IOSR Journal of Mathematics (IOSRJM)
Volume 1, Issue 1 (May-June 2012), PP 18-35
www.iosrjournals.org

Exact value of pi (π) = $17 - 8\sqrt{3}$

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Abstract : In this paper, I show that exact value of pi (π) is $\mathbf{17 - 8\sqrt{3}}$ or $\mathbf{17 - \sqrt{192}}$.
I found that π is an algebra . My findings are based on geometrical constructions, arithmetic calculation and algebraic formula & proofs.

Introduction

Pi (π) is a mathematical constant that is the ratio of a circles circumference to its diameter equal to area of circle divided by to radius square. I found that by this method, the constant Pi (π) is approximately equal to 3.14359..........

π is an irrational number. Its decimal representation never ends and never repeats. The digits in the decimal representation of Pi (π) appear to be random. Pi (π) is found in many formulae concerning circles, ellipse, spheres, number theory, statistics, mechanics and electromagnetism.

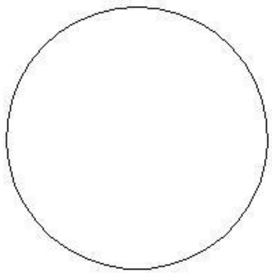
I compute the exact value of pi(π). I use geometric constructions and algebraic calculations to find Value of Pi(π).

Our proof involves geometric, arithmetic & algebraic identities/formulae. I can prove it by using geometric , arithmetic & algebraic methods, 100 & more examples , 40 types of proofs .

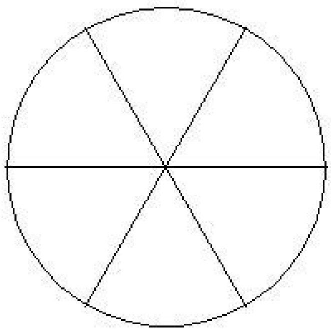
Here, myself is giving you the summary of Pi(π) value research .

Construction

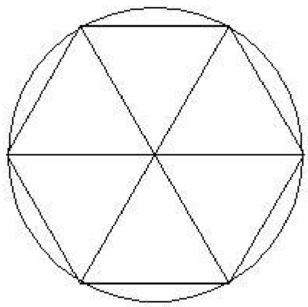
First draw a circle



With the help of rounder divide the circumference in six equal parts, draw three diameters.



Then join the end points of the diameters, so that we get six equilateral triangles with sides equal to radius i.e. Regular hexagon is formed.



Then draw regular twelve sided polygon (Dodecagon) i.e. make twelve equal sectors of the circle.

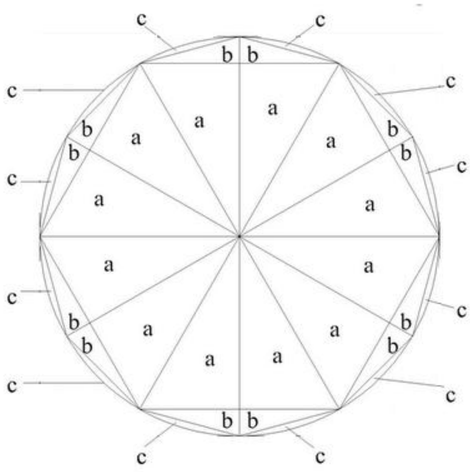


Figure Number – 1

Then draw a square so that circle is inscribed in a square , then give names a, b, c, d to different parts shown in following figure.

Basic Figure

Draw the square such that circle is totally inside in the square.

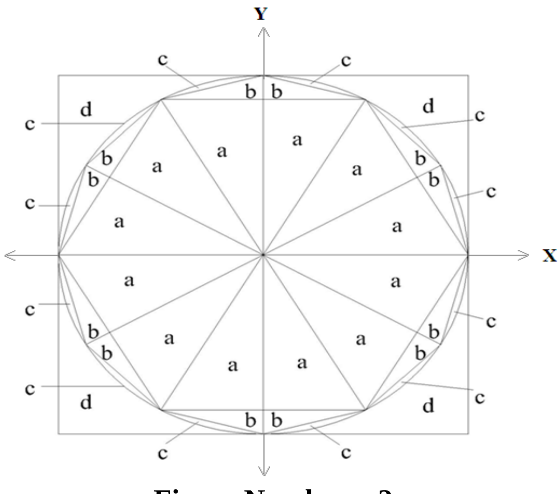


Figure Number – 2

Note :- 1) Let a , b, c, d each part shows area .

2) Every Figure in the proof is symmetric about X & Y axis .

From above figure we get ,

$12a + 12b$ = Area of Dodecagon (12 sides Polygon) ------- (1)

$12a + 12b + 12c$ = Area of circle $= \pi r^2$ ------- (2)

$12a + 12b + 12c + 4d$ = Area of Square $= 4r^2$ ------- (3)

$12c + 4d$ = Outside area of the Dodecagon ------- (4)

1st Modification in basic figure

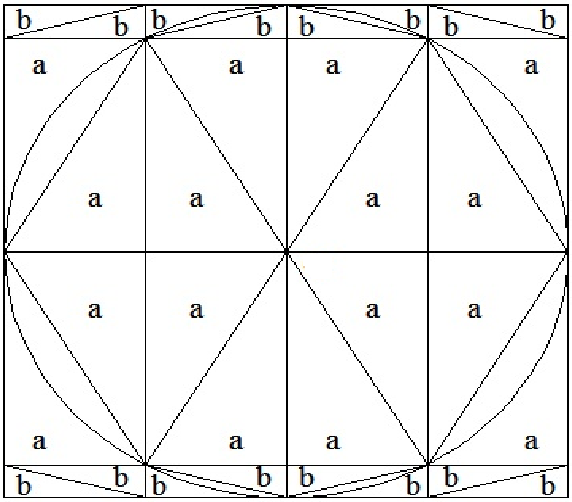


Figure Number – 3

From Figure – 3 ,

$16a + 16b$ = Area of Square $= 4r^2$ ----------(5)

