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# M-Artex Spaces over Monoids and Cap-Quotient and Cup-Quotient M-Artex Spaces

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***Abstract:*** *We introduce Monoid Artex Spaces or M-Artex Spaces over Monoids. We define SubM-Artex Spaces of an M-Artex Space over a Monoid. We give examples for M-Artex Spaces over Monoids and SubM-Artex Spaces. We define a Cap-Quotient Space and a Cup-Quotient Space and we prove these Quotient Spaces are M-Artex Spaces Over monoids.*
***Keywords:*** *M-Artex Spaces, Cap- Quotient and Cup-Quotient M-Artex Spaces*

## I. Introduction

Two binary operations are there in a lattice. In a monoid only one binary operation is there. But there are two binary operations in a Bi-monoid. In an Artex Space over a bi-monoid the two binary operations of the lattice agree with the two binary operations of a bi-monoid in so many axioms to form new theories. We have many propositions and results in the theory of Artex Spaces over bi-monoids. We could define special Artex Spaces over bi-monoids namely Complete Artex Spaces over bi-monoids, Upper bounded Artex Spaces over bi-monoids, Lower Bounded Artex Spaces over bi-monoids, Bounded Artex Spaces over bi-monoids, Distributive Artex Spaces over bi-monoids, Complemented Artex Spaces over bi-monoids and Boolean Artex Spaces over bi-monoids. Many Propositions and results have been found in the Artex Spaces over bi-monoids, Special Artex Spaces over bi-monoids and Boolean Artex Spaces over bi-monoids. However there are so many examples which are left by the stronger condition of inequalities involving the first binary operation written in the order denoted by +, that is, because of the axioms $ma \wedge na \leq (m+n)a$ and $ma \vee na \leq (m+n)a$. Therefore, if we consider a monoid instead of considering a bi-monoid together with a lattice, we can have so many examples and we can form a new theory. This motivated us to form a new theory called Monoid Artex Spaces Over Monoids. When we define Monoid Artex Spaces Over Monoids, we had in mind that there should be a relation between the existing theory of Artex Spaces Over Bi-monoids and the new theory of Monoid Artex Spaces Over Monoids. As a result of our thinking all the Artex Spaces Over Bi-monoids ( M, + , . ) have become Monoid Artex Spaces Over the Monoids ( M, . ). We hope the theory of Monoid Artex Spaces Over Monoids will form a new chapter and it will be more useful. The theory of Lattices and Boolean Algebra together with monoids has given us the new theory Monoid Artex Spaces Over Monoids. In this theory of Monoid Artex Spaces Over Monoids we have found beautiful results so that to form the theory of Monoid Artex Spaces Over Monoids an exiting and recreational theory.

As a development of it we have proved some propositions which will be useful to further development of the theory of Artex Spaces over bi-monoids. The study of Quotient Spaces in Group theory and in Vector Spaces over fields motivated us to define Cap-Quotient M-Artex Spaces Over Monoids and Cup-Quotient M-Artex Spaces Over Monoids. These Cap-Quotient M-Artex Spaces Over Monoids and Cup-Quotient M-Artex Spaces Over Monoids agree with the existing Quotient Spaces in many concepts and operations of the Quotient Spaces.

## II. Preliminaries

2.1.1 Lattice : A lattice is a partially ordered set ( $L,\leq$ ) in which every pair of elements $a,b \in L$ has a greatest lower bound and a least upper bound.
The greatest lower bound of a and b is denoted by $a \wedge b$ and the least upper bound of a and b is denoted by $a \vee b$

2.1.2 Lattice as an Algebraic System : A lattice is an algebraic system $(L, \wedge, \vee)$ with two binary operations $\wedge$ and $\vee$ on L which are both commutative, associative, and satisfy the absorption laws namely
$a \wedge (a \vee b) = a$ and $a \vee (a \wedge b) = a$
The operations $\wedge$ and $\vee$ are called cap and cup respectively, or sometimes meet and join respectively.

