

## Finite element analysis of deployable space structures

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**Abstract:** This paper presents the stress analysis for predicating the behavior of flat thin inflatable membrane structure in MATLAB. This is having square shaped with a thickness in millimeter fabricated using the various smart materials. Within structural member, it is subjected to pre-stress rather than bending or moments. The deployable structure has the low weight (minimal mass to achieve high acceleration); large area & durable (easily withstand the temperature changes, micrometeoroid hazards in outer space). The objective of this paper is to optimize the smart material for the space satellite technology so that the light weight inflatable structure attracts in satellite application. The observations show the good agreement of finite element results.

**Keywords:** inflatable, material property, membrane shape, Pre-stress, thermal stresses, static displacement

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### I. Introduction and background

The deployable space structures consist of thin polymer films that offer a wider range of packaging configuration than structures with traditional deployment mechanisms. Due to the flexibility of such deployable structure like shell or membrane shows greater importance for space application and hold great promise [1]. Even in the field of architectures and civil engineering, both pre-stressed membranes and cable networks constitute a very remarkable group [2-4]. Many structures are in the developing stage and the materials that are meant to serve to make these applications possible are not yet within reach. A number of commercial films (Kapton, Mylar & Teflon) used in ground test and analysis are space proven. Experimental investigation of the vibration testing of an inflated thin torus using smart material had been done and remarkable dynamic response obtained. Future missions depend much on new discoveries, mainly in material manufacturing [5-7]. There are several different space applications in which the use of thin membrane structures are used or being considered. Due to their light weight, high strength-to-weight ratio and ease of stowing and deploying, the membranes are especially attractive for space applications. Inflatable reflectors, space-based radar, space based communication systems such as antennae and solar power collection panels on spacecraft, etc are the examples included [8]. The membrane material also play vital role in the numerical analysis even though it was assumed inextensible and its weight was neglected in determination of the equilibrium shape.

It has been observed that the membrane's mass density is of little influence on the computed natural frequencies [9]. The finite elements and boundary elements are used to model and compute physical stresses and thermal stresses of a single-anchor inflatable dam [10]. The pressure in an inflatable structure can also play a critical role in the suppression but here also the membrane material play very vital role [11]. Literature that exists on 'pure' structural membrane components has concentrated mostly on inflated components such as beams [12], torus (Main), and inflated lenticular concentrators [13]. The dynamics of the membrane themselves are of great interest though, as it is the membrane itself that is performing the 'useful' work, and in some applications they could be attached to more traditional aerospace structures. Therefore improving understanding the behavior of the membranes appears to be important [14-16]. The performance efficiency of the reflective surfaces depends not only on the geometric accuracy of the surface but also on its material characteristics. The behavior of lightweight structures is afflicted considerably by the surrounding medium. Thus, spacecraft structures should be tested in a vacuum chamber, but this would be too costly for a large structure. The efficiency and stability of the membrane structures depends on their dynamic controls in the deployed configuration, thus it is necessary to have a detailed understanding of feasible characteristics of these membrane structures [17]. This study makes impact on finding the thermal aspect on the flat membrane using the various smart materials.

A membrane is essentially a thin shell with no flexural stiffness. Consequently a membrane cannot resist any compression at all. However, membrane theory accounts for tension and compression stresses. In membrane theory only the in-plane stress resultants are taken into account [18]. Finite element analysis of membrane structures for small deformations can be found in [19]. The basic classical plate model helps to analysis criteria of the boundary condition for the FE model of membrane [20]. This paper present the modal analysis for predicating the behavior of square shaped inflatable membrane structure with a thickness in millimeter using the various smart materials which optimally within structural member subjected to pre-stressed rather than bending or moments. The reason behind the choosing of the membrane structure is due to low weight (minimal mass to achieve high acceleration), large area & durable (easily withstand the temperature changes,

micrometeoroid hazards in outer space). The Finite Element Analysis results obtained using continuum membrane element. An important objective of this paper is to optimize the smart material for the space satellite technology so that the light weight inflatable structure attracts in satellite application.

