

Analysis of deep beam- A parametric study using Finite Element Method

Ms. Shubhangi chafale¹, Dr. K.R.C. Reddy²

¹Lecturer, Department of Civil Engineering, G. H. Raisoni Academy of Engineering & Technology, Nagpur

²Associate Professor, Department of Civil Engineering, Kavikulguru Institute of Technology and Science, Ramtek-441106, Dist. Nagpur, India, krreddy22@rediffmail.com, 09422127868 (M)

Abstract: The analysis of deep beam by finite element method is presented in this paper, which involves the convergence study of deflection, bending stress and shear stresses at a critical point of the beam. There are several analytical tools available for analyzing of a deep beam. Among all the available analytical methods, finite element analysis (FEA) offers a better option.

For the analysis of deep beam SFEAP software developed by a Professor of IIT Bombay is used. The beam is analyzed by discretising it in to rectangular elements of various aspect ratios changing from 2 to 4. The results of deflection, bending stress and shear stresses are noted and compared with the true solution which is obtained from the strength of material principles.

It is found that, the results obtained by analytical procedure and by SFEAP software are deviating by about 40% when the beam is slender. When the beam is deep the results obtained by both the methods are almost matching each other. As the number of elements increased from 20 to 40 elements in the discretization of the beam, has no impact on the results of deflection, bending stress and shear stress. The results obtained are same in all the cases when the beam is slender. As the depth of the beam increased all these parameters have gradually decreased.

Keywords: Deep beam, aspect ratio, convergence study, discretisation, bending stress.

I. INTRODUCTION

In the finite element analysis it is possible to take the elements of any size and shape depending upon the nature of the problem. Especially, the size of the element varies from the location of the importance to that of the location away from importance. Normally, in the process of approximation of the problem aspect ratio of the elements play a vital role in getting the proper results.

It is a fact that in the finite element analysis, as the number of elements increases the result comes closer to the true results. The effect of high aspect ratio in multiphase field model is studied by Burman et al [2]. Maximal independent set technique for degrading the aspect ratio is derived by the Miller [3]. But he discussed only for triangular meshes. Later on experimental study was made of the effect of elements with large aspect ratios on the solution of second order elliptic partial differential equations is studied by Rice [4].

The development of novel software (ADBUFEM) for analysis of deep beams is introduced by Enem et al [5]. In this study isoparametric element are used mostly as a result of their versatility and simplicity. Nonlinear analysis using the finite element method to predict the ultimate load and mode of failure for reinforced concrete deep beams with web openings is analyzed by Khalaf [6]. Later on analytical study of reinforced concrete simply supported deep beam subjected to two point loads to study the behavior of deep beam is introduced by Niranjana and Patil [7].

By studying the literature as mentioned above it is found that the refinement of mesh and in some cases higher orders displacement functions are used for getting the good results from the finite element analysis (FEA). The application of FEA for analysis of deep beam, stress distribution in deep beam with different aspect ratio, is not seen much in the literature. In the present study it is aimed to do a parametric study on the deep beam by varying depth of beam and with different aspect ratios of elements.

II. ANALYSIS OF DEEP BEAM

Simplified Finite Element Analysis Program (SFEAP) given by Desai et al (2011) is used for the analysis of beams in the present study. To obtain the analytical solution, a procedure is based on the strength of material principles are used.

III. DETAILS OF DEEP BEAM

The beam that is considered for the analysis is of length, $L=10\text{m}$ subjected to a point load of 20kN is shown in Fig. 1. The beam is fixed at one end and free at the other end. The material properties such as modulus of elasticity and Poisson's ratio are considered as $E=2 \times 10^5 \text{N/mm}^2$ and $\mu=0.2$ respectively. Thickness of beam is considered as, $t=20 \text{mm}$. For the analysis purpose the beam is taken as plane stress problem lying in xy plane subjected to the loading along the y -axis as shown in the figure.