Divide by 4 to above equation we get ,

$4a + 4b = r^2$ ----------(6)

From equation (3) & (5)

$12a + 12b + 12c + 4d = 16a + 16b$

$12a + 12b + 12c + 4d - 16a - 16b = 0$

$-4a - 4b + 12c + 4d = 0$

i.e $4a + 4b = 12c + 4d$

But by using equation (6)

$4a + 4b = 12c + 4d = r^2$ ----------(7)

Divided by 4 to above equation we get ,

$a + b = 3c + d = \frac{r^2}{4}$ ----------(8)

2nd Modification in basic figure

Find X part area

Construction :- We divide Square into 16 equal part . Each sub square having area a + b.
Part X area is unknown.

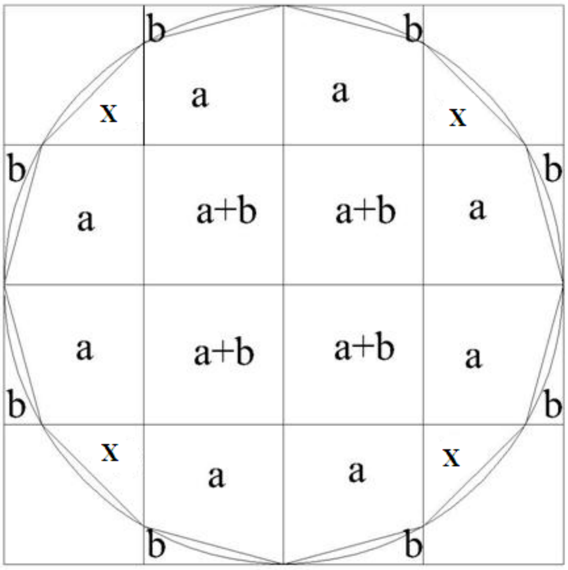


Figure Number – 4

From Above Figure ,

Area of Dodecagon = 12a +4b + 4X **--------(9)**

From Equation (1) & (9) ,

12a + 4b + 4X = 12a + 12b

4X = 12a +12b -12a -4b

4X = 8b

Divide by 4 to above equation we get ,

X = 2b

Area of X part is 2b .

i.e our figure becomes,

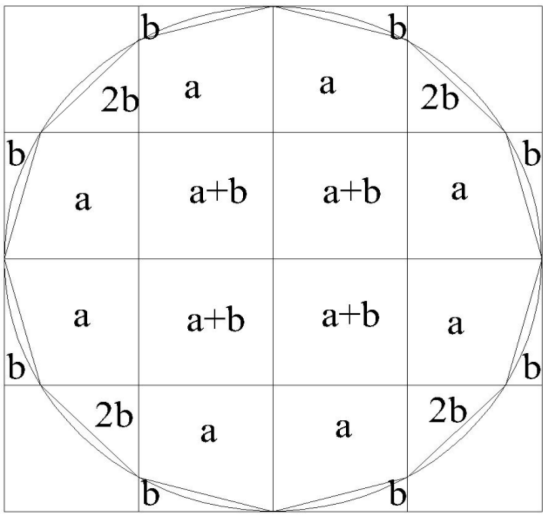


Figure Number – 5

Modification in figure no. 5

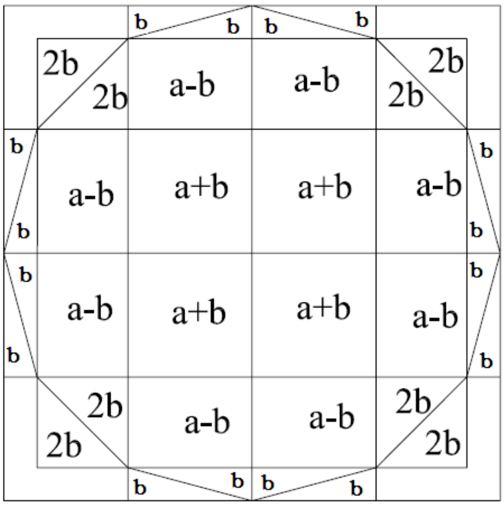


Figure Number – 6

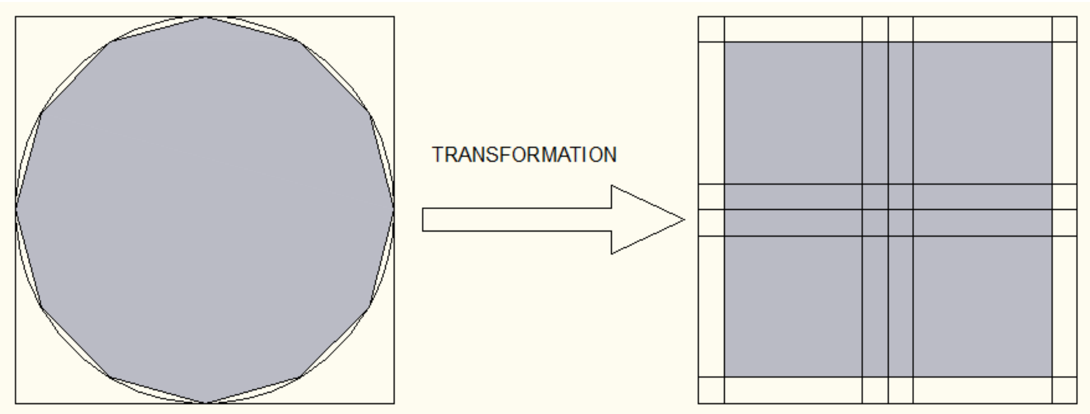
From Figure No. 6 we get ,

Inside Square area = 8(a - b)+4(a + b) +8(2b)

Inside Square area = 12a + 12b .