## III. Monoid Artex Space Over a Monoid

3.1 Definition : Monoid Artex Space Over a Monoid : Let ( N, . ) be a monoid with the identity element e. A non-empty set A is said to be a Monoid Artex Space Over the Monoid ( N, . ) if
1.(A, $\wedge$ , V) is a lattice and 2.for each m$\in$N and a$\in$A, there exists an element ma $\in$ A satisfying the following conditions :

(i) m(a $\wedge$ b) = ma $\wedge$ mb
(ii) m(a V b) = ma V mb
(iii) (m.n)a = m.(na), for all m,n$\in$N and a,b$\in$A
(iv) e.a = a , for all a $\in$ A.

Note 3.1.1 : If A is a Monoid Artex space over a Monoid N, then we can simply express this as A is an M-Artex Space over N. That is, any Monoid Artex Space A over a monoid N can be said as an M-Artex Space over N.
Note 3.1. 2 : The multiplication ma is called a Monoid multiplication with an artex element or simply a Monoid multiplication in A.
Unless otherwise stated A remains as an M-Artex space over a monoid N.

3.2.0 Examples
3.2.1 Let N = {1,2,3,…} and Z be the set of all integers.
Then ( N, . ) is a Monoid , where . is the usual multiplication.
( Z, $\leq$ ) is a lattice in which $\wedge$ and V are defined by a $\wedge$ b = mini {a,b} and a V b = maxi {a,b} , for all a,b$\in$Z.
Clearly for each m$\in$N and for each a$\in$Z, there exists ma$\in$Z.
Also, (i) m(a $\wedge$ b) = ma $\wedge$ mb
(ii) m(a V b) = ma V mb
(iii) (m.n)a = m.(na)
(iv) 1.a = a , for all m,n$\in$N, and a,b$\in$Z
Therefore, Z is a Monoid Artex Space Over the monoid N
3.2.2 As defined in Example 3.2.1, Q, the set of all rational numbers is an M-Artex space over N
3.2.3 As defined in Example 3.2.1, R, the set of all real numbers is an M-Artex space over N

## IV. SubM-Artex Spaces

4.1 Definition : SubM-Artex Space : Let (A, $\wedge$ ,V) be an M-Artex space over a monoid (N, . )
Let S be a nonempty subset of A. Then S is said to be a SubM-Artex space of A if (S, $\wedge$ ,V) itself is an M-Artex Space over N.
Example 4.1.1 As in Example 3.2.1, Z is an M-Artex Space over N = {1,2,3,…..} and N is a subset of Z.
Also N itself is an M-Artex space over N under the operation defined in Z
Therefore, N is a SubM-Artex space of Z.
Example 4.1.2 As in Example 3.2.1, Q is an M-Artex Space over N= {1,2,3,…..} and Z is a subset of Q.
Clearly Z itself is an M-Artex Space over N and therefore Z is a SubM-Artex Space of Q.
N is also a SubM-Artex Space of Q.

## V. Cap-Quotient Monoid Artex Space Over a monoid

5.1.1 Definition : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a subset of A. Let a $\in$ A. Define a ^ S = { a ^ s / s $\in$ S}. Then a ^ S is called a Cap-coset of S in the Monoid Artex Space A or a Product-Coset of S in the Monoid Artex Space A.
5.1.2 Definition : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A. For each a $\in$ A, define a ^ S. Then the set of all Cap-cosets of S in the Monoid Artex Space A is denoted by A/S^.
Proposition 5.2.1 : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A.
For each a $\in$ A, define a ^ S. Let A/S^ be the set of all Cap-cosets of S in the Monoid Artex Space A.
Define the Cap ^ , Cup V operations and the monoid multiplication on A/S^ by the following :
For a,b $\in$ A and m $\in$ N, define (a ^ S) ^ (b ^ S) = (a ^ b) ^ S and (a ^ S) V (b ^ S) = (a v b) ^ S
and the monoid multiplication by m(a ^ S) = ma ^ S.
Then ( A/S^ , ^ , V ) is an M-Artex Space over the monid (N, . ).
Proof : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A.
For each a $\in$ A, define a ^ S = { a ^ s / s $\in$ S}.
Let A/S be the set of all Cap-cosets of S in the Monoid Artex Space A.
Define the Cap ^ , Cup V operations and the monoid multiplication on A/S^ by the following :
For a,b $\in$ A and m $\in$ N, define (a ^ S) ^ (b ^ S) = (a ^ b) ^ S and (a ^ S) V (b ^ S) = (a v b) ^ S
and the monoid multiplication by m(a ^ S) = ma ^ S.
Let A' = A/S^
Let x,y $\in$ A' = A/S^. Then x = a ^ S and y = b ^ S, for some a, b $\in$ A

