IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE)
e-ISSN: 2278-1684, p-ISSN: 2320-334X
PP 01-07
www.iosrjournals.org

A Study on Optimized Design of a Spur Gear Reduction Unit

Faisal.S. Hussain[1], Syed Mohiuddin[2], Sajid Siddiqui[3], R.N. Dehankar[4]
[1]*HOD, Department of Mechanical Engineering, Priyadarshini - Polytechnic, Nagpur-440016*
[2, 3, 4]*Assistant Professor, Anjuman College of Engg. and Technology,Nagpur-440001*
Email. [1]fshppn@gmail.com , [2]syedmohi13@gmail.com , [3]sajid_150278@yahoo.com

ABSTRACT : *The field of gear design is an extremely broad and complex area, and a complete coverage in any research work is not possible. In this work only parallel axis spur gear reduction unit which is the type, probably encountered most often in general practice, has been considered. A review of relevant literature in the areas of optimized design of spur gear indicates that compact design of spur gears involves a complicated algebraic analysis. A series of iterations is normally required to arrive at a practical combination of pinion teeth and module from their theoretical values. The present work describes the development of such a design methodology and diagnostic tool for determining the modes of failure for spur gear and also the causes of these failures have been studied. The ray diagram is also considered for finding out the minimum diameter and maximum transmission range. The focus is on developing a design space which is based on module and pinion teeth by using a simple logical statement in computer software. This is a much simplified approach for obtaining practical values of the module and pinion teeth for an optimum minimum centre distance between the two transmission shafts. Attention has been devoted to determine the exact mode of failure which dictated the design at the optimum conditions corresponding to the minimum centre distance for the design of gear reduction unit with minimum dimensions.*

Keywords: *Spur Gear Reduction Unit, Optimal Design, Structural Diagram, Ray Diagram, Modes of Failure.*

I. INTRODUCTION

For transmission of power, gears have been in use since the dawn of civilization. Gears in the early stages were in the form of cylindrical discs having surface irregularities . These primitive transmission tools were adequate to meet the operational needs of those days. Ancient engineers were aware of the desired performance parameters such as a gear ratio, center distance and available power source (water current, wind, horse power) and used them to define the gear parameters (diameters, number and shape of the teeth). They then manufactured gears using available materials, technology and tools.From these facts it is seen that it is necessary to vary ratio of the speed of rotation of the vehicle wheels (and thereby the vehicle speed) to the speed of rotation of the engine.

II. PRESENT WORK

The present work is an attempt for the study of optimized design of spur gear reduction unit. The transmission system of a machine tool is the main component, which consists of gears and shafts. The gears are of different diameters and number of teeth but the module of the two mating gears should be same. The study on the optimized design of reduction unit is very important as new machines, which are compact and operate at very high speed, require gears of minimum volume with same fatigue strengths and same reliability. Thus following study has been conducted with an objective to incorporate various parameters like bending, pitting and scoring mode of failures of gears to design an optimized gear pair.

1. This study of spur gear reduction unit has been carried out to find out the optimized design of the gearbox for minimum distance between transmission shafts for different gear pairs.
2. The ray diagrams have been incorporated to make gear design more feasible with respect to the transmission ratio and number of teeth used in gearbox.
3. In this present study of the optimized design, the various aspects of the design of gears and various modes of failures have also been taken into consideration.

III. PROBLEM DEFINITION

The optimization function can be used to obtain the best performance of a set of gears based on the boundary conditions. One of the most common applications of this function is to optimise a set of gears to a given centre distance for design strength.The objective of the present work is to develop a methodology from

the gear geometry relationships and to study optimized design of spur gear reduction set with the help of ray diagrams. The design objectives chosen are:
To minimize the volume of the reduction unit i.e. to minimize the product of pinion number of teeth and module for a given gear ratio.
To diagnose the mode of failure in bending and surface fatigue.
To incorporate ray diagram for optimized design of spur gear reduction unit.
This approach is dependent on design space based on module. The present work has been started by developing ray diagram to find out the best transmission ratio and speeds on the different shafts of the gearbox. Then both of these parameters are used as input values of the design space generation program. Various researchers had worked for the designing of the reduction unit, from which they concluded that design is based on the fatigue failures of the gears. The main aim of the present work is to determine the optimized design of the spur gear reduction unit by utilizing the ray diagrams.