From equation (1)

Area of Dodecagon (12 sides Polygon) = Inside square area = 12 a +12 b -------(10)



Area of Dodecagon (12 sides Polygon) **Inside square area**

Figure Number – 7

Area Bounded between two squares = Area of outer square – Area of inner square

= (16a +16b) - (12a +12b)

Area Bounded between two squares = 4a + 4b **-------(11)**

Divide Area Bounded between two squares into four equal parts .

i.e. **one part value is a + b** .

Modification in figure no. 6
Find X part area

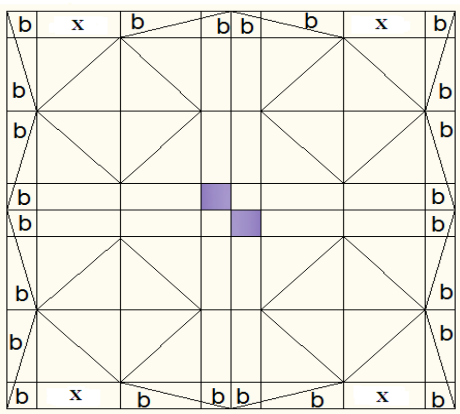


Figure Number – 8

From Above Figure we get ,
Area Bounded between two squares = 24b + 4x **---------(12)**
From equation (11) & (12) we get ,
24b + 4X = 4a + 4b
4X = 4a + 4b – 24 b
4X = 4a – 20b
Divide by 4 to above equation we get,
X = a – 5b
Area of X part is a – 5b .

i.e our figure 7 becomes,

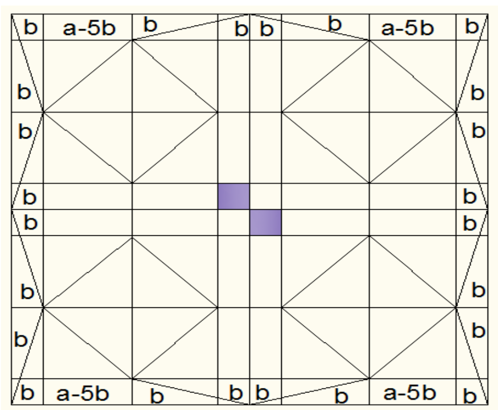
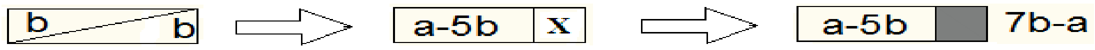


Figure Number – 9

Find X part area



From above figure ,
2b = (a – 5b) + X
X = 2b – a + 5b
X = 7b – a
Area of X part is 7b – a .

<u>Symmetric aera of part d inside & outside the circle</u>

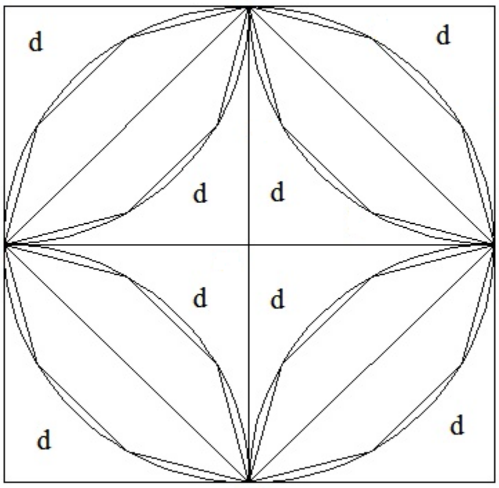


Figure Number – 10

From above figure we get following two results ,

1) **4(Corner's ¼ th circle) = Area of Circle**
2) **Inside area of 4d = Out of circle area of 4d .**

Modification in figure no. 6

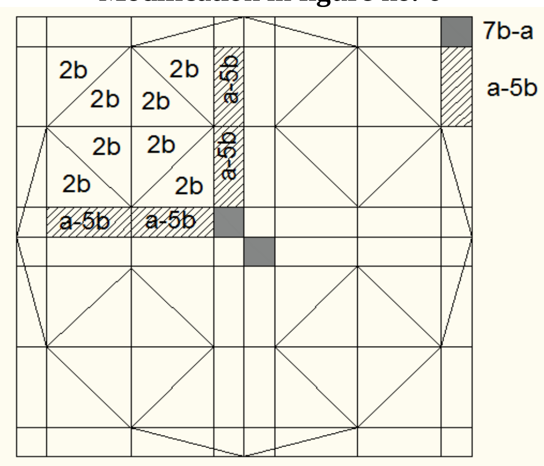
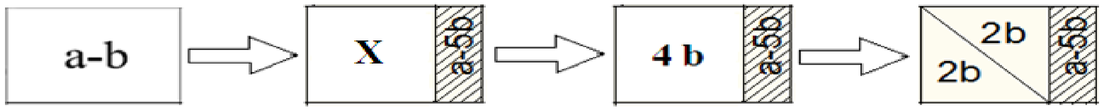


Figure Number – 11

Relation between figure 6 & figure 11

<u>Find X part area</u>



From above figure

$a - b = X + a - 5b$

$X = a - b - a + 5b$

$X = 4b$

Area of X part is 4b .

Relation between figure 6 & figure 11

<u>Find X part area</u>

From above figure

$a + b = X + 2(a - 5b) + 7b - a$

$a + b = X + 2a - 10b + 7b - a$

$X = a + b - 2a + 10b - 7b + a$

$X = 4b$

Area of X part is 4b .

