

Optimal synthesis of a Path Generator Linkage using Non Conventional Approach

Mr. Monish P. Wasnik¹, Prof. M. K. Sonpimple², Prof. S. K. Undirwade³
¹M-Tech MED Priyadarshini College of Engineering, Nagpur
^{2,3}Mechanical Engineering Dept. Priyadarshini College of Engineering, Nagpur

ABSTRACT: This paper studies the solution methods of optimal synthesis of a Path Generator Linkage using Non – Conventional Approach. The method is defined by using Harmony Search Method and a common kind of goal function which is used to find the appropriate dimension and to minimize the error and find the best mechanism with accurate Solution. The possibility of extending is the advantage of this method. Unlike others we do not consider input angle as design variable, because in those cases when many precision points are available, computation will increase without having exact solution. So we divided the path to some section and find minimum error between desired points and design points. Using this method, we can easily decrease the path error and processing time.

I. INTRODUCTION

Mechanisms which compose some connected rigid members are exclusively used in the area of mechanical engineering to transfer energy from one member to another. To improve the atlas of mechanisms with a lot of curves to solve the mechanism problems. These methods are easy and fast to use but offer a low precision rate. Dimensional synthesis can be classified as motion generation, function generation and path or trajectory generation. Both graphical and analytical methods have been used for dimensional synthesis. Using precision points which are traced by a mechanism is also classified but such methods are relatively restrictive because of their low precision rates and cannot be used when we have variety of precision point's mechanism. By increasing the power of computers, numerical methods are used commonly to minimize the goal function. They used some optimization methods to optimize the goal function, the error between the points trace by the coupler and its desire trajectory. But all the solutions have a disadvantage of falling if the solutions appear in a local minimum.

In this paper the approach presented to the synthesis of mechanism deals with Harmony Memory Search Method algorithm and we can compare it with other solution

II. HARMONY MEMORY SEARCH METHOD

The HS algorithm conceptualizes a behavioral phenomenon of musicians in the improvisation process, where each musician continues to experiment and improve his or her contribution in order to search for a better state of harmony. It is first given by Geem & Kim [17]. This section describes the HS algorithm based on the heuristic algorithm that searches for a globally optimized solution.

2.1 Basic Algorithm

The procedure for a harmony search, which consists of steps 1-5.

Step 1. Initialize the optimization problem and algorithm parameters.

Step 2. Initialize the harmony memory (HM) .

Step 3. Improvise a new harmony from the HM .

Step 4. Update the harmony memory.

Step 5. Repeat steps 3 and 4 until the termination criterion is satisfied.

These steps are explained below:

Step 1: Initialize the optimization problem and algorithm parameters. First, the optimization problem is specified as follows:

Minimize $f(X)$ subjected to $x_i \in X, i=1, 2, \dots, N$

Where $f(X)$ is an objective function; X is the set of decision variables; N is the number of decision variable; X is the set of the possible range of values for each decision variable, that is $x_i^L \leq x \leq x_i^U$, and x_i^L and x_i^U are the lower and the upper bounds for each decision variables, respectively. The algorithm requires several parameters: Harmony memory size (HMS), Maximum number of

improvisations (NI) Harmony Memory Consideration Rate (HMCR), pitch adjusting rate (PAR), Bandwidth vector used in (bm).

Step 2: The HM matrix is initially filled with as many randomly generated solution vectors as the HMS, as well with the corresponding function values of each random vector, f(X). This is shown below:

Step-3 New Harmony

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_{N-1}^2 & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \dots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \dots & x_{N-1}^{HMS} & x_N^{HMS} \end{bmatrix}$$

vector, $X=(x'_1, x'_2, x'_3, \dots, \dots, x'_N)$, is

improvised based on the following three mechanisms: (1) random selection, (2) memory consideration, and (3) pitch adjustment. In the random selection, the value of each decision variable, in the New Harmony vector is randomly chosen within the value ranges with a probability of (1-HMCR). The HMCR, which varies between 0 to 1, is the rate of choosing one value from historical values stored in the HM, and (1-HMCR) is the rate randomly selecting one value from the possible range of values.