(i) $x \wedge y = (a \wedge S) \wedge (b \wedge S)$
$= (a \wedge b) \wedge S$
Since A is an M-Artex space over N, $a,b \in A$ implies $a \wedge b \in A$
Therefore $(a \wedge b) \wedge S \in A' = A/S^\wedge$
That is, $x \wedge y \in A' = A/S^\wedge$
Therefore, $\wedge$ is a binary operation in $A/S^\wedge$
(ii) $x \vee y = (a \wedge S) \vee (b \wedge S)$
$= (a \vee b) \wedge S$
Since A is an M-Artex space over N, $a,b \in A$ implies $a \vee b \in A$
Therefore $(a \vee b) \wedge S \in A' = A/S^\wedge$. That is, $x \vee y \in A' = A/S^\wedge$
Therefore, V is a binary operation in $A/S^\wedge$
(iii) $x \wedge y = (a \wedge S) \wedge (b \wedge S)$
$= (a \wedge b) \wedge S$
$= (b \wedge a) \wedge S$ ( since A is an M-Artex space, $a \wedge b = b \wedge a$ )
$= (b \wedge S) \wedge (a \wedge S)$
$= y \wedge x$
Therefore, the operation $\wedge$ in $A/S^\wedge$ is commutative.
(iv) $x \vee y = (a \wedge S) \vee (b \wedge S)$
$= (a \vee b) \wedge S$
$= (b \vee a) \wedge S$ ( since A is an M-Artex space, $a \vee b = b \vee a$ )
$= (b \wedge S) \vee (a \wedge S)$
$= y \vee x$
Therefore, the operation V in $A/S^\wedge$ is commutative.
(v) Let $x,y,z \in A' = A/S^\wedge$
Then $x = a \wedge S$ and $y = b \wedge S$ and $z = c \wedge S$, for some $a, b,c \in A$
Now, $x \wedge (y \wedge z) = (a \wedge S) \wedge ((b \wedge S) \wedge (c \wedge S))$
$= (a \wedge S) \wedge ((b \wedge c) \wedge S))$
$= (a \wedge (b \wedge c)) \wedge S)$
$=((a \wedge b) \wedge c)) \wedge S)$ ( since A is an M-Artex space, $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ )
$= ((a \wedge b) \wedge S) \wedge (c \wedge S))$
$= ((a \wedge S) \wedge (b \wedge S)) \wedge (c \wedge S)$
$= (x \wedge y) \wedge z$
Therefore the operation $\wedge$ in $A/S^\wedge$ is associative.
(vi) Now, $x \vee (y \vee z) = (a \wedge S) \vee ((b \wedge S) \vee (c \wedge S))$
$= (a \wedge S) \vee ((b \vee c) \wedge S))$
$= (a \vee (b \vee c)) \wedge S)$
$=((a \vee b) \vee c)) \wedge S)$ ( since A is an M-Artex space, $a \vee (b \vee c) = (a \vee b) \vee c$ )
$= ((a \vee b) \wedge S) \vee (c \wedge S))$
$= ((a \wedge S) \vee (b \wedge S)) \vee (c \wedge S)$
$= (x \vee y) \vee z$
Therefore, the operation V in $A/S^\wedge$ is associative.
(vii) Let $x,y \in A' = A/S^\wedge$. Then $x = a \wedge S$ and $y = b \wedge S$, for some $a, b \in A$
Now, $x \wedge (x \vee y) =(a \wedge S) \wedge ((a \wedge S) \vee (b \wedge S))$
$= (a \wedge S) \wedge ((a \vee b) \wedge S))$
$= (a \wedge (a \vee b)) \wedge S$
$= a \wedge S$ ( since A is an M-Artex space, by absorption law $a \wedge (a \vee b) = a$ )
$= x$
Now, $x \vee (x \wedge y) =(a \wedge S) \vee ((a \wedge S) \wedge (b \wedge S))$
$= (a \wedge S) \vee ((a \wedge b) \wedge S))$
$= (a \vee (a \wedge b)) \wedge S$
$= a \wedge S$ ( since A is an M-Artex space, by absorption law $a \vee (a \wedge b) = a$ )
$= x$
Therefore, the absorption laws are satisfied in $A/S^\wedge$
Hence $A/S^\wedge$ is a lattice.
(viii) Let $m \in N$ and $x,y \in A/S^\wedge$
Then $x = a \wedge S$ and $y = b \wedge S$, for some $a,b \in A$.
Now, $m(x \wedge y) = m((a \wedge S) \wedge (b \wedge S))$
$= m((a \wedge b) \wedge S)$
$= m(a \wedge b) \wedge S$
$= (ma \wedge mb) \wedge S$ (since A is an M-Artex space over N, $m(a \wedge b) = ma \wedge mb$)