Modification in figure no. 11

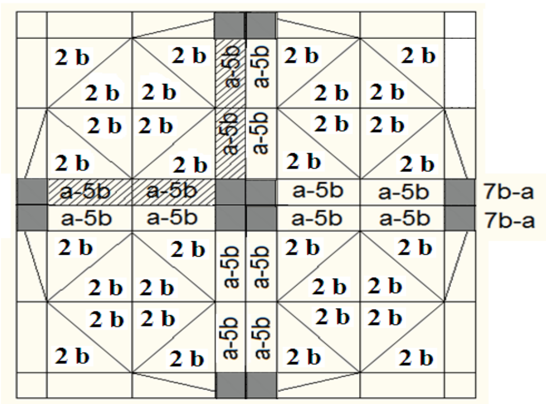


Figure Number – 12

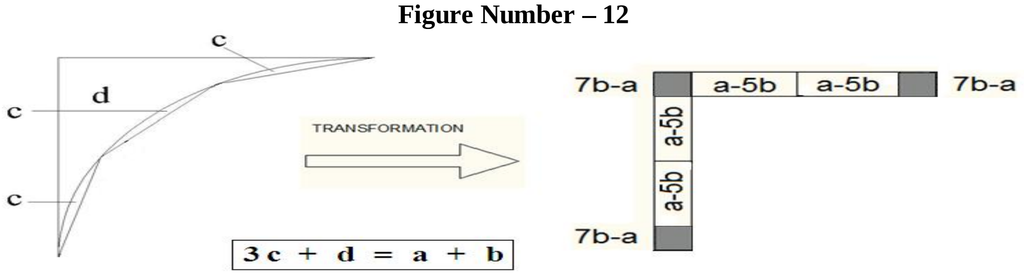
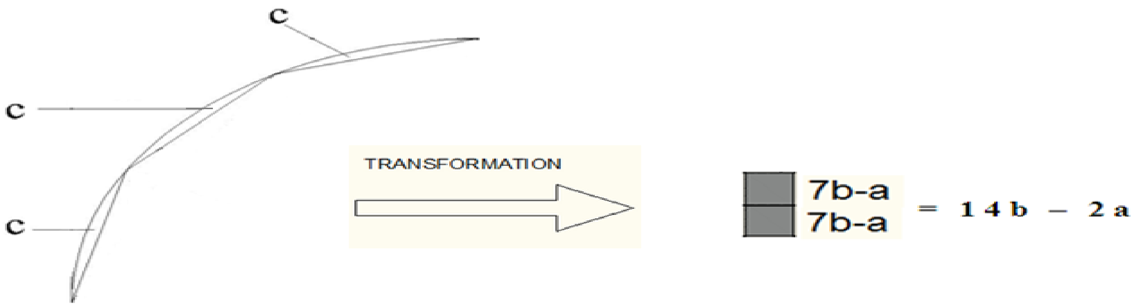


Figure Number – 13



$$3c = 2(\blacksquare) = 2(7b - a) = 14b - 2a$$

$$\text{i.e} \quad 12c = 8(\blacksquare) = 8(7b - a) = 56b - 8a$$

[Refer equation (**)]

Figure Number – 14

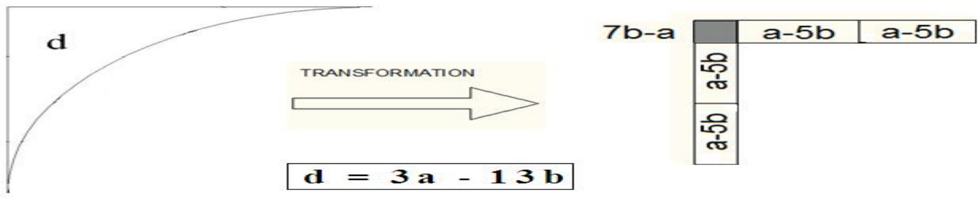


Figure Number – 15

Some important relations From figure 11.

1) Area of Square = 32(2b) + 32(a – 5b) + 16(7b - a)
= 64b + 32a – 160b +112b - 16a

Area of Square = 16a + 16b

2) Area of Inside square = 32(2b) + 16(a – 5b) + 4(7b - a)
= 64b + 16a -80b +28b – 4a

Area of Inside square = 12a +12b

Area of Inside square = Area of Dodecagon = 12a + 12b.

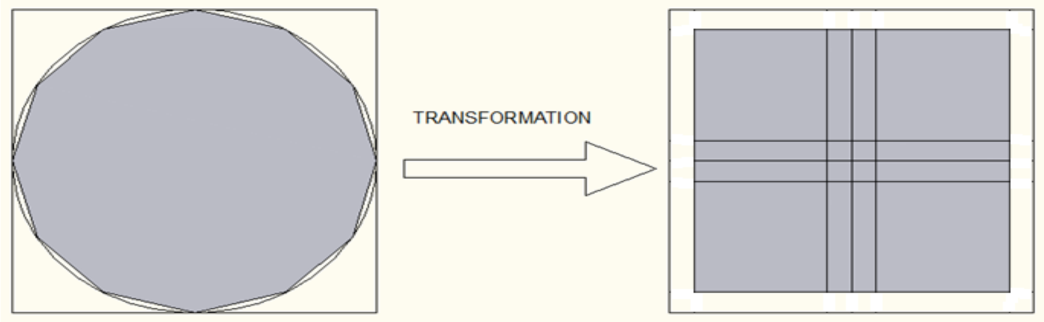
3) Area in between two squares = 16(a – 5b) +12(7b - a)
= 16a – 80b +84b -12a

Area in between two squares = 4a + 4b

4) Area in between two squares = Area of outer square - Area of Dodecagon
= 12a +12b +12c + 4d - 12a – 12 b