IV. ANALYTICAL FORMULATION OF THE DESIGN SPACE EQUATIONS

The application of optimization techniques in the designing external spur gears requires formulation of an objective function and the constraints inequality equation. The pinion and gear addenda are directly proportional to module for standard tooth profiles, and the contact ratio and roll angles are functions only of the number of pinion teeth, the gear ratio, the pressure angle and the addendum ratios. The objective of the development model of the present problem is to minimize the pinion diameter subjected to the requirement, that,the gear set does not fail in bending, there should not be any involute interference and the contact stresses do not exceed certain limits during

3. RAY DIAGRAM AND MODE OF FAILURES

3.1 STRUCTURAL DIAGRAM

It is a graphical tool which is used to find the range ratio of transmission groups. These structural diagrams are drawn from the structural formulas. So the structural diagrams give information about:The number of shafts in the gear box,The number of gears on each shaft,The order of changing transmissions in individual groups to get the desired spindle speed, and,The transmission range and characteristics of each group

3.2 RAY DIAGRAM

The structural diagram only depicts the range ratio of transmission groups but gives no information about transmission ratios. So it is necessary to plot the speed chart to determine the transmission ratio. The line joining points of adjacent shafts in a speed chart depict the transmission ratios. If:
1. The line is horizontal, it corresponds to transmission ratio i=1, i.e. no speed change.
2. The line is inclined upward, it depicts i>1, i.e., speed increase.
3. The line is inclined downward, it depicts i<1, i.e., speed reduction.
While plotting the speed chart it is desirable to have the minimum transmission ratio, i.e. maximum speed reduction.

3.3 MODES OF FAILURES

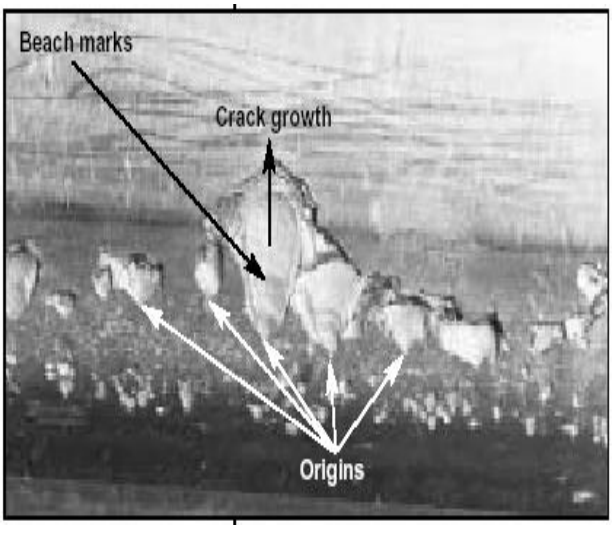


Figure.3.1: Bending fatigue fracture of gear teeth **Figure.3.2: Surface fatigue fracture of gear teeth**

The four most common failure modes are bending fatigue, contact fatigue, wear, and scuffing.

The most commonly used stress ratio is R, the ratio of the minimum stress to the maximum stress (S_{min}/S_{max}).

1. If the stresses are fully reversed, then R = -1.
2. If the stresses are partially reversed, R = a negative number less than 1.
3. If the stress is cycled between a maximum stress and no load, R = zero.
4. If the stress is cycled between two tensile stresses, R = a positive number less than 1.

Variations in the stress ratios can significantly affect fatigue life as shown in figure 3.1and 3.2. The presence of a mean stress component has a substantial effect on fatigue failure. When a tensile mean stress is added to the alternating stresses, a component will fail at lower alternating stress than it does under a fully reversed stress. Figure3.3 shows a hypothetical two-dimensional nonlinear design space formed in m- N1 plane where the constraint surfaces, are the inequality constraints $gj(C) \leq 0$.