= (ma ^ S) ^ (mb ^ S)
= m(a ^ S) ^ m(b ^ S)
= mx ^ my

(ix) Now, m(x v y) = m((a ^ S) V (b ^ S))
= m((a v b ) ^ S)
= m(a v b) ^ S
= (ma v mb) ^ S (since A is an M-Artex space over N, m(a v b) = ma v mb)
= (ma ^ S) V (mb ^ S)
= m(a ^ S) V m(b ^ S)
= mx V my

(x) Let m,n  N and x  A/S^
Then x = a ^ S, for some a  A
Now (m.n)x = (m.n)(a ^ S)
= (m.n)a ^ S
= m(na) ^ S ( since A is an M-Artex space over N, (m.n)a = m(na),for all m,n  N and for all a  A)
= m((na) ^ S)
= m(n(a ^ S))
= m(nx)

(xi) Let x  A/S^
Then x = a ^ S, for some a  A
Let e be the identity of the nomoid ( N, . )
Then e.x = e(a ^ S)
= e.a ^ S
= a ^ S, ( since A is an M-Artex space over N, e.a = a, for all a  A)
= x.

Hence A/S^ is a Monoid Artex Space over the Monoid ( N, . ).

5.2.2 Remark : Let us consider the Monoid Artex space (A/S^ , ^ , V ) over a monoid ( N, . ).
Let x  A/S^
Then x = a ^ S, for some a  A
Now, x ^ x = (a ^ S) ^ (a ^ S)
= (a ^ a) ^ S
= a ^ S ( since A is an M-Artex space over N, a ^ a = a )
= x
Therefore, every element of A/S^ is idempotent with respect to the binary operation ^ .
Now, x V x = (a ^ S) V (a ^ S)
= (a v a) ^ S
= a ^ S ( since A is an M-Artex space over N, a v a = a )
= x
Therefore, every element of A/S^ is idempotent with respect to the binary operation V.

5.2.3 Definition : A Cap-Quotient Monoid Artex Space Over a Monoid N or A Product-Quotient Monoid Artex Space Over a monoid N : Let ( N, . ) be a monoid. Let ( A, ^ , v ) be a Monoid Artex space over the monoid (N, . ). Then the Monoid Artex Space ( A/S^ , ^ , V ) is called a Cap-Quotient Monoid Artex Space Over the monoid N or a Product-Quotient Monoid Artex Space Over the monoid N.

