.26	14.84	14.84	14.84	14.84
4	44.62	30.26	30.26	14.84	14.84	14.84
5	54.25	30.27	30.26	30.26	14.84	14.84
6	54.51	30.27	30.27	30.26	30.26	14.85

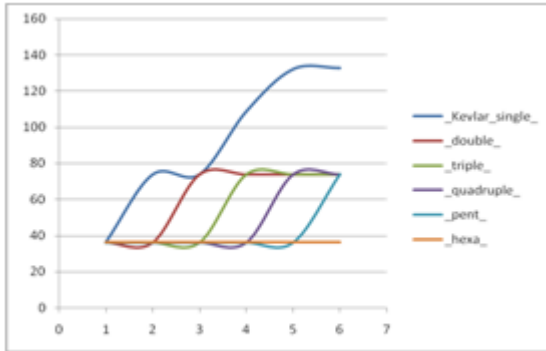
**Table 3:** Vibration Analysis of Mylar Inflatable Membrane

Mode shape	Natural Frequencies (in Hz) in different layers					
	Single	Double	Triple	Quad	Penta	Hexa
1	28.11	28.11	28.11	28.11	28.11	28.11
2	57.34	28.11	28.11	28.11	28.11	28.11
3	57.34	57.34	28.22	28.12	28.12	28.12
4	84.55	57.34	57.34	28.12	28.12	28.12
5	102.79	57.36	57.34	57.34	28.12	28.12
6	103.29	57.36	57.36	57.34	57.34	28.13

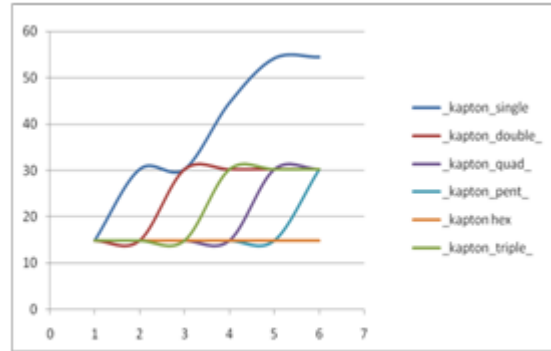
**Table 4:** Out plane deflection of Kevlar Inflatable Membrane

Mode shape	Maximum Out Plane deflection (in mm) in different layers					
	Single	Double	Triple	Quad	Penta	Hexa
1	1.083	1.083	1.083	1.083	1.083	1.083
2	1.059	1.066	1.066	1.066	1.066	1.066
3	1.059	1.098	1.056	1.056	1.056	1.056
4	0.969	1.098	1.045	1.056	1.056	1.056
5	1.053	1.087	1.045	1.095	1.056	1.056
6	1.301	1.087	1.076	1.095	1.094	1.057

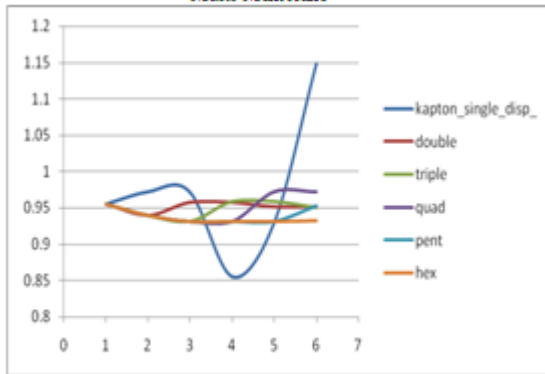
It is clearly visible from the TABLE 2 and TABLE 3 that the natural frequencies experience damping with increase in the number of layers and the reach a constant saturation value in the case of 6 layered packing. From TABLE 4, it can be clearly seen that the aggregate maximum out plane deviation is least in the case of 6 layer packing. The each graph on natural frequency vs. mode shape is plotted for single to hexa layers and it is noted that natural frequency decreases with increasing layers also the natural frequency increases as we increase the mode number.