Area in between two squares = 12c + 4d

Area in between two squares = Outside area of the Dodecagon



Outside Area of Dodecagon **Area in between two squares**

Figure Number – 16

5) Area of Circle = 32(2b) + 16 (a - 5b) + 12 (7b - a)
= 64b +16a – 80b + 84b – 12a

Area of Circle = 4a + 68b **---------(13)**

Area of Circle Transformation

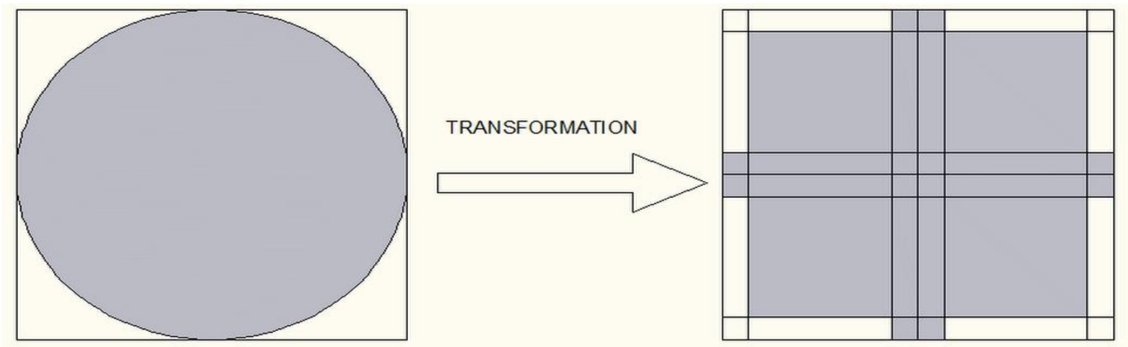


Figure Number – 17

Area of Circle Transformation

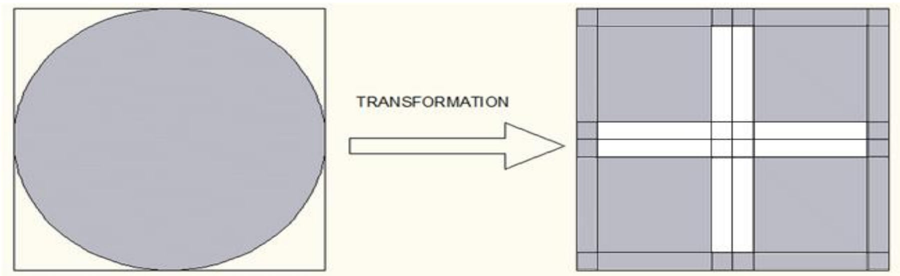


Figure Number – 18

6) Part 4d Area :-

4d = 16(a – 5b) + 4(7b – a)
= 16a – 80b + 28b – 4a
4d = 12a – 52b
Divide by 4.

d = 3a – 13 b

Area of part d = 3a – 13b --------(14)

Important Formulae :-

1) **Area of Square = 12a +12b +12c +4d = 16a + 16b**
2) **Area of Circle = 12a +12b +12c = 4a + 68b**
3) **Area of part 3c = 14b – 2a**
4) **Area of part d = 3a - 13b**

Verification :-

1) Area of square = Area of Circle + Area of Part 4d
= 4a + 68b + 4(3a – 13b)
= 4a + 68b +12a – 52b

Area of square = 16a + 16b

2) Area of square = Area of Dodecagon + Area of part 12c + Area of part 4d
= 12a + 12b +12c + 4 (14b – 2a) + 4 (3a – 13b)
= 12a +12b +56b – 8a +12a - 52b

Area of square = 16a + 16b

3) Area of Circle = Area of Dodecagon + Area of part 12c
= 12a + 12b +4(14b – 2a)
= 12a + 12b + 56b - 8a

Area of Circle = 4a + 68b

For Further Proof we use Value of Area of Circle = 4a + 68b

Theorm :- **Exact value of π (Pi) is $17 - 8\sqrt{3}$.**

Proof :- We Know Standard formula for

Area of Circle = πr^2 ----(i)

And we get ,

Area of Circle = 4a + 68b ----(ii)

From equation (i) & (ii), we get

$\pi r^2 = 4a + 68b$

Add 64a in both sides , we get

$\pi r^2 + 64a = 4a + 68b + 64a$

$\pi r^2 + 64a = 68a + 68b$

$\pi r^2 + 64a = 68 (a+b)$

From equation (8)

$\pi r^2 + 64a = 68 \left(\frac{r^2}{4} \right)$

$\boldsymbol{\pi r^2 + 64a = 17 r^2}$ ----(iii)

From Basic Figure ,

64a = 32 equilateral triangle

Area of equilateral triangle = $\frac{\sqrt{3}}{4} (side)^2$

Side of equilateral triangle = radius of circle = r .

$\therefore$ Area of equilateral triangle = $\frac{\sqrt{3}}{4} (r)^2$

32(Area of equilateral triangle) =32 $\left(\frac{\sqrt{3}}{4} (r)^2\right)$

$\therefore$ **64a = 32(Area of equilateral triangle) =** $\boldsymbol{8\sqrt{3}\, r^2}$

Put this value in equation (iii) , we get

$\pi r^2 + 8\sqrt{3} r^2 = 17 r^2$

$\pi r^2 = 17 r^2 - 8\sqrt{3} r^2$
$\pi r^2 = (17 - 8\sqrt{3}) r^2$
Divide by r^2
$\therefore$ We get ,
$\boldsymbol{\pi = 17 - 8\sqrt{3}}$ **or** $\boldsymbol{\pi = 17 - \sqrt{192}}$
or $\boldsymbol{\pi}$ **=3.14359353944….**

Hence the Proof.