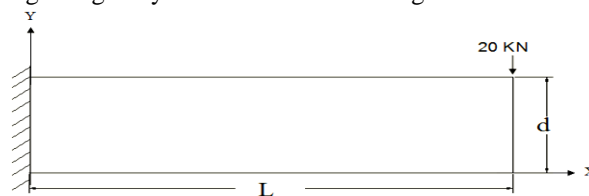


Figure 1: Model of deep beam

The beam is discretized by using 2D four noded rectangular elements by varying the aspect ratio. In the first model the beam is divided into 20 elements with aspect ratio of elements as 2.0 as shown in Fig. 2. In the second case the beam is divided into 30 elements with aspect ratio of elements as 3.0 as shown in Fig. 3 and in the third case the beam is divided into 40 elements with aspect ratio of elements as 4.0 as shown in Fig. 4

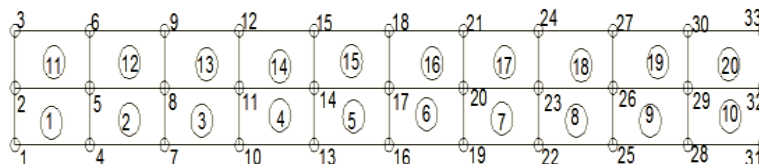


Figure 2: Beam model with 20 elements

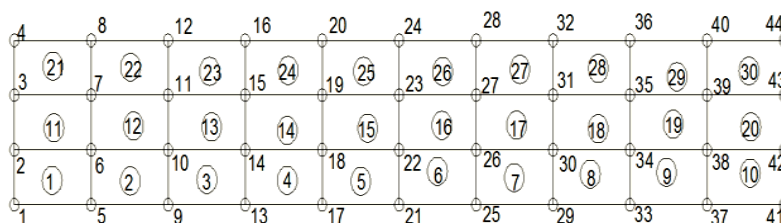


Figure 3: Beam model with 30 elements

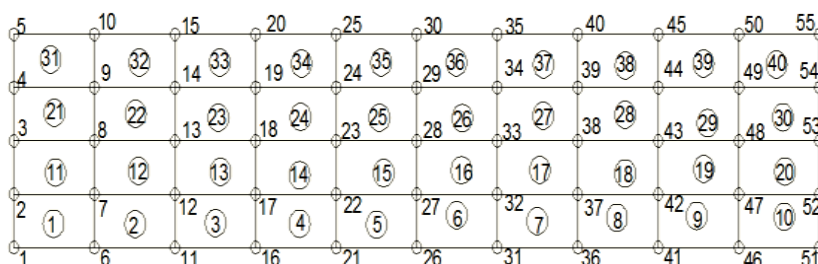


Figure 4: Beam model with 40 elements

IV. ANALYTICAL RESULTS

The analytical results of the above beam are obtained from the strength of material principles. The analytical results are shown in Table 1. It is found that as the depth of the beam increases the magnitudes of the deflection, bending stress and shear stresses are reduced.

Table 1: Analytical results

Sr. No	Depth (mm)	Bending stress, σ (N/mm ²)	Shearing stress, q (N/mm ²)	Deflection \bar{y} (mm)
1	1000	6.000	0.149	2.000
2	2000	1.500	0.075	0.250
3	4000	0.375	0.037	0.031
4	6000	0.166	0.025	0.009
5	8000	0.093	0.018	0.004
6	10000	0.060	0.015	0.002

V. FEA RESULTS

To obtain the finite element analysis (FEA) results of the deep beam, the SFEAP software is used. The length of beam is fixed as 10m, where as the depth of beam is considered as 1m, 2m, 3m, 4m, 6m, 8m and 10m. In the first case the beam with depth 1m is considered and modeled with discretizing it into 20 elements with aspect ratio of elements as 2.0 and in the second stage it is discretized into 30 elements with aspect ratio of 3.0 and finally it is discretized into 40 elements with aspect ratio of 4.0, then the results have been obtained.

5.1. Analysis of beam of depth 1m with the elements of aspect ratio 2.0

The beam is first analyzed with 20 elements with the aspect ratio of elements as 2.0 as shown in Fig. 2 and the results are obtained. The deflection along the length of beam using SFEAP software in comparison with the analytical deflection is shown in Table 2 and the deflected profile of the beam is shown in Fig. 5.