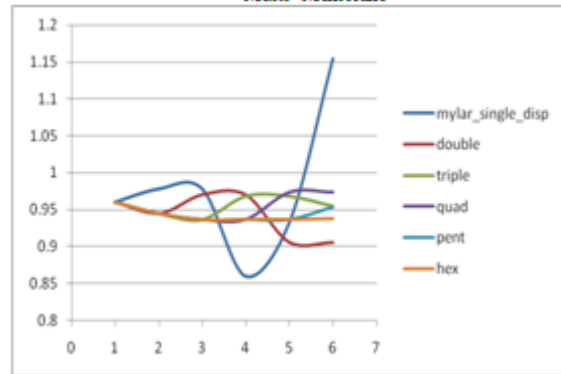
Graph 1: Natural Frequencies obtained for Kevlar Multi-Membrane



Graph 2: Natural Frequencies obtained for Kapton Multi-Membrane



Graph 3: Max. Out Plane deflection of Kapton Multi-Membranes



Graph 4: Max. Out Plane deflection of Mylar Multi-Membranes

The Graph 1-4 on mode shape vs. natural frequency and mode shape vs. out plane deflection is plotted for single to hexa layers and it is noted that natural frequency decreases with increasing layers also the natural frequency increases as we increase the mode number. Graph 1 and Graph 2 depict the saturation frequency obtained at 6 layered packing for both Kapton and Kevlar membranes. The advantage gained for Hex layer in terms of Minimum Aggregate Maximum out Plane deflection is depicted clearly in Graphs 3-4 for Kapton and Mylar Membranes. It is also seen that Kapton has the minimum aggregate out plane deflection as compared to Mylar and Kevlar

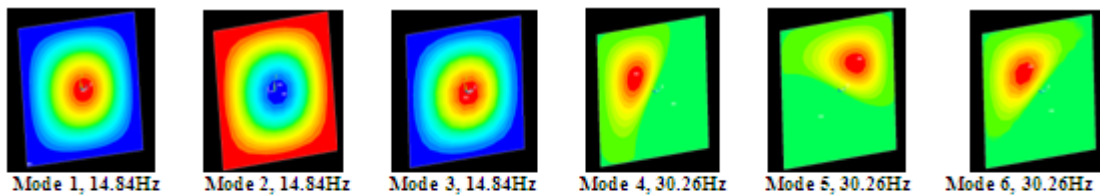


Figure 3 (a) Mode Shapes, Triple Layered KAPTON

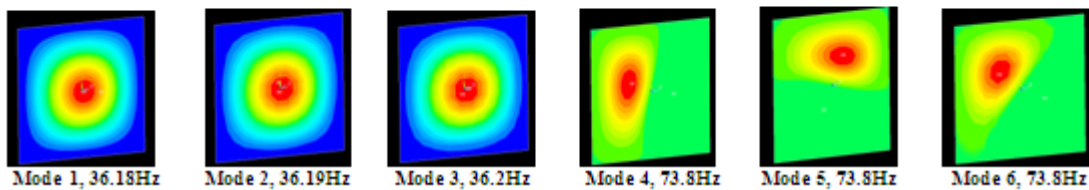


Figure 3 (b) Mode Shapes, Triple Layered KEVLAR



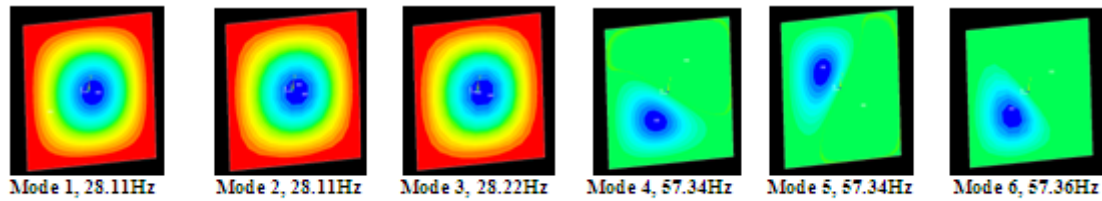


Figure 3 (c) Mode Shapes, Triple Layered MYLAR

Fig. 3 shows the mode shapes of kapton, Kevlar, Mylar materials for triple layer. The saturation of out plane displacements and natural frequency occurs in penta as well as in hexa layer geometry and Kapton membrane undergoes the least displacements. Thus the five and six layered Kapton material can be used as adaptive membrane for effective shape control, but according to the weight factor in ultra light aerospace structures penta layer is suitable.