### II. Membrane materials

The variety of materials with improved combinations of properties is more necessary for the space scientist / Engineer to select their specific mission requirement. The smart materials are listed in the TABLE 1 [22].

**Table 1:** properties of smart structures

Material	Density(Kg/m <sup>3</sup> )	Young's Modulus(GPa)	Poisson's Ratio
Mylar	1390	8.81	0.35
Kevlar	790	11.9	0.3
Kapton	1420	2.5	0.34

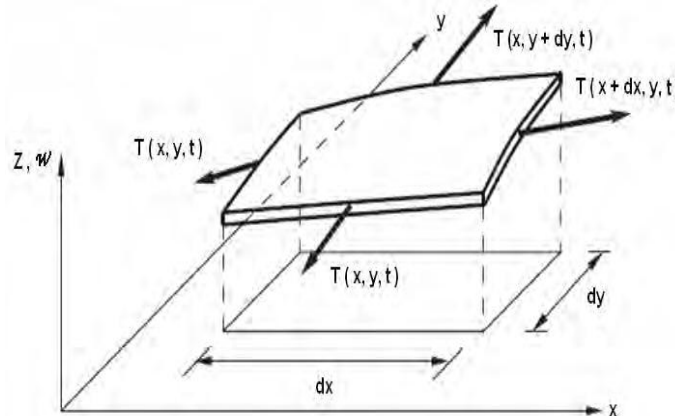
### III. Dynamics of Membrane

Membrane structures consist of thin membrane in two dimensional pre-tensioned elastic form or fabric as a major structural element. A membrane has no compression or bending stiffness, therefore it has to be pre stressed to act as a structural element. In practice, any two-dimensional elastic continuum resists bending moment. However, if the tension is large and the curvatures are small, the effect of bending moment can be neglected. Thus, the membrane can be imagined as an extension of the string to two dimensions.

For analysis of membrane structure following assumptions are made as;

- i. Effect of gravity on the membrane is negligible.
- ii. Displacement is only in vertical direction.
- iii. Membrane is thin enough to neglect its volume and only consider its area.
- iv. Magnitude of pre-stress remains constant and mass density assumed uniform throughout the membrane.

A membrane is a planar two-dimensional pre-tensioned elastic continuum, where two space coordinates have been considered to represent points of the membrane. In the Cartesian coordinate system as shown in Fig. 1, the x – y plane represents the un-deformed configuration of the membrane. The configuration of the membrane at any time t will be represented by the field variable w(x, y, t). It was assumed in all analysis that the displacements of all points of the membrane are small and always perpendicular to the x – y plane and the thickness h of the membrane remains constant. The membrane is homogeneously stretched by a tension T, given as force per unit length and ρ as the density of the material [23-25].



**Figure 1:** An infinitesimal element of a membrane in a Cartesian coordinate system

Equation (1) represents the plain stress matrix and strain matrix. The strain is derivative of displacement matrix

$$D = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{Bmatrix} = \begin{Bmatrix} \frac{du}{dx} \\ \frac{dv}{dy} \\ \frac{du}{dy} + \frac{dv}{dx} \end{Bmatrix} \dots\dots\dots (1)$$

$$U = \{x, y\}^T \quad \sigma = \{\sigma_x, \sigma_y, \tau_{xy}\}^T \quad \epsilon = \{\epsilon_x, \epsilon_y, \gamma_{xy}\}^T \quad \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [D] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{Bmatrix} \dots\dots\dots (2)$$

$$\begin{aligned}
 x(\xi, \eta) &= \sum_{i=1}^4 (N_i x_i) & y(\xi, \eta) &= \sum_{i=1}^4 (N_i y_i) \\
 u(\xi, \eta) &= \sum_{i=1}^4 (N_i u_i) & v(\xi, \eta) &= \sum_{i=1}^4 (N_i v_i) \dots\dots\dots (3)
 \end{aligned}$$