$$x'_i = \begin{cases} x'_i \in \{x'_i, \dots, \dots, x_i^{HMS}\} & \text{if } rand(-1,1) < HMCR, \text{ with Probability } HMCR \\ x'_i \in X_i & \text{otherwise, with probability } (1 - HMCR) \end{cases}$$

The value of each decision variable obtained by the memory consideration is examined to determine whether it should be pitch-adjusted. This operation uses the PAR parameter, which is the rate original pitch obtained in the memory consideration is kept with a probability of HMCR . (1-PAR). If the pitch adjustment decision for is made with the probability of PAR, x'_i is replaced with $x'_i \pm rand(-1,1) \times bw$, where bw is an arbitrary distance bandwidth for the continuous design variable, and adjustment is applied to each variable as follows:

$$x'_i = \begin{cases} x'_i \pm u(-1,1. bw) & \text{With probability } PAR \text{ (that if } rand(-1,1) < PAR \\ x'_i & \text{with probability } (1 - PAR) \end{cases}$$

Step 4. Update the HM. If the New Harmony vector is better than the worst harmony vector in the HM, based on the evaluation of the objective function value, the New Harmony vector is included in the HM, and the existing worst harmony vector is excluded from the HM.

Step 5. If the stopping criterion (or maximum number of improvisation) is satisfied, the computation is terminated. Otherwise, steps 3 and 4 are repeated.

3.FORMULATION OF WORK

3.1. COUPLER POINT COORDINATES

In the problem of four-bar linkage synthesis there is some number of precision points to be traced by the coupler point P. To trace the coupler point, the dimension of the links (a, b, c, d, L_x, L_y) is to be determined along with the input crank angle θ_2 , so that the average error between these specified precision points (P_{x_di}, P_{y_di}), (where i=1,2,...N with N as number of precision points given) and the actual points to be traced by the coupler point P gets minimized. The objective or error function can be calculated when the actual traced points (P_{x_d}, P_{y_d}) is evaluated which is traced by the coupler point P with respect to the main coordinate from X, Y as shown in Fig.3.1.

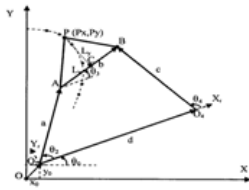


Fig.3.1 Four-Bar Linkage With ABP As Coupler Link

The position vector of the coupler point P reference frame X_r, Y_r can be expressed as a vector equation:

$$r_p = a + L_x + L_y$$

This can be represented in its components according to:

$$P_{x_r} = a \cos \theta_2 + L_x \cos \theta_3 + L_y (-\sin \theta_3)$$

$$P_{y_r} = a \sin \theta_2 + L_x \sin \theta_3 + L_y (\cos \theta_3)$$

Here, for calculation the coupler point coordinates (P_x, P_y) , we have to first compute the coupler link angle θ_3 using the following vector loop equation for the four-bar linkage:

$$a \vec{b} + \vec{c} + d \vec{0}$$

This equation also can be expressed in its components with respect to relative coordinates:

$$a \cos \theta_2 + b \cos \theta_3 - c \cos \theta_4 - d = 0$$

$$a \sin \theta_2 + b \sin \theta_3 - c \sin \theta_4 = 0$$

For this equation following two solutions are obtained:

$$\theta_3^1 = 2 \tan^{-1} \left(\frac{-E + \sqrt{E^2 - 4DF}}{2D} \right)$$

$$\theta_3^2 = 2 \tan^{-1} \left(\frac{-E - \sqrt{E^2 - 4DF}}{2D} \right)$$

where $E = -2 \sin \theta_2$ $D = \cos \theta_2 - k_1 + k_4 \cos \theta_2 + k_5$

$F = k_1 + (k_4 - 1) \cos \theta_2 + k_5$

These solutions may be (i) real and equal (ii) real and unequal and (iii) complex conjugates. If the discriminant $E^2 - 4DF$ is negative, then solution is complex conjugate, which simply means that the link lengths chosen are not capable of connection for the chosen value of the input angle θ_2 . This can occur either when the link lengths are completely incapable of connection in any position. Except this there are always two values of θ_3 corresponding to any one value of θ_2 . These are called, (i) crossed configuration (plus solution) and (ii) Open configuration of the linkage (minus solution) and also known as the two circuits of the linkage. The other methods such as Newton-Raphson solution technique can also be used to get approximate solution for θ_3 . The position of coupler P, with respect to world coordinate system XOY is finally defined by:

$$P_x = x_0 + P_{x_r} \cos \theta_0 - P_{y_r} \sin \theta_0$$

$$P_y = y_0 + P_{x_r} \sin \theta_0 - P_{y_r} \cos \theta_0$$

3.2 position Error as Objective Function

The objective function is usually used to determine the optimal link lengths and the coupler link geometry. In path synthesis problems, this part is the sum squares which computes the position error of the distance between each calculated precision point P_{xi}, P_{yi} and the desired points P_{xdi}, P_{ydi} which are the target points indicated by the designer. This is written as:

$$f(X) = \sum_{i=1}^N [(P_{xdi} - P_{xi})^2 + (P_{ydi} - P_{yi})^2]$$

Where X is set of variables to be obtained by minimizing this function. Some authors have also considered additional objective functions such as the deviation of minimum and maximum transmission angles α_{\min} and α_{\max} from 90° , for all the set of initial solutions considered.

4. The constraints of the linkage

The synthesis of the four-bar mechanism greatly depends upon the choice of the objective function and the equality or the inequality constraints which is imposed on the solution to get the optimal dimensions. Generally the objective function is minimized under certain conditions so that the solution is satisfied by a set of the given constraints. The bounds for variables considered in the analysis are treated as one set

of constraints, while the other constraints include: Grashof condition, input link order constraint and the transmission angle constraint.

4.1. Grashof criterion

For Grashof criterion, it is required that one of the links of mechanism, should revolve fully by 360° angle. There are three possible Grashof linkages for a four-bar crank chain: (a) Two crank-rocker mechanisms (adjacent link to shortest is fixed) (b) One double crank mechanism (shortest link is fixed) and (c) One double rocker mechanism (opposite to shortest link is fixed). Of all these, in the present task, only crank-rocker mechanism configuration is considered. Here, the input link of the four-bar mechanism to be crank. Grashof criterion states that the sum (L_s+L_l) of the shortest and the longest links must be lesser than the sum (L_a+L_b) of the rest two links. That is:

$$(L_s+L_l) <= L_a+L_b$$

$$\text{Or } 2(L_s+L_l) <= a+b+c+d$$

$$\text{Or } g_1 = 2(L_s+L_l) <= (a+b+c+d) - 1 <= 0$$

In the present work violation is defined as follows:

$$\text{Grashof's } = 1 \text{ if } g_1 > 0$$

$$\text{Or } = 0 \text{ if } g_1 <= 0$$

4.2 Input link angle order constraint

Usually a large combination of the mechanisms exists that generates the coupler curves passing through the desired points, but those solutions may not satisfy the desired order. To ensure that the final solution honors the desired order, testing for any order violation is imposed. This is achieved by requiring that the direction of rotation of the crank as defined by the sign of its angular increments $\Delta\theta_2^i = (\theta_2^i - \theta_2^{i-1})$, between the two position i and $i-1$, where $i = 3, 4, 5, \dots, N$, have same direction as that between the 1st and the 2nd positions

$(\theta_2^2 - \theta_2^1)$. That checks the following:

$$\text{Is } \text{sign}(\Delta\theta_0) == \text{sign}(\theta_2^2 - \theta_2^1) \text{ for all } i = 3 \text{ to } N$$

$$\text{Where } \text{sign}(Z) = 1 \text{ if } Z >= 0$$

$$= -1 \text{ if } Z < 0$$

If this condition is not satisfied the solution is rejected.