## VI. Conclusion

In the field of engineering application, thin membrane structures with very light materials are demandable due to non flexural stiffness and optimally within structural member subjected to pre-stressed rather than bending or moments. In this paper, the dynamic behavior of the square shaped flat thin membrane is being analyzed in terms of the mode shape and natural frequency using the different types of smart materials such as Kevlar, Kapton and Mylar. This analysis makes more effective to select the smart material in the space technology. With this study, now it can be easily said that Kapton is the best material for membrane structures as it give least out plane deflections. It is also concluded that a higher number of layers in the packing of membranes is highly beneficial for damping of vibrations and can be used strategically for space gossamer structures. The best configuration is achieved for a six layer packing but considering the high complexity in manufacturing and the high costing involved in the fabrication of these membranes, quad or penta layer packing is the most feasible solution for practical purposes.

## Acknowledgements

The authors are truly thankful to Mr Anil C Mathur, Group Head, Antenna Systems Mechanical Group, Mechanical Engineering Systems Area, Space Applications Centre, ISRO, Ahmedabad and Mr. Kripa Shanker Singh, Scientist/Engineer-SD, Antenna Systems Mechanical Group(ASMG), Mechanical Engineering Systems Area(MESA),Space Applications Centre (ISRO), Ahmedabad for their valuable and significant suggestions which substantially improved the manuscript.

## References

- [1]. D.S. Wakefield, Engineering analysis of tension structures: theory and practice, Journal of Engineering and Structures, Vol.-21, 1999, 680–690.
- [2]. S. C. Gajbhiye, S. H. Upadhyay and S. P. Harsha, Vibration Analysis of an Inflatable Torus, AIAA JOURNAL, Vol. 51, No. 6, 2013
- [3]. S.C. Gajbhiye, S.H. Upadhyay, S.P. Harsha, Finite element analysis of an inflatable torus considering air mass structural element, advances in Space Research 53 2014,163–173.
- [4]. C.H. Jenkins, Gossamer Spacecraft: Membrane and Inflatable Structures Technology for Space Applications, Progress in Astronautics and Aeronautics, Vol.-191, AIAA, Inc., New York.
- [5]. C.H. Jenkins, Nonlinear dynamic response of membranes: state of the art-update, Journal of Applied Mechanical Revised. Vol.-9(10), 1996, 41–48.
- [6]. Mysore, G.V., Liapis S.I., Dynamic analysis of single-anchor inflatable dams, Journal of Sound Vibration, Vol. 215 (2), 1998, 251–272.
- [7]. Choura, S., Suppression of structural vibrations of an air-inflated membrane dam by its internal pressure, Journal of Computational Structure Vol.-65(5), 1997, 669–677.
- [8]. Slade, Kara N., Tinker, Michael L., Analytical and Experimental Investigation of the Dynamics of Polyimide Inflatable Cylinders, Paper AIAA- 99-1518, Proceedings of the 1999 SDM Conference, St. Louis, MO, 1999.
- [9]. Jenkins, C.H. Kondareddy, S., Dynamics of a Seamed Precision Membranes, Paper AIAA-99-1522, Proceedings of the 1999 SDM Conference, St. Louis, MO, 1999.
- [10]. J.G. Valdés, J.Miquel, E.Oñate, Non linear finite element analysis of orthotropic and pre-stressed membrane structures, International Center for Numerical Methods in Engineering (CIMNE), Technical University of Catalunya, Gran Capitán, Campus Nord,08034 Barcelona, Spain, 2009.
- [11]. O.C.Zienkiewicz, R.L.Taylor, The Finite Element Method (fourth Vol.-1, 2 McGraw-Hill, London, 1989).
- [12]. Eric J. Ruggiereo, Garret T. Bonnema, and Daniel Inman, Application of SIS and MIMO analysis techniques on a membrane mirror satellite, Centre for Intelligent Material System and Structures (Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0261, USA.
- [13]. S. Kukathasan and S. Pellegrino, Vibration of pre stressed membrane structures in Air, AIAA, 2002, 13-68.
- [14]. Srivastava A., Mishra B. K. and Jain S. C., Effect of enclosed fluid on the dynamics of an inflated torus, Journal of Sound and Vibration, Vol.-309, 2008, 320–329.