Now we find formula for area of part c & d :-
From equation (2) & (13) , we get
12a + 12b+ 12c = 4a + 68b
12a + 12b + 12c – 4a – 68b = 0
8a – 56b + 12c = 0
12c = 56b – 8a
Divide by 4 , we get
3c = 14b – 2a --------- **(15)**

From figure (16) & (17) we get ,
Area of part 12c = Area of circle - Inside Square
= 4a + 68b – 12a – 12b
12c = 56b - 8a
Divide by 4 , we get
3c = 14b – 2a **-----------(**)**

<u>Area of part d</u> :-
From **Basic figure**
Area of part 4d = Area of square - area of circle
= 16a + 16b - (4a + 68b)
= 16a + 16b - 4a – 68b
4d = 12a – 52b
Divide by 4
d = 3a – 13b

Area of part d = 3a – 13b

<u>Area of Circle Equivalent equations are</u> :-
Area of Circle = 12a + 12b + 12c
Area of Circle = 16a + 16b - 4d
Area of Circle = 64b + 12c + 4d
Area of Circle = 16a – 16b + 18c
Area of Circle = 128b – 24c – 4d
Area of Circle = 120b - 8a + 4d
Area of Circle = 96b - 6c
Area of Circle = 12d + 48c
Area of Circle = 4a + 68b **-----------(16)**

We can construct geometric figures of each & every equations . also on sbstitution of values of 3c & value of d in each equation we conclude the constant value is 4a + 68b

Area of circle + 4d = Area of square
Area of circle + 4d = 16a + 16b

Arithmetic formula for area of part a,b,c,d :-

Area of part a :-

From Basic Figure ,
Area of part 2a = Area of equilateral triangle
$= \frac{\sqrt{3}}{4}\,(side)^2$
Side of equilateral triangle = radius of circle
∴ Area of part 2a = $\frac{\sqrt{3}}{4}\,(r)^2$
Divided by 2 , we get
∴ Area of part a = $\frac{\sqrt{3}}{8}\,(r)^2$
∴ **Area of part a = $0.125(\sqrt{3})\, r^2$ ----------(17)**

Area of part b :-

From equation (8) ,we get

$a + b = \frac{r^2}{4}$
∴ $b = \frac{r^2}{4} - a$
∴ $b = (0.25)\, r^2 - (0.125(\sqrt{3}))\, r^2$
∴ **Area of part b = $[\,0.25 - 0.125(\sqrt{3})\,]\, r^2$ ---------(18)**

Area of part 3c :-

From equation (15) , we get
$3c = 14b - 2a$
Substitute a & b values , we get
$3c = 14\,(\,0.25 - 0.125(\sqrt{3})\,)\, r^2 - 2(0.125(\sqrt{3}))r^2$
$3c = (\,3.5 - 1.75(\sqrt{3}) - 0.25(\sqrt{3})\,)\, r^2$
∴ **Area of part 3c = $(\,3.5 - 2\sqrt{3}\,)\, r^2$ ---------(19)**

Area of part d :-

From equation (14) , we get
$d = 3a - 13b$
Substitute a & b values , we get
$d = 3(0.125(\sqrt{3})\, r^2) - 13(\,0.25 - 0.125(\sqrt{3})\,)\, r^2$
$d = [\,0.375(\sqrt{3}) - 3.25 + 1.625(\sqrt{3})\,]\, r^2$
∴ **Area of part d = $(\,2\sqrt{3} - 3.25\,)\, r^2$ ----------(20)**

<u>Formulae</u> :-

1) **Area of part a = $[0.125(\sqrt{3})]\, r^2$**
2) **Area of part b = $[\,0.25 - 0.125(\sqrt{3})\,]\, r^2$**
3) **Area of part 3c = $[\,3.5 - 2\sqrt{3}\,]\, r^2$**
4) **Area of part d = $[\,2\sqrt{3} - 3.25\,]\, r^2$**

If we put this a, b, c, d values in any arithmetic equation i.e in equation (16)
We always get the constant value $\pi = 17 - 8\sqrt{3}$ or $\pi = 17 - \sqrt{192}$.

∴ Finaly I conclude that value of $\pi = 17 - 8\sqrt{3}$ or $\pi = 17 - \sqrt{192}$
or 3.14359........... .