4.3 Transmission Angle Constraint

For a crank-rocker mechanism generally the best results the designers recognize when the transmission angle is close to 90 degree as much as possible during entire rotation of the crank. Alternatively, the transmission angle during entire rotation of crank should lie between the minimum and maximum values. This can be written as one of the constraints as follows. First of all, the expressions for maximum and minimum transmission angles for crank-rocker linkage are defined.

$$\mu_{max} = \cos^{-1} \frac{b^2 - (d+a)^2 + c^2}{2bc}$$

$$\mu_{min} = \cos^{-1} \left[\frac{b^2 - (d-a)^2 + c^2}{2bc} \right]$$

The actual value of transmission angle at any crank angle θ_2^i is given by:

$$\mu = \cos^{-1} \left[\frac{b^2 - a^2 - d^2 + c^2 + 2ad \cos \theta_2^i}{2bc} \right]$$

The condition to be satisfied is: $\mu_{min} \leq \mu \leq \mu_{max}$

The constraint given by above equations are handled by penalty method. That is the non-dimensional constraint deviation is directly added to the objective function for minimization.

For example, constraint eq. if not satisfied, the penalty term is given as follows:

$$\text{Trans} = \sum_{i=1}^N (1 - \text{Trans min})(\mu_i - \mu_{min})^2 + (1 - \text{Trans max})(\mu_i - \mu_{max})^2$$

Where

$$\text{Transmin} = \text{sign}(b^2 + c^2 - (d - a)^2 - 2bc \cos \mu_{min})$$

$$\text{Transmax} = \text{sign}(2bc \cos\mu_{\max} - b^2 + c^2 + (a + d)^2)$$

4.4. Variable Bounds

All variables considered in the design vector should be defined within prespecified minimum and maximum values. Often, this depends on the type of problem. For example, if we have 19 variables in a 10 point optimization problem, all the variables may have different values of minimum and maximum values. Generally, in non-conventional optimization techniques starting with set of initial vectors, this constraint is handled at the beginning itself, while defining the random variable values. That is we use the following simple generation rule:

$$X = X_{\min} + \text{rand} (X_{\max} - X_{\min})$$

Where rand is a random number generator between 0 and 1.

4.5 Overall optimization problem

The objective function is the sum of the error function and the penalties assessed to violation the constraints as follows:

$$F(k) = f(X) + W1 \square \text{Grashof} + W2 \square \text{Tran},$$

Whereas W1 is the weighting factor of the Grashof’s criteria and W2 is the weighting factor of the Transmission angle constraints .these additional terms acts as scaling factors to fix the order of magnitude of the different variables present in the problem or the objective function.

III. RESULTS AND DISCUSSION

5. Path Synthesis:

The efficiency and accuracy of the proposed are verified by studying three method cases (for more than five target points) from the literature. Three cases are explained :

- (1) 6 points (15 variables)
- (2) 10 points (19 variables)

Different parameters are used. It includes HS algorithm .

Number of variable NVAR=15, maximum no of iteration Maxitr =10000, harmony memory size HMS=30, harmony memory consideration rate HMCR=0.95, maximum pitch adjustment rate PARmax=0.9, minimum pitch adjustment rate PARmin=0.4, bandwidth minimum = 0.0001, bandwidth maximum=1.

5.1 Six Points Path Generation and 15 design variables:

The first case is a path synthesized problem with given six target points arranged in a vertical line without prescribed timing.

Design variables are: $X = [a, b, c, d, ly, lx, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_0, x_0, y_0]$ Target Points: [(20,20),(20,25),(20,30),(20,35),(20,40),(20,45)]

Limits of the variable: a, b, c, d $\in [5, 60]$

$$Lx, ly, x_0, y_0 \in [-60, 60]$$

$$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6 \in [0, 2\pi]$$

The synthesized geometric parameters and the corresponding values of the precision points (Pxd, Pyd) and the traced points by the coupler point (Px,Py) and the difference between them are shown in table 1 and table 2 respectively . Although the constraint of the sequence of the input angles during the evolution is ignored in this case .The accuracy of the solution in case 1 has been remarkably improved using the present method. Fig(5.2) shows the convergence graph of HS algorithm .Fig(5.3) shows the six target points and the coupler curve obtained using the harmony memory search method with NVAR=15,Maxitr=10000, HMS=30,HMCR=0.95,PARmax=0.9,PARmin=0.4,bwmin=0.0001,bwmax=1. The out-put of the variables values as shown below.