Shape function for 4 node quadrilateral element is given in (4)

$$\begin{aligned}
 N_1 &= \frac{1}{4}(1 + \xi)(1 - \eta) & N_2 &= \frac{1}{4}(1 + \xi)(1 + \eta) \\
 N_3 &= \frac{1}{4}(1 - \xi)(1 + \eta) & N_4 &= \frac{1}{4}(1 - \xi)(1 - \eta) \dots\dots\dots (4)
 \end{aligned}$$

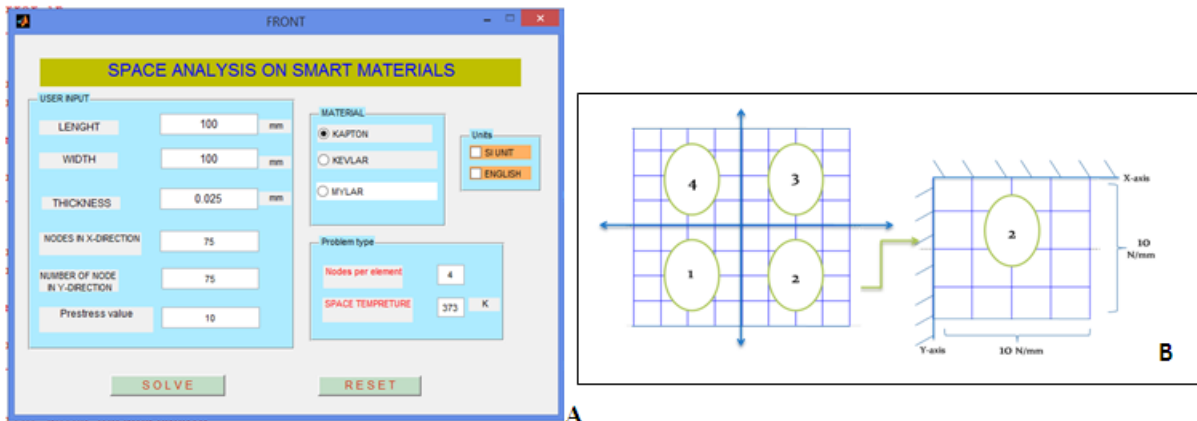
$$\{\sigma\} = [d]\{\varepsilon\} - \{\sigma_T\}$$

$$\sigma_{11T} = \sigma_{22T} = \frac{E_{YOUNG'S} \alpha_{CTE} T}{(1 - \nu)} \quad \{\sigma_T\} = \{\sigma_{11T} \quad \sigma_{22T} \quad 0\}^T \dots\dots\dots (5)$$

**Nomenclature:**

- FEA : Finite Element Analysis
- $\sigma$  : Stress
- (x, y) : Coordinates of membrane
- T : Tension for stretching in term of force per unit length
- h : Thickness of the membrane in m
- $\rho$  : Mass density of the membrane in kg/m<sup>3</sup>
- $\varepsilon$  : Strain
- $N_1$  : Shape function
- D : Plain stress or plain strain matrix
- $\sigma_T$  : Thermal stress (MPa)
- T : Temperature field (K)
- $\alpha_{CTE}$  : Coefficient of thermal expansion (1.6\*E-6)

In Fig. 2(b) the quarter element is considered for the purpose of analysis because of symmetric problem. The boundary condition shows clearly that upper and left edges are fixed and bottom and right edges are uniformly stretched. This inflatable membrane is used at satellite antenna in space. Air packed antenna, made of membrane having zero bending stiffness. As the fluid pressure increases, membrane gets uniformly stretched at the edges. So that pre stress variation is considered in analysis. When antenna roam in space at dark place temperature T= 0 k & in solar region T= 393k.