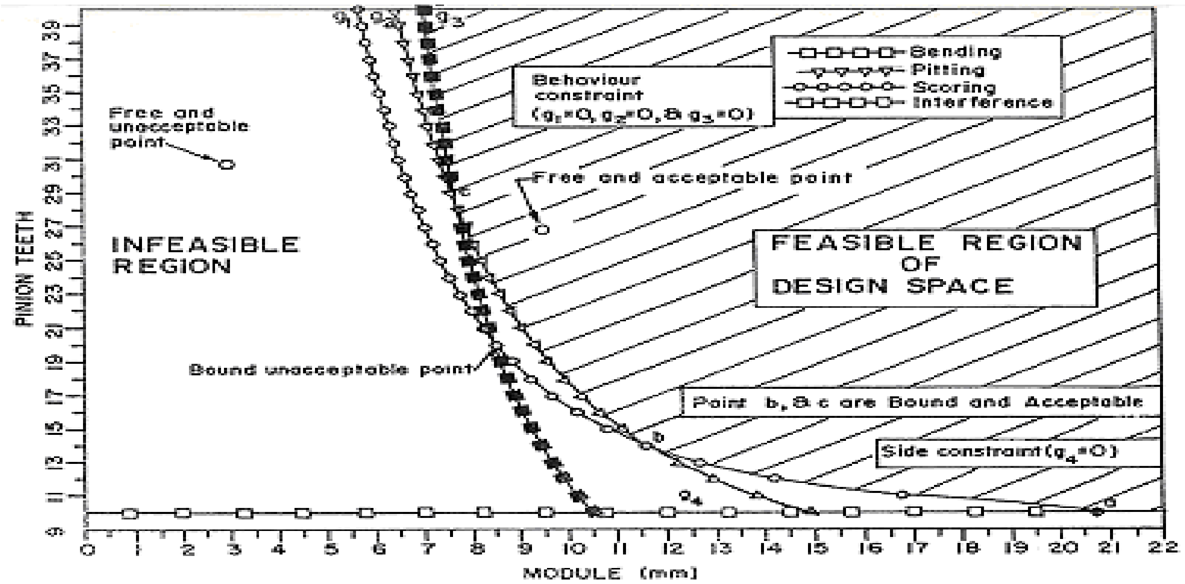


Figure3.3: Design space for compact spur gear

3.4 DESIGN METHODOLOGY

The strategies are developed in the design methodology to execute the following two different tasks:

1.The centre distance is minimized, and the corresponding feasible pinion teeth number and standard module size are fixed.

2.The modes of failure are diagnosed and the dominant ode of failure is located.

The input parameters are divided into two groups, one, is defined by the requirement of the problem, and the second one is defined by the designer to obtain the final results to the best of accuracy and quality according to the required design.

These concepts have been implemented in software developed for the design of a spur gear pair. The flow chart of this computer program has been outlined. The change of mode of failure with respect to the pinion number of teeth and module can easily be observed in a design space.

V. RESULTS AND DISCUSSION

The structural diagram only depicts the range ratio whereas with the help of ray diagram the transmission ratio of all transmissions and the rpm values of gear box shafts can be determined.

To illustrate the design procedure for optimal gear pair, the following example has been considered. Here the gear box having twelve speed steps, that is z=12, has been realized in three stages that is u=3. The maximum input speed (n_{max}) has been taken as 1000 rpm and geometric progression ratio (Φ) has been taken as 1.41. The motor speed (n_m) is 1440 rpm. The twelve speed steps may be distributed in three stages as discussed below:

Possible speed steps arrangements for z=12.

(a) $z = p_1(X_1)p_2(X_2)p_3(X_3)$; $X_1 = 1, X_2 = 2, X_3 = p_1p_2 = 6$
$z =$ 2(1) 3(2) 2(6)
(b) $z = p_1(X_1)p_2(X_3)p_3(X_2)$; $X_1 = 1, X_2 = p_1 = 2, X_3 = p_1p_3 = 4$
$z =$ 2(1) 3(4) 2(2)
(c) $z = p_1(X_2)p_2(X_1)p_3(X_3)$; $X_1 = 1, X_2 = p_2 = 3, X_3 = p_2p_1 = 6$
$z =$ 2(3) 3(1) 2(6)
(d) $z = p_1(X_3)p_2(X_1)p_3(X_2)$; $X_1 = 1, X_2 = p_2 = 3, X_3 = p_2p_3 = 6$
$z =$ 2(6) 3(1) 2(3)
(e) $z = p_1(X_2)p_2(X_3)p_3(X_1)$; $X_1 = 1, X_2 = p_3 = 2, X_3 = p_3p_1 = 4$
$z =$ 2(2) 3(4) 2(1)
(f) $z = p_1(X_3)p_2(X_2)p_3(X_1)$; $X_1 = 1, X_2 = p_3 = 2, X_3 = p_3p_2 = 6$
$z =$ 2(6) 3(2) 2(1)

Let us now analyze structural formulae (a) and (e) that are drawn above.