Table 5.1. Synthesized Results For Six Target Points Problem

a	b	c	d	ly	Lx	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_0	x0	y0
10.565		26.477	33.613	-	15.914	4.698	5.166	5.701	6.108	0.199	0.672	0.769	25.964	18.057
2	46.0859	6	7	8.234	4	8	6	1	5	7	9	1	3	7

Table 5.2. Percentage Error Of The Coupler Link And The Precision Points

px	pxd	px-pxd	(px-pxd) ²	PY	PYD	(PY-PYD) ²	%Error in x	%Error in y
18.7297		-		20.9235		0.9235	0.85293	
3	20	1.2703	1.61359	5	20	5	15.1392	39.951

20.9740 2	25	-4.026	16.2085	24.3770 8	25	- 0.6229	0.38803 6	47.9811	26.946
21.3923 8	20	1.3923 8	1.93872	29.8954 5	30	- 0.1046	0.01093 1	16.594	4.524
20.5503	20	0.5503	0.30283	34.9462 7	35	- 0.0537	0.00288 7	6.5583	2.3230
19.8659 4	20	- 0.1341	0.01797	39.9980 9	40	- 0.0019	3.65E- 06	1.5981	0.0821
18.9822	20	- 1.0178	1.03592	44.3950 4	45	-0.605	0.36598 1	12.13	26.17

Table 5.3. Actual Points Which Is Traced By The Coupler Link And The Precision Points

Px	Pxd	Py	Pyd	Px
18.72973	20	20.92355	20	18.72973
20.97402	25	24.37708	25	20.97402
21.39238	20	29.89545	30	21.39238
20.5503	20	34.94627	35	20.5503
19.86594	20	39.99809	40	19.86594
18.9822	20	44.39504	45	18.9822
Fitness Function Value = 22.74				

Ten Points Path Generation and 19 design variables:

Design variables are:

$X = [a, b, c, d, l_y, l_x, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}, x_0, y_0]$

Target Points: [(20,10), (17.66,15.142), (11.736,17.878), (5,16.928), (0.60307,12.736), (0.60307, 7.2638), (5, 3.0718), (11.736, 2.1215), (17.66, 4.8577), (20,10)]

Limits of the variable: $a, b, c, d \in [5,80]$

$l_x, l_y, x_0, y_0 \in [-80,80]$

$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \dots, \theta_{10} \in [0,2\pi]$

The synthesized geometric parameters and the corresponding values of the precision points (Pxd, Pyd) and the traced points by the coupler point (Px,Py) and the difference between them are shown in table 3 and table 4 respectively. Although the constraint of the sequence of the input angles during the evolution is ignored in this case. The accuracy of the solution in case 1 has been remarkably improved using the present method. Fig(5.5) shows the convergence graph of HS algorithm, fig (5.6) shows the ten target points and the coupler curve obtained using the harmony memory search method with NVAR=18, Maxitr=10000, HMS=30, HMCR=0.95, PARmax=0.9, PARmin=0.4, bwmin=0.0001, bwmax=1.

Table 5.4 Synthesized Results For Ten Target Points Problem

Table 5.5. Actual Points Which Is Traced By The Coupler Points And The Precision Points

Px	Pxd	Py	Pyd
19.356	20	10.1177	10
17.676	17.66	16.0281	15.14
11.538	11.736	19.493	17.87
5.0531	5	17.9153	16.92
1.254	0.603	12.9509	12.73
0.8418	0.603	8.62267	7.26
4.5868	5	2.6323	3.07
Fitness Function Value = 11.43			

IV. CONCLUSIONS

Even this work has concentrated on path synthesis part with some important constraints, some more constraints like mechanical advantage of the linkage, and flexibility effects can be also considered to get the accuracy. Also as in hybrid synthesis approach, the same linkage may be adopted both for path synthesis applications as well as motion synthesis applications. The objective function should be modified so as to

get a different optimum link dimensions. Finally fabrication of the proto-type of this linkage may be done to know the difference between theoretically obtained coupler coordinates and actual values achieved.