The bending stress across a section at the fixed support where the maximum bending moment is acting is obtained and presented in Table 3 and the variation of the bending stress in comparison with analytical results is shown in Fig. 6. The shear stress across a section is shown in Table 4 and the profiles of the shear stress variation across the section are shown in Fig. 7.

Table 2: Deflection of beam with 20 elements

Length from free end (mm)	Deflection by SFEAP (mm)	Analytical deflection (mm)
0	0	0
2000	-0.080	-0.016
4000	-0.294	-0.128
6000	-0.609	-0.432
8000	-0.991	-1.023
10000	-1.407	-2.000

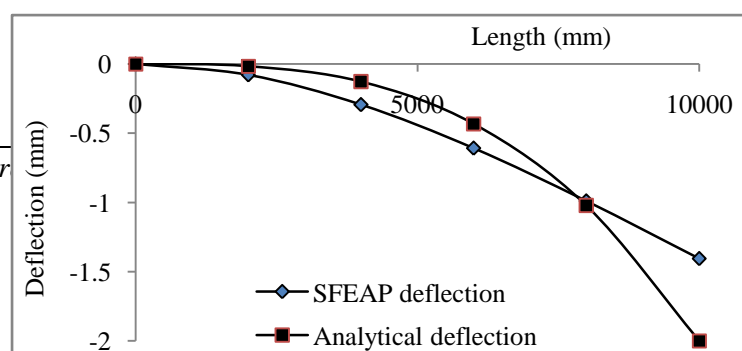


Figure 5: Deflection of beam with 20 elements

Table 3: Bending stress distribution

Analytical bending stress (N/mm ²)	Depth (mm)	Bending stress by SFEAP (N/mm ²)
-6.000	-500	-3.232
0	0	0
6.000	500	3.232

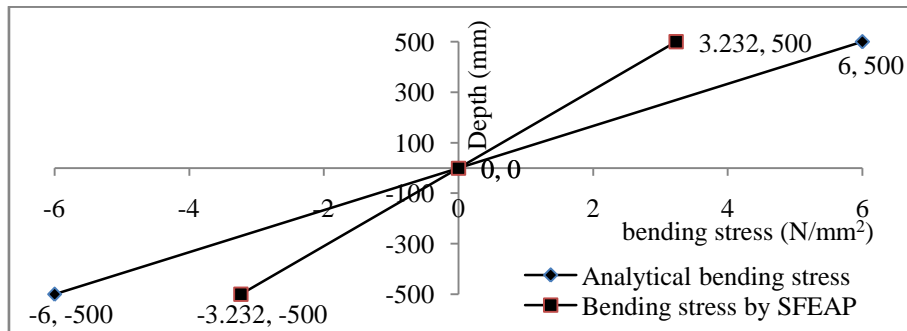


Figure 6: Bending stress distribution diagram

Table 4: Shear stress distribution

Analytical shear stress (N/mm ²)	Depth (mm)	Shear stress by SFEAP (N/mm ²)
0	500	0
0.149	0	1.081
0	-500	0

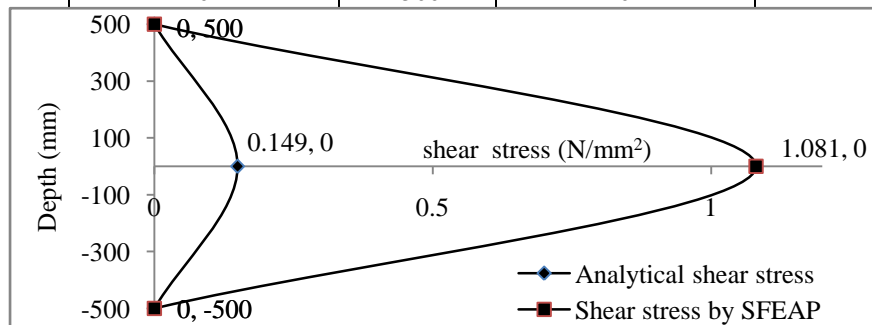


Figure 7: Shear stress distribution diagram

5.2. Analysis of beam of depth 1m with the elements of aspect ratio 3.0 and 4.0

The beam is also analyzed with 30 elements with the aspect ratio of elements as 3.0 and the results are obtained. The deflection profile and the bending stress distribution and the shear stress distribution across the same section as that mentioned in art. 5.1 have been obtained. In this case also the profile pattern is found as shown above. The maximum values of the various parameters obtained with the elements of aspect ratio 3.0 are shown in Table 5.