## VI. Cup-Quotient Monoid Artex Space Over a monoid

6.1.1 Definition : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a subset of A. Let a  A. Define a v S = { a v s / s  S}. Then a v S is called a Cup-Coset of S in the Monoid Artex Space A or a Sum-Coset of S in the Monoid Artex Space A.

6.1.2 Definition : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A. For each a  A, define a v S. Then the set of all Cup-cosets of S in the Monoid Artex Space A or Sum-Cosets of S in the Monoid Artex Space A is denoted by $A/S^v$.

Proposition 6.2.1 : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A. For each a  A, define a v S. Let $A/S^v$ be the set of all Cup-cosets of S in the Monoid Artex Space A. Define the Cap ^ , Cup V operations and the monoid multiplication in $A/S^v$ by the following :
For a,b  A and m  N, define (a v S) ^ (b v S) = (a ^ b) v S and (a v S) V (b v S) = (a v b) v S and the monoid multiplication by m(a v S) = ma v S. Then ( $A/S^v$ , ^ , V ) is an M-Artex Space over the monid ( N, . ).

Proof : Let (A, ^ , v) be an M-Artex space over a monoid ( N, . ) and S be a SubM-Artex space of A.
For each a  A, define a v S = { a v s / s  S}.
Let $A/S^v$ be the set of all Cup-Cosets of S in A.

Define the Cap ^ , Cup V operations and the monoid multiplication in $A/S^v$ by the following :
For a,b □ A and m □ N, define (a v S) ^ (b v S) = (a ^ b) v S
(a v S) V (b v S) = (a v b) v S
and the monoid multiplication by m(a v S) = ma v S.
Let A' = $A/S^v$
Let x,y □ A' = $A/S^v$
Then x = a v S and y = b v S, for some a, b □ A
(i) x ^ y = (a v S) ^ (b v S)
= (a ^ b) v S
Since A is an M-Artex space over N, a,b □ A implies a ^ b □ A
Therefore (a ^ b) v S □ A' = $A/S^v$
That is, x ^ y □ A' = $A/S^v$
Therefore, ^ is a binary operation in $A/S^v$
(ii) x V y = (a v S) V (b v S)
= (a v b) v S
Since A is an M-Artex space over N, a,b □ A implies a v b □ A
Therefore (a v b) v S □ A' = $A/S^v$
That is ,x V y □ A' = $A/S^v$
Therefore, V is a binary operation in $A/S^v$
(iii) x ^ y = (a v S) ^ (b v S)
= (a ^ b) v S
= (b ^ a) v S ( since A is an M-Artex space, a ^ b = b ^ a )
= (b v S) ^ (a v S)
= y ^ x
Therefore, the operation ^ in $A/S^v$ is commutative.
(iv) x V y = (a v S) V (b v S)
= (a v b) v S
= (b v a) v S ( since A is an M-Artex space, a v b = b v a )
= (b v S) V (a v S)
= y V x
Therefore, the operation V in $A/S^v$ is commutative.
(v) Let x,y,z □ A' = $A/S^v$
Then x = a v S and y = b v S and z = c v S, for some a, b,c □ A
Now, x ^ (y ^ z) = (a v S) ^ (( b v S) ^ (c v S))
= (a v S) ^ (( b ^ c) v S))
= (a ^ ( b ^ c)) v S)
=((a ^ b) ^ c)) v S) ( since A is an M-Artex space, a ^ (b ^ c) = (a ^ b) ^ c )
= ((a ^ b) v S) ^ (c v S))
= ((a v S) ^ ( b v S)) ^ (c v S)
= (x ^ y ) ^ z
Therefore the operation ^ in $A/S^v$ is associative.
(vi) Now, x V (y V z) = (a v S) V (( b v S) V (c v S))
= (a v S) V (( b v c) v S))
= (a v ( b v c)) v S)
=((a v b) v c)) v S) ( since A is an M-Artex space, a v (b v c) = (a v b) v c )
= ((a v b) v S) V (c v S))
= ((a v S) V ( b v S)) V (c v S)
= (x V y ) V z
Therefore, the operation V in $A/S^v$ is associative.
(vii) Let x,y □ A' = $A/S^v$
Then x = a v S and y = b v S, for some a, b □ A
Now, x ^ (x V y) =(a v S) ^ ((a v S) V (b v S))
= (a v S) ^ ((a v b) v S))
= (a ^ (a v b)) v S
= a v S ( since A is an M-Artex space, by absorption law a ^ (a v b) = a )
= x
Now, x V (x ^ y) =(a v S) V ((a v S) ^ (b v S))
= (a v S) V ((a ^ b) v S))
= (a v (a ^ b)) v S
= a v S ( since A is an M-Artex space, by absorption law a v (a ^ b) = a )