Formula (a) **Between shafts I and II**, $\frac{i_{max}}{i_{min}} = \phi^{(p_1-1)X_1} = \phi^{(2-1)1} = \phi$; **Between shafts II and III**, $\frac{i_{max}}{i_{min}} = \phi^{(p2-1)X2} = \phi^{(3-1)2} = \phi^4$; **Between shafts III and IV**, $\frac{i_{max}}{i_{min}} = \phi^{(p_3-1)X3} = \phi^{(2-1)6} = \phi^6$

Hence maximum transmission range in the speed box is $\phi^{X_{max}} = \phi^6$

Formula (e)**Between shafts I and II**, $\frac{i_{max}}{i_{min}} = \phi^{(p_1-1)X2} = \phi^{(2-1)2} = \phi^2$; **Between shafts II and III**, $\frac{i_{max}}{i_{min}} = \phi^{(p2-1)X3} = \phi^{(3-1)4} = \phi^8$; **Between shafts III and IV,** $\frac{i_{max}}{i_{min}} = \phi^{(p3-1)X_1} = \phi^{(2-1)1} = \phi$

Hence maximum transmission range in the speed box is $\phi^{X_{max}} = \phi^8$. Similarly the remaining four structural diagrams can also be analyzed. The value of Φ should be less that 1.41 so all six formulae qualify for selection as far as consideration of factor (A) has been concerned.Then all formulae has bee analyzed by considering factor (B). A comparison of the six structural formulae reveals that (a) is better than the rest because if satisfy the condition $X_1 < X_2 < X_3$.Let us now plot the speed chart by considering the first structural formula i.e.

(a) $z = p_1(X_1)p_2(X_2)p_3(X_3)$; $X_1 = 1, X_2 = 2, X_3 = p_1p_2 = 6$
$z =$ 2(1) 3(2) 2(6) The transmission ratios that provide the two new speed values must lie in the followrange $i_{max} = 2$; $i_{min} = \frac{1}{4}$

Table 4.1Various parameters obtained from Ray Diagram

Shaft No.	Pinion No.	Speed	Gear Ratio	Torque
I	1.1	1000	1.5	600.5 N-m
	1.2	1000	2	600.5 N-m
II	2.1	710	1	600.5 N-m
	2.2	710	2	600.5 N-m
	2.3	710	4	600.5 N-m

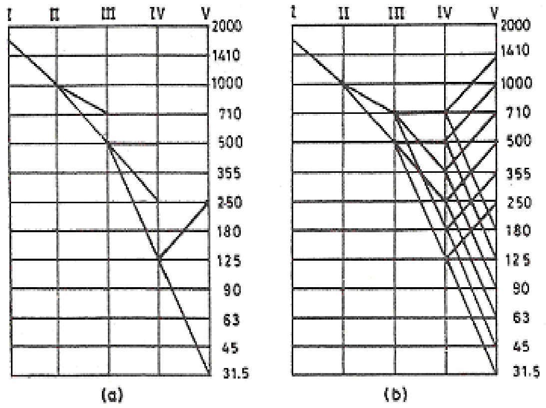


Figure 4.1: (a) Ray diagram (b) Speed chart

	2.4	**500**	**1**	**600.5 N-m**
	2.5	**500**	**2**	**600.5 N-m**
	2.6	**500**	**4**	**600.5 N-m**
III	**3.1**	**710**	**4**	**600.5 N-m**
	3.2	**500**	**4**	**600.5 N-m**
	3.3	**355**	**4**	**600.5 N-m**
	3.4	**250**	**4**	**600.5 N-m**
	3.5	**180**	**4**	**600.5 N-m**
	3.6	**125**	**4**	**600.5 N-m**
IV	**4.1**	**1410**	**2**	**600.5 N-m**
	4.2	**1000**	**2**	**600.5 N-m**
	4.3	**710**	**2**	**600.5 N-m**
	4.4	**500**	**2**	**600.5 N-m**
	4.5	**355**	**2**	**600.5 N-m**
	4.6	**250**	**2**	**600.5 N-m**

VI. VALIDATION OF THE PRESENT APPROACH

To illustrate the design technique, an example is considered for the comparison of the present work with the work done previously. The example is selected from a paper presented by Kader et al.