**Figure 2:** (a) Front End development in MATLAB for analysis of smart materials (b) Axisymmetric boundary condition

In matrix laboratory, front end development as shown in Fig. 2(a), is created by graphical user interface coding for user friendly program, all input data, material selection, value of properties of membrane are merged in front end development. This is connected to back end program by function calling. Here some code of meshing of plate is written to understand the syntax and formulation of FEM in MATLAB.

```

[node,element] = meshRectangularRegion(pt1,pt2,pt3,pt4,nnx,nnx,elemType)
uln = (nnx-1)*nny+1; % upper left node number
    
```

```
urn = nnx*ny; % upper right node number
lru = nny; % lower right node number
K(sctrB,sctrB) = K(sctrB,sctrB) + thk*B'*C*B*W(kk)*det(J0);
```

where variable K is used for stiffness assembly matrix B is shape function matrix and C is plane stress matrix in this code the meshRectangular region is external function which is called by main program when user press the solve button in front end development .

#### IV. Result And Discussion

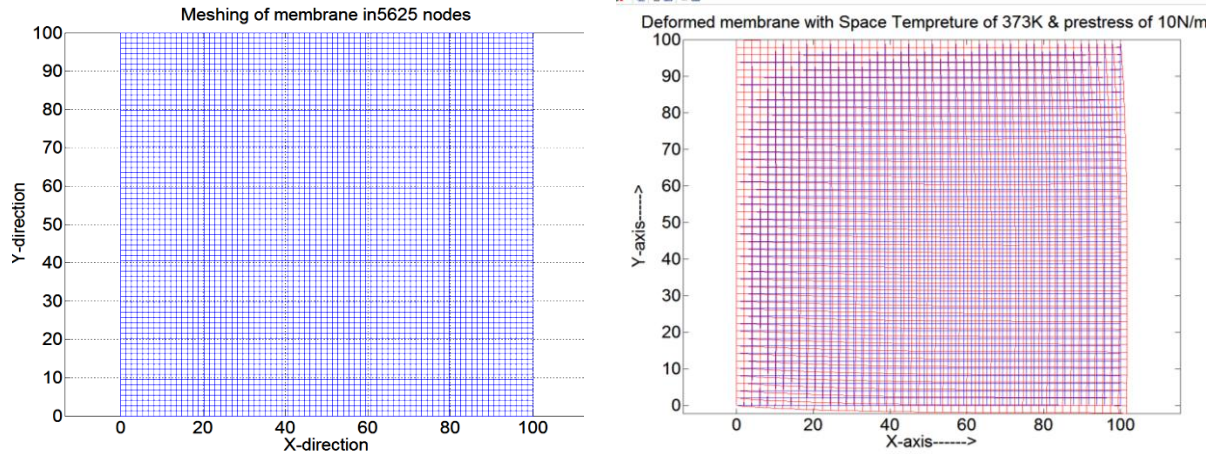


Figure 3: a) Meshing of membrane and Deformed & undeformed shape of membrane

Fig.3 shows the meshing of membrane of size 100 X 100 mm in 5625 nodes .It shows deformed and undeformed shape of the membrane with space temperature 373K and pre stress of 10N/m .Degree of Freedom