The beam is finally analyzed with 40 elements with the aspect ratio of element as 4.0 and the results are obtained and the maximum values of various parameters are presented in the Table 5.

5.3. Analysis of beam of depth 2 m, 4 m, 6 m, 8 m and 10 m

The same beam as mentioned above of length 10m is analyzed with variation of depth as 2m to 10 m. by using the same meshes as used in the case of beam with depth 1m with 20 elements, 30 elements and 40 elements. The table 5 shows the results of the beam with varied depth.

Table 5: Variation of deflection of beam with depth increasing

Depth of beam (mm)	No. of elements	Deflection (mm)	Bending stress (N/mm ²)	Shear stress (N/mm ²)
1000	20	1.407	3.232	1.081
	30	1.413	3.452	1.092
	40	1.415	3.677	1.096
2000	20	0.229	1.032	0.084
	30	0.230	1.132	0.091
	40	0.231	1.185	0.096
4000	20	0.032	0.276	0.053
	30	0.033	0.308	0.056
	40	0.033	1.185	0.096
6000	20	0.011	0.124	0.027
	30	0.012	0.141	0.028
	40	0.011	0.150	0.029
8000	20	0.005	0.070	0.018
	30	0.005	0.080	0.018
	40	0.005	0.086	0.019
10000	20	0.003	0.045	0.013
	30	0.003	0.052	0.013
	40	0.003	0.056	0.013

As the depth of the beam increases the analytical deflection and the deflection by SFEAP software are showing similar kind of trend. As the depth of beam increases the deflections are decreased in either case. However the deflection of beam does not make any difference with number of elements. In all the cases of aspect ratio the deflection is found same.

The maximum bending stress values are noted across the section the analytical solution and by SFEAP software as shown in Table 5. The similar kind of variation is found in bending stress also. The maximum bending stress with depth of beam 1m is noted as 6N/mm² by analytical procedure and about 3.6N/mm² by SFEAP software. As the depth of beam increases the bending stress values are decreased from 6N/mm² to 0.06N/mm² by analytical procedure. And by SFEAP software, the bending stress with depth of beam 1m noted as 3.67N/mm² and with depth of beam 10m it is noted as 0.56N/mm²

The maximum shear stress distribution across a section is noted by increasing the depth of the beam. The maximum shear stress, the analytical procedure is found as 0.149N/mm^2 with depth of the beam 1m and 0.015N/mm^2 with depth of the beam 10m. The results by SFEAP software is also shown similar kind of trend by decreasing the shear stress as the depth of the beam increased

It is found that the parameters such as deflection, bending stress and shear stress have not changed with increased of number of elements. That is the aspect ratio has no impact on the results. The beam is also analyzed by increasing the depth of the beam. As the depth of the beam increases the parameters have decreased by analytical procedure and the SFEAP software.

VI. CONCLUSION

In the present study, the parametric study of a deep beam by analytical procedure and the SFEAP software developed by Desai et al (2011) is performed. The parametric study includes understanding the variation of the deflection along the length of the beam, the variation of the bending stress across a section where the maximum bending moment is acting and the variation of the shear stress across a section where the maximum shear force is acting.

The parametric study also includes the change of parameters noted down by increasing the depth of the beam and by changing the aspect ratio of the elements from 2.0 to 4.0. From the parametric study of deep beam the conclusions drawn are as follows:

- The results obtained by analytical procedure and by SFEAP software are deviating by about 40% when the beam is slender. When the beam is deep the results obtained by both the methods are almost matching each other.
- As the number of elements increased from 20 to 40 elements in the discretization of the beam, has no impact on the results of deflection, bending stress and shear stress. The results obtained are same in all the cases.
- As the depth of the beam increased the parameters such as deflection, bending stress and shear stress have gradually decreased.
- Hence, the present method of analysis is more relevant and useful for the analysis of deep elements.

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