Table4.2 Comparision of work

Savage et al		**Present Work**		
N_1	C_F	N_1	mst	C_F
32	152	18	3	117

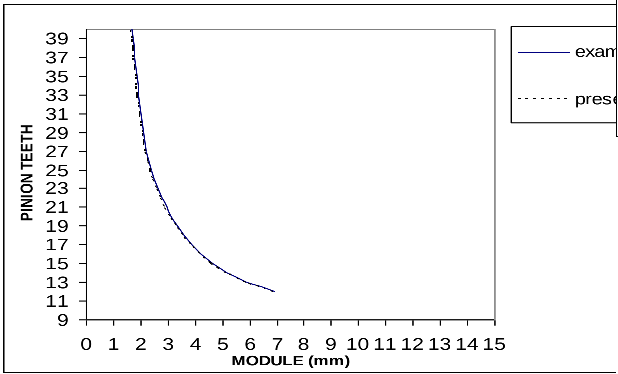
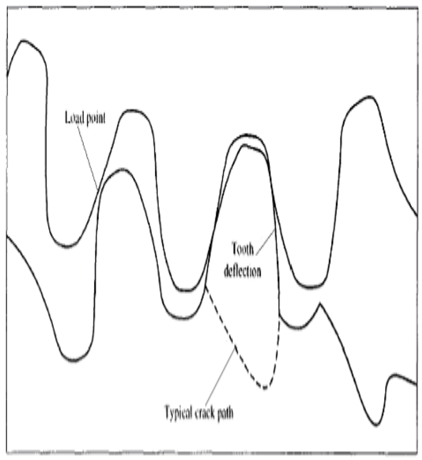
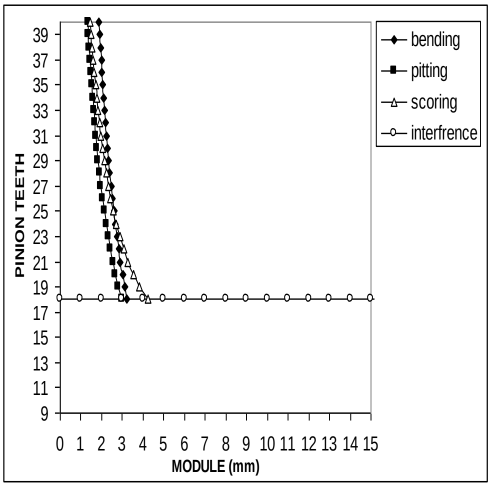


Figure 4.2 Comparison between m_{max} curves, Torque=600.5(N-m), n1=1000, m_G =1.5



Cf- Surface Condition Factor
mst-Standard module
N-Number of teeth

The optimal design is calculated for 25 pressure angle, pinion torque 113000 N-mm surface strength 1380 N/mm^2, bending strength of 414 N/mm^{2,} elastic modulus if 205000 N/mm^{2,} Poisson's ratio of 0.25, external mesh and standard teeth.Now by considering same data, a comparison is done with the results developed by Savage et al. the best solution in the table below. The optimal values of number of teeth ($N_{1)}$ and optimal centre distance (C_F) are shown. It is concluded that the best solution is for a standard module 3 mm which gives the minimum centre distance of 117 mm. The difference in the optimum centre distance values in the above examples may be due to the difference in the values of conditional factors 'K' and 'C' chosen in the present work and the referred work. In these figures different graphs and tables are developed for different torques and different pressure angles. Figures 4.3 give design space for medium torque values of 100.5 N-m with different pressure angles i.e. 20 and 25. and Figure 4.4 shows the design space for a high torque value for same conditions.