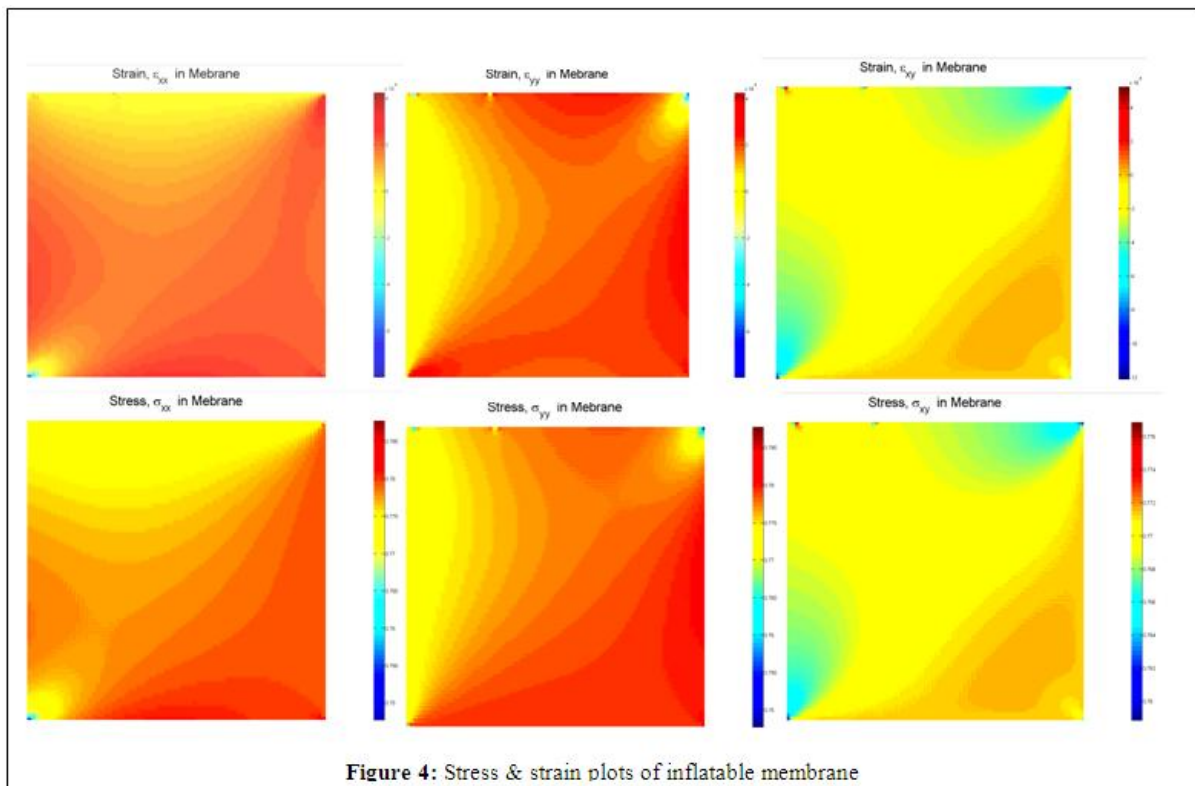


Figure 4: Stress & strain plots of inflatable membrane

is 2.The left and Upper part of the square membrane is fixed and right and bottom part of membrane is uniformly stretched by 10 N/m. The result in Fig. 4 is combination of Physical as well as Thermal Stresses.

The Fig 4 shows various stress and strain Plots of Kapton membrane in MATLAB. In Fig. 4 various stress strain plots shows the wrinkled part in it's out plane direction

The back end development means the editor window of MATLAB where entire program of huge lines are written on the basis of finite element formulation of membrane theory. Here the membrane of size 100x100 mm with quadrilateral element, meshed with size 75 is shown in Fig.3.

**Table 2:** Physical and thermal stresses developed in Kapton material at 373k

Sr. No.	Thickness in micro-m	Pre stress values				
		10N/m	20N/m	30N/m	40N/m	50N/m
1	25	13.46	19.21	24.96	30.21	36.46
2	50	10.58	13.46	16.34	19.22	22.09
3	75	9.63	11.54	13.46	15.38	17.29
4	100	9.15	10.58	12.02	13.46	14.9

TABLE 2 shows the maximum physical and thermal stresses occurred in entire kapton membrane by max\_stress (:,:,)) command which gives the highest value of stress in entire membrane. Membrane is uniformly meshed and due to symmetric problem and equal x & y directional traction forces, deformation and stresses are same in both directions. The yield point of kapton is 40Mpa at 373k and results shows that at maximum pre stress condition the values are below the yield point.