<u>Supportive Work related to value of pi (π)</u>

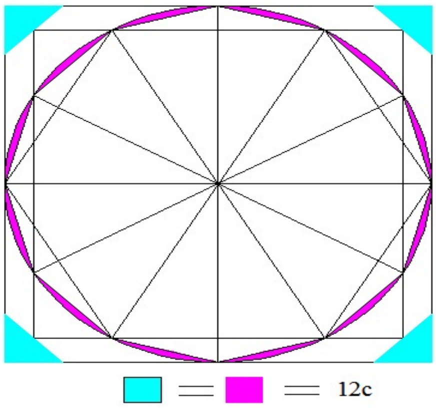


Figure Number – 19

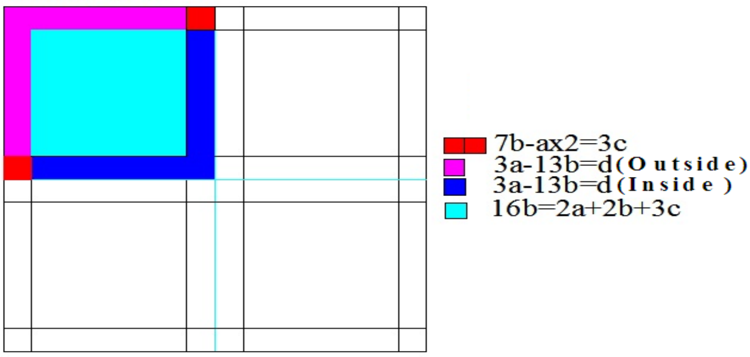


Figure Number – 20

Basic Figure 1

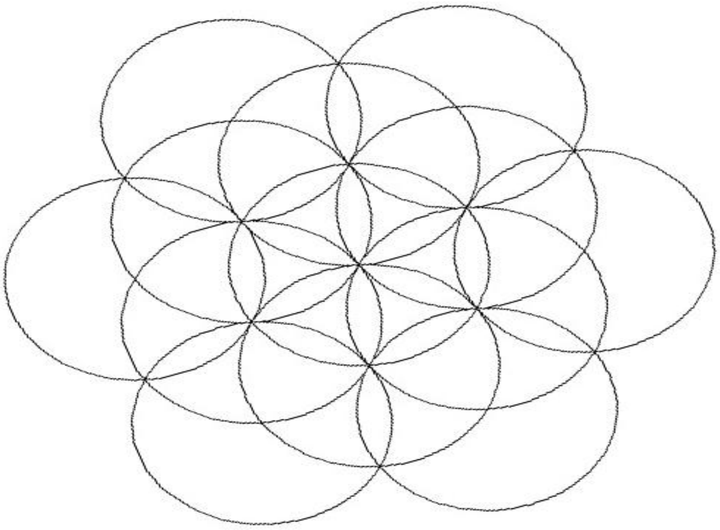


Figure Number – 21

<u>Modification in Basic figure 1</u>

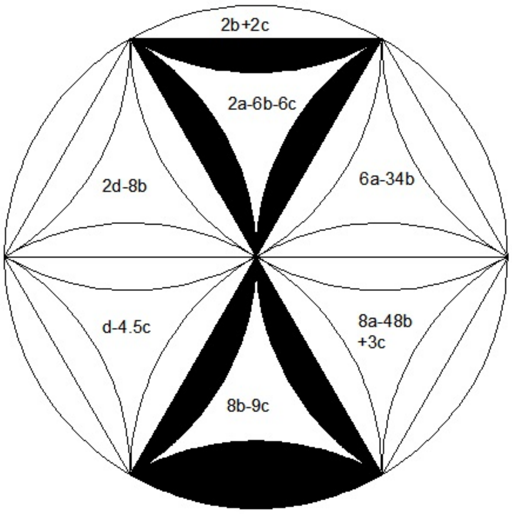


Figure Number – 22

Some Important formulae :-

a + b = d + 3c

3c = 14b – 2a

d = 3a – 13b

By using c & d values

We can verified 6 equations given in the hexagon .

Verification :-

Assume that , X can take any real value .

1) X Numbers of equilateral Triangle = X(multiple of any one equation in hexagon) +[(3X)(2b + 2c)]

For Example :-

For X = 10, we choose one equation in hexagon , 6a - 34b.

10 Numbers of equilateral Triangle = 20a **--------From Basic figure**

X Numbers of equilateral Triangle = X(multiple of any one equation in hexagon) +[(3X)(2b + 2c)]

10 Numbers of equilateral Triangle = 10(6a – 34b) +[(30)(2b + 2c)]

= 60a – 340b + 60b + 60c

= 60a - 280b + 60c i.e 60c = 20 (3c)

= 60a – 280b + 280b – 40a. = 20(14b – 2a)

10 Numbers of equilateral Triangle = 20a (From Basic Figure) = 280b – 40a

2) (4a + 68b)(X/6)=(X/6)(Area of Circle)= X(multiple of any one equation in hexagon) +[(4X)(2b+ 2c)]

For Example :-

For X = 12, i.e (X / 6) = (12 / 6) = 2 . we choose one equation in hexagon , 6a - 34b.

2 (Area of Circle) = 2(4a + 68b) = 8a + 136b -------- By using equation (13)

(X/6)(Area of Circle) = X(multiple of any one equation in hexagon) +[(4X)(2b+ 2c)]

2 (Area of Circle) = 12(6a – 34b) +[(48)(2b + 2c)]

= 72a – 408b + 96b + 96c

= 72a - 312b + 96c i.e 96c = 32 (3c)

= 72a – 312b + 448b – 64a. = 32(14b – 2a)

2 (Area of Circle) = 8a + 136b = 448b – 64a

<u>Three circles having Same radius touches each other</u>

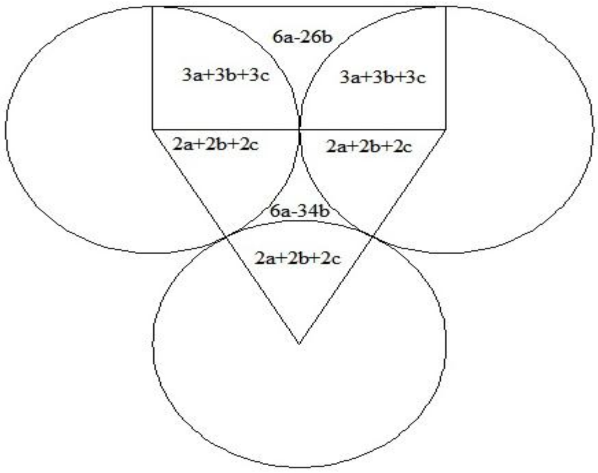


Figure Number – 23

1) **Area of Circle = 2(3a +3b +3c) + 3(2a +2b +2c)**
   Area of Circle = 12a + 12b + 12c
2) **Area of circle = Area of Triangle + Area of rectangle – (6a - 34b) – (6a -26b)**
   = 8a + (8a + 8b) – (6a - 34b) – (6a -26b)
   Area of circle = 4a + 68b