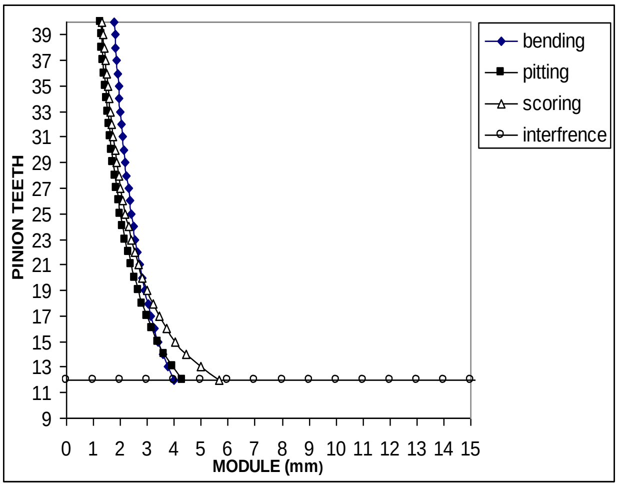


Figure 4.3and 4.4 : Feasible design space, Torque=100.5(N-m), n1=1120, m_G =2, $\Phi = 20^\circ$ and $\Phi = 25^\circ$

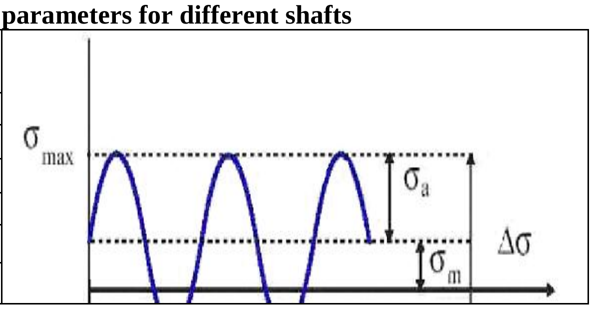
VII. CONCLUSIONS AND FUTURE WORK

Based on the present studies the following conclusions are drawn:

1. This study of spur gear reduction unit can be carried out to find out the optimized design of the gearbox for compact machine tools.
2. In this present study of the optimized design, the various aspects of the design of gears and various modes of failures are taken into consideration.
3. The ray diagrams are incorporated to make the design more feasible with respect to the transmission ratio and number of teeth used in gearbox.
4. The mode of failure curve in a design space shifts quite appreciably as torque increases. It is further seen that the design space which included the mode of failures curves is not showing any change in the behavioral pattern when the pressure angle is changed from 20° to 25°, other things remaining the same.
5. It is observed that for higher number of pinion teeth, module is dictated by the bending mode of failure and a gear thus could be sized upon by a module value obtained from Lewis equation itself; for gears with low number of teeth the scoring mode of failure dominates and for medium number of pinion teeth the pitting mode of failure, by and large, dictated the optimum design.
6. As is obvious the optimal feasible centre distance for 25° pressure angle spur gear would be lower that that for 20° pressure angle case under similar conditions, due to basic involute geometry of the tooth profile system.
7. It is further observed that large pressure angle gears have smaller value of pinion teeth and larger values of standard modules corresponding to optimal feasible centre distance in every case. This clearly indicates that the lower number of teeth rather than the higher module is dictating the reduced size of gear sets.

Table 5.1 Optimized parameters for different shafts

Shaft No.	Speed	m_G	N1	Mst	Cf
I	1000	1.5	18	7	137.5
	1000	2	21	5	165
II	710	1	30	4	120
	710	2	21	5	165
	710	4	19	6	275
III	710	4	19	6	275



	500	**4**	**19**	**6**	**275**	
	250	**4**	**19**	**6**	**275**	
	1410	**0.5**	**18**	**6**	**171**	
IV	**1000**	**0.5**	**21**	**5**	**165**	
	500	**0.5**	**21**	**5**	**165**	

VIII. SCOPE OF FUTURE WORK

The present work includes study of the optimized design of the spur gear reduction unit by incorporating ray diagrams. The study of the problem addressed in this work can be further extended in the following directions:

1. In the present study the optimization is concentrated on the spur gear reduction unit. Study could be further undertaken for the helical gear reduction unit.
2. No attempt was made to study the stub tooth system. Further work could be directed in the direction of optimization of the optimal centre distance of such tooth system.

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