**Table 3** Physical and thermal stresses developed in Kevlar material at 373k

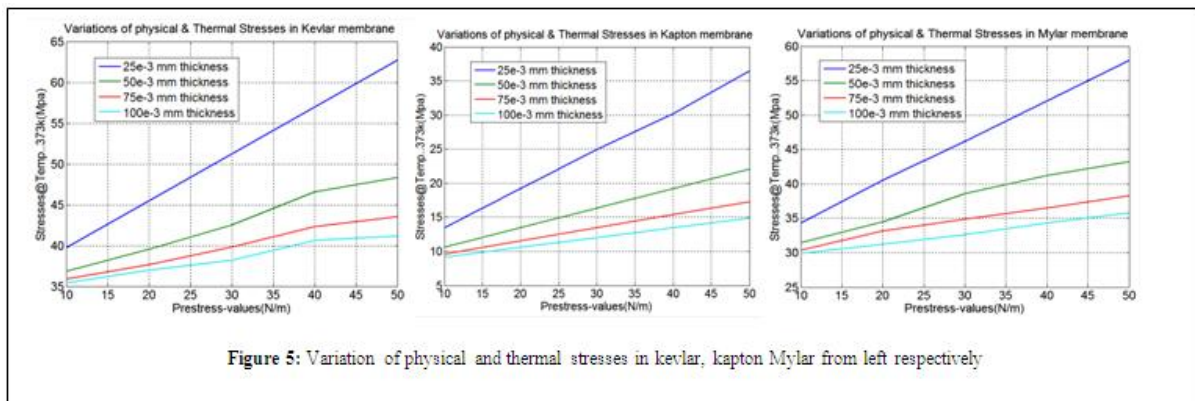
Sr. No.	Thickness in micro-m	Pre stress values				
		10N/m	20N/m	30N/m	40N/m	50N/m
1	25	39.75	45.51	51.25	57	62.75
2	50	36.87	39.56	42.56	46.56	48.37
3	75	35.91	37.65	39.86	42.36	43.58
4	100	35.43	36.98	38.23	40.65	41.18

Same boundary conditions are applied for Kevlar membrane and results are tabulated in TABLE 3. The yield point for Kevlar at 373k is 47-51 Mpa, but at the max. Pre stress value, physical & thermal stresses exceeds the value of yield. In TABLE 4, the stress values of Mylar membrane exceed 45Mpa yield stress. Among three materials, only kapton is within the range from the zero to yield point.

**Table 4** Physical and thermal stresses developed in Mylar material at 373k

Sr. No.	Thickness in micro-m	Pre stress values				
		10N/m	20N/m	30N/m	40N/m	50N/m
1	25	34.33	40.53	46.15	52.07	57.98
2	50	31.46	34.52	38.56	41.25	43.2
3	75	30.35	33.15	34.85	36.5	38.27
4	100	29.89	31.25	32.65	34.35	35.81

In Fig.5 the graph shows the pre stress value vs. stresses generated in the membrane. The graph shows, that lowest thickness value of all material posses quit linear response for the various pre stress values. As the thickness of membrane is increases the stresses reduces. Initially when thickness changes from 25 to 50 micrometer then the stress value of Kevlar reduces by 14% and kapton by 16% because of twice thickness increment of membrane. After that the thickness increases 25 micrometer step by step at each time.



**Figure 5:** Variation of physical and thermal stresses in kevlar, kapton Mylar from left respectively



## V. Conclusion:

Effect of space environment on adaptive membrane is evaluated by finite element analysis. Various stresses are estimated at different pre stress values among these kapton posses good stability in space environment and pre stress values. Kevlar and Mylar materials posses its stress values near to its yield point so it cannot be used for space reflector. In the field of engineering application, thin membrane structures with very light materials are demandable due to non flexural stiffness and optimally within structural member subjected to pre-stressed rather than bending or moments. In this paper, the static behavior of the square shaped flat thin membrane is being analyzed in terms of the physical and thermal stresses using the different types of smart materials such as Kevlar, Kapton and Mylar. Using the pre-stressed of 10 N/m to the outer edges along the plane is applied and the encaster boundary condition to the inner edges of the flat square-shaped membrane shows the symmetric variation. This analysis makes more effective to selects the smart material in the space technology.

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