V. ACKNOWLEDGEMENT

I avail this opportunity to extend my hearty indebtedness to my guide PROF.M.K. Sonpimple, Prof. S.K. Undirwade Mechanical Engineering Department, for his valuable guidance, constant encouragement and kind help at different stages for the execution of this dissertation work.

I also express my sincere gratitude to him for extending their help in completing this project.

I take this opportunity to express my sincere thanks to my project guide and co-guide for co-operation and to reach a satisfactory conclusion

REFERENCES

Optimal Synthesis of Planar Mechanisms with PSO Algorithms

Author: 1)S.Alinia 2)M.Ghadimi 3) H.D Kaliji

Journal: World Applied sciences Journal18 (2):268-273, 2012

Objective: This Paper studies the solution methods of optimal synthesis of planer mechanism. The method defined by using PSO Technique and a common kind of goal function which is used to find the appropriate dimensions and to minimize the error and find the best mechanism with accurate solution.

Optimization of Watt's Six-Bar Linkage to Generates Straight and Parallel Leg Motion

Author:Hamid Mehdigholi and Saeed Akbarnejad

Journal: Journal of Humanoids, Vol.1, No.1, (2008), ISSN 1006-7290, pp.11-16

Objective: This paper considers optimal synthesis of a special type of four-bar linkages. Combination of this optimal four-bar linkage with on of its cognates' and elimination of two redundant cognates will result in a Watts's six-bar mechanism, which generates straight and parallel motion. This mechanism can be utilized for legged machines. The advantage of this mechanism is that the leg remains straight during it's contact period and because of its parallel motion, the legs can be wide as desired to increase contact area and decrease the number of legs required to keep body's stability statically and dynamically.

Optimal Synthesis of Crank Rocker Mechanism for Point to Point Path Generation

Author:1)Subhash N Waghmare, 2) Roshan B. Mendhule, 3) M.K.Sonpimple

Journal:International Journal of Engineering Inventions ISSN:2278-7461,Vol 1,2Sept.(2012), PP:47-55

Objective: The concept of orientation structure error of the fixed link and present a new optimal synthesis method of crank rocker linkages for the path generation. The orientation structure error of the fixed link effectively reflects the overall difference between the desired and generated path. In this paper Genetic Algorithm is used for the formulation of work and the desired output is made by the same.

Kinematics Design of a Planer Parallel Controllable Mechanism Based on Particle Swarm Optimization Algorithm.

Author:1) Ke Zhang 2) Shengze Wang

Journal: Journal of Computers, Vol.6, No.6, Jue 2011

Objective: Planer parallel controable mechanism, which is a combination of two types of motor and mechanism, has better flexible transmission behavior. In this paper, the kinematics analysis for a planer parallel controllable five-bar mechanism is introduced. In order to improve kinematic performance of the controllable mechanism an optimization design for the mechanism is performed with reference to kinematics objective function. A hybrid optimization algorithm which combines Particle Swarm Optimization (PSO) with Matlab Optimization tool box is proposed to solve the optimal design problem with constrained condition.

Optimal synthesis of mechanisms with genetic algorithms

Author: 1) J.A. Cabrera ,2) A. Simon, 3)M. Prado

Journal: Mechanism and Machine Theory 37 (2002) 1165–1177

Objective: This paper deals with solution methods of optimal synthesis of planar mechanisms. A searching procedure is defined which applies genetic algorithms based on evolutionary techniques and the type of goal function. Problems of synthesis of four-bar planar mechanisms are used to test the method, showing that solutions are accurate and valid for all cases. The possibility of extending the method to other mechanism type is outlined. The main advantages of the method are its simplicity of implementation and its fast convergence to optimal solution, with no need of deep knowledge of the searching space.