= x

Therefore, the absorption laws are satisfied in A/$S^v$

Hence A/$S^v$ is a lattice.

(viii) Let m □ N and x,y □ A/$S^v$

Then x = a v S and y = b v S, for some a,b □ A.

Now, m(x ^ y) = m((a v S) ^ (b v S))
= m((a ^ b ) v S)
= m(a ^ b) v S
= (ma ^ mb) v S (since A is an M-Artex space over N, m(a ^ b) = ma ^ mb)
= (ma v S) ^ (mb v S)
= m(a v S) ^ m(b v S)
= mx ^ my

(ix) Now, m(x v y) = m((a v S) V (b v S))
= m((a v b ) v S)
= m(a v b) v S
= (ma v mb) v S (since A is an M-Artex space over N, m(a v b) = ma v mb)
= (ma v S) V (mb v S)
= m(a v S) V m(b v S)
= mx V my

(x) Let m,n □ N and x □ A/$S^v$

Then x = a v S, for some a □ A

Now (m.n)x = (m.n)(a v S)
= (m.n)a v S
= m(na) v S ( since A is an M-Artex space over N, (m.n)a = m(na), for all m,n □ N and for all a □ A)
= m((na) v S)
= m(n(a v S))
= m(nx)

(xi) Let x □ A/$S^v$

Then x = a v S, for some a □ A

Let e be the identity element of the nomoid ( N, . )

Then e.x = e(a v S)
= e.a v S
= a v S, ( since A is an M-Artex space over N, e.a = a, for all a □ A)
= x.

Hence A/$S^v$ is a Monoid Artex Space over the Monoid ( N, . ).

6.2.2 Remark : Let us consider the Monoid Artex Space (A/$S^v$ , ^ , V ) over a monoid ( N, . ).

Let x □ A/$S^v$ . Then x = a v S, for some a □ A

Now, x ^ x = (a v S) ^ (a v S)
= (a ^ a) v S
= a v S ( since A is an M-Artex space over N, a ^ a = a )
= x

Therefore, every element of A/$S^v$ is idempotent with respect to the binary operation ^ .

Now, x V x = (a v S) V (a v S)
= (a v a) v S
= a v S ( since A is an M-Artex space over N, a v a = a )
= x

Therefore, every element of A/$S^v$ is idempotent with respect to the binary operation V.

6.2.3 Definition : A Cup-Quotient Monoid Artex Space Over a Monoid N or a Sum-Quotient Monoid Artex Space Over a Monoid N : Let ( N, . ) be a monoid. Let (A, ^ , v) be a Monoid Artex space over the monoid ( N, . ). Then the Monoid Artex Space ( A/$S^v$ , ^ , V ) over the monoid N is called a Cup-Quotient Monoid Artex Space Over the monoid N or a Sum-Quotient Monoid Artex Space Over the Monoid N.

## VII. CONCLUSION :

The theory of Lattices and Boolean Algebra together with monoids has given us the new theory Monoid Artex Spaces Over Monoids. The study of Quotient Spaces in Group theory and in Vector Spaces over fields motivated us to define Cap-Quotient M-Artex Spaces Over Monoids and Cup-Quotient M-Artex Spaces Over Monoids. We hope this paper will motivate the researcher to bring new useful results in the theory of Monoid Artex spaces over Monoids.

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