Four circles having Same radius touches each other

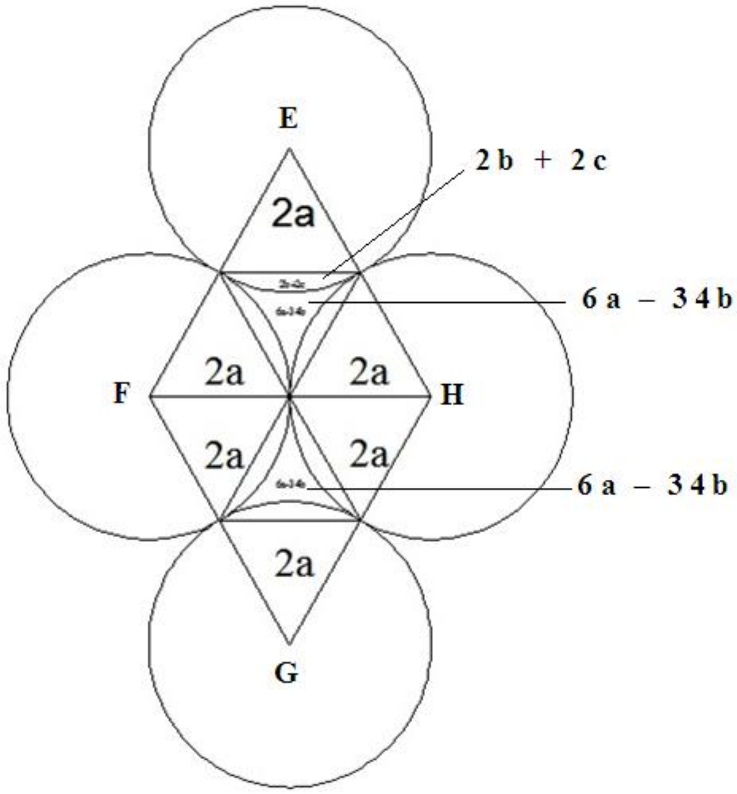


Figure Number – 24

From Above Figure we get following some results ,

1) **Area of circle = 6 (2a + 2b + 2c) = 12a + 12b + 12c**
2) **Area of circle = Area of (□ EFGH) - 2(6a - 34b)**

= 8(2a) – 2(6a – 34b)

= 16a - 12a + 68b

Area of circle = 4a + 68b

Note :- 6a – 34b = 2a – 3 (2b + 2c)

Basic Figure 2

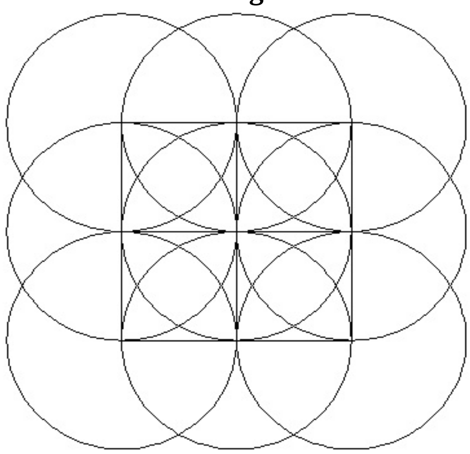


Figure Number – 25

Basic Figure 3

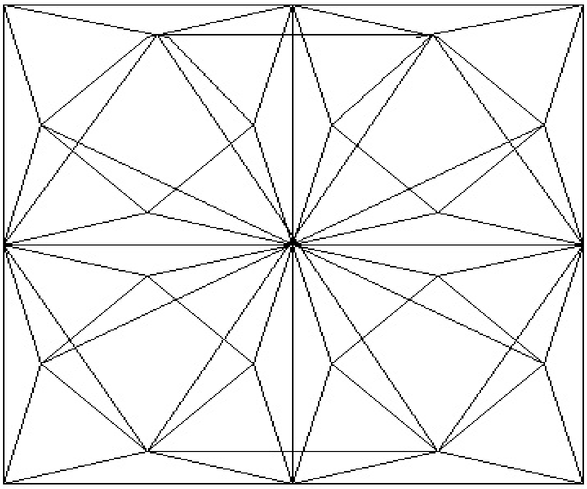


Figure Number – 26

<u>Basic Figure 4</u>

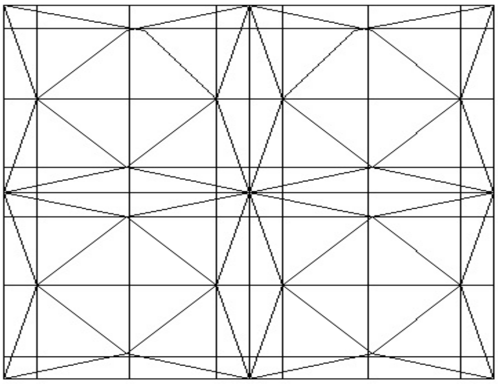


Figure Number – 27

Conclusions :-

We can find area of circle in terms of a,b,c,d formula for any circle having radius r .

<u>Formulae</u> :-

1) **Area of part a** = $[0.125(\sqrt{3})]\ r^2$
2) **Area of part b** = $[\ 0.25 - 0.125(\sqrt{3})\ ]\ r^2$
3) **Area of part 3c** = $[\ 3.5\ -\ 2\sqrt{3}\ ]\ r^2$
4) **Area of part d** = $[\ 2\sqrt{3}\ - 3.25\ ]\ r^2$

I conclude that pi (π) is an algebra & Exact value of pi (π) is ,

$\pi = 17 - 8\sqrt{3}$ or $\pi = 17 - \sqrt{192}$ or 3.14359353944.. .

References :-

[1] Basic algebra & geometry concepts .
[2] Histry of Pi(